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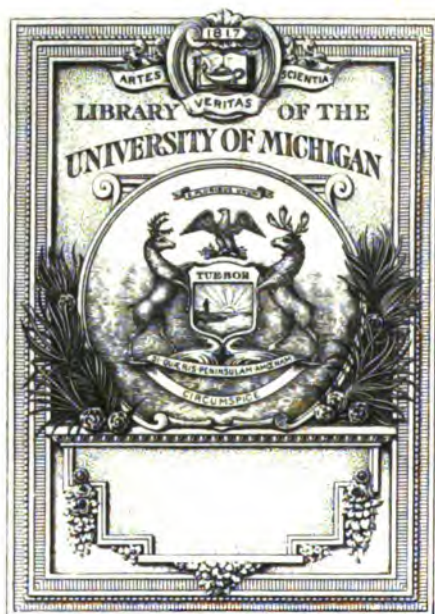
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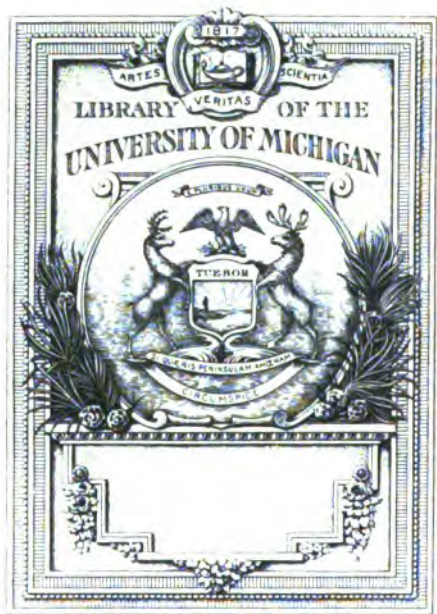
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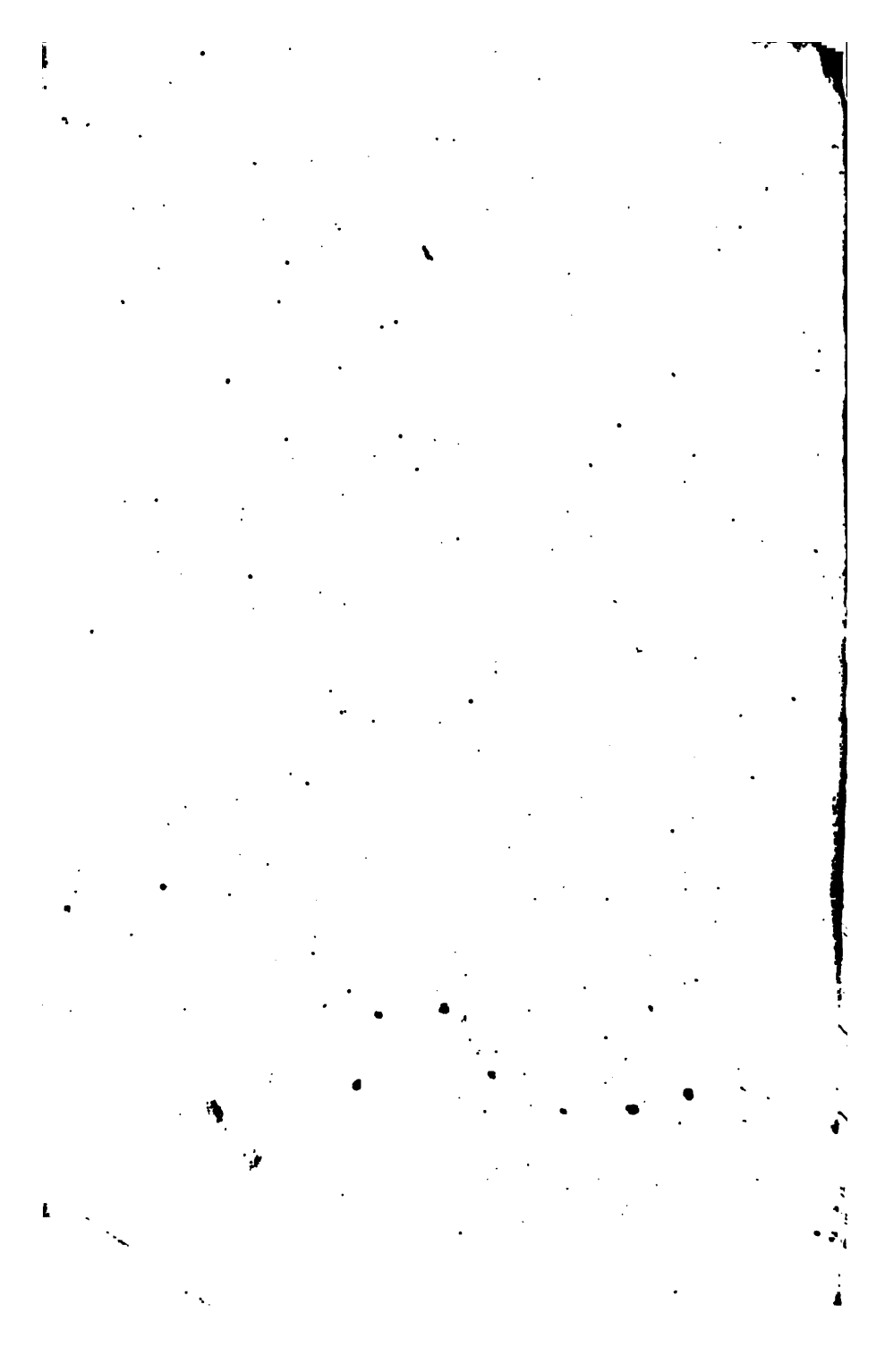
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THE
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PLAIN and SPHERICAL
TRIGONOMETRY:

WITH ITS
Application and Use in the following
Parts of MATHEMATICKS;

VIZ.

- I. NAVIGATION in all its Kinds; as *Plain Sailing*,
Mercator's Sailing, *Middle Latitude*, and *Parallel*
Sailing.
- II. ASTRONOMY; wherein all the Problems relating to
the Doctrine of the Sphere are solved.
- III. PROJECTION of the SPHERE in *Plano*.
- IV. GEOGRAPHY.
- V. FORTIFICATION.
- VI. MENSURATION of Heights and Distances, both
accessible and inaccessible.
- VII. DIALLING, Arithmetical and Instrumental, on
all sorts of Planes.

By WILLIAM HAWNEY, Author of *The*
Compleat Measurer.

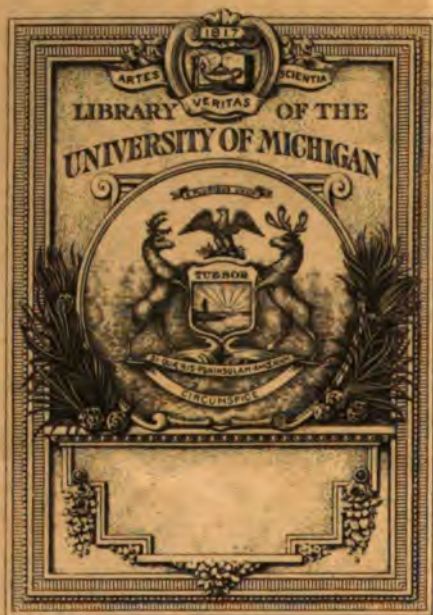
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P. 181. l. 13. r. $45^{\circ}.$ P. 237. l. 24. r. *Angles.* P. 279. l. 5.
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and 319. the running Title must be, *Sailing by Middle Latitude.*
P. 342. l. 10. r. *Aldebaran.* P. 367. l. 3. r. *find.* P. 378. l. 2.
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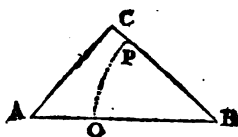
PART I.

CHAP. I.

Definitions and Affections of Triangles.

1. **O**F Triangles there are two kinds, *Plain* and *Spherical*; either of which consists of six Parts; namely, Three Sides, and as many Angles, as in the Figure ABC,

the three Sides are AB, AC and CB; and the three Angles are ABC, ACB, and BAC. Where Note, That if an Angle be express'd by three Letters, the middlemost Letter respecteth the Angular Point. If I would express the Angle C, I would say ACB, or BCA; and the Angle B is expressed by ABC, or CBA.



2. Any two sides of a Triangle are called the containing sides of the Angle contained, or comprehended between them, as the sides AB and AC are the containing sides of the Angle BAC.

3. Every side of a Triangle is the subtending side of that Angle which is opposite unto it. As in the Triangle ABC, the side AB subtends the Angle ACB, and AC subtends the Angle B. Where Note, The greatest Side always subtends the greatest Angle,

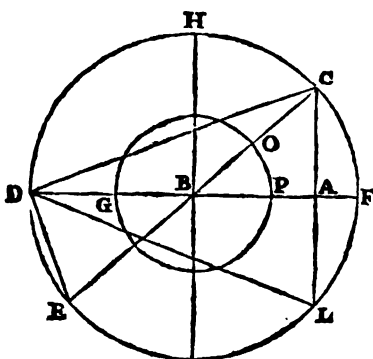
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the least Side the least Angle, and equal Sides subtend equal Angles.

4. The Measure of an Angle is the Arch of a Circle, described upon the angular Point, and intercepted between the two Sides containing the Angle. As in the Triangle ABC, the Measure of the Angle ABC is the Arch OP.

5. Every Circle is divided into 360 degrees, and every degree into 60 minutes, and every minute into 60 seconds, &c. which degrees are so much the greater, by how much the Circle is greater; and the Arches which contain the same number of Degrees in equal Circles are equal, but in unequal Circles they are term'd like Arches. As the Arches CF and DE are equal, being equal parts of the same Circle. But the Arches CF and OP are like Arches; that is, as CF contains 40 parts in the great Circle, so is OP 40 parts in the lesser Circle POG. See the following Figure.



6. A Quadrant (or Quarter) of a Circle is an Arch of 90 Degrees, as the Quadrant HF.

7. The Complement of an Arch less than a Quadrant, is so much as the Arch wanteth of a Quadrant; as the Complement of the Arch CF 40 Degrees, is the Arch CH 50 Degrees.

8. The Semi-circle is an Arch of 180 Degrees.

9. The Complement of an Arch to a Semi-circle, is so much as that Arch wants of 180 degrees. As the Complement of the Arch DHC 140 degrees, is CF 40 degrees.

10. The

Chap. 1. Plain Trigonometry.

3

10. The opposite Angles made by the crossing of two Lines are equal, as the Angle DBE is equal to the Angle ABC.

11. An Angle is either Right or Oblique.

12. A right Angle is that whose Measure is 90 degrees, as FBH.

13. An oblique Angle is either Acute, or Obtuse.

14. An acute Angle is that whose Measure is less than 90 degrees; as the Angle ABC, or the Angle ACB.

15. An obtuse Angle is that whose Measure is more than 90 degrees, as the Angle DBC.

16. All Angles coming (or meeting) together in one point upon a right Line; all of them taken together are equal to two right Angles, as the Angles DBH, HBC, and CBA meeting all together in the point B, upon the Line DF, are equal to two right Angles DBH, HBF.

17. A Triangle hath some of its Sides equal, or else they are all unequal.

18. A Triangle of some unequal Sides, is either Equicrural, or Equilateral.

19. An Equicrural (called also an Isosceles) Triangle, is equiangled at the Base. And if a Perpendicular be let fall from the meeting of the equal Sides, it cuts the Base and the Angle opposite thereunto into two equal parts; as in the Equicrural Triangle DLC, the Perpendicular DA cuts the Base CL into two equal parts in A, and also bissecteth the Angle CDL.

20. An Equilateral Triangle is that whose Sides are all equal, and whose Angles contain each 60 degrees.

21. A Triangle is Right-angled, or Oblique-angled.

22. A Right-angled Triangle is that which hath one right Angle, as the Triangle CBA Right-angled at A. (See the former Figure.)

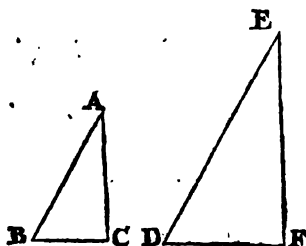
23. An Oblique-angled Triangle is that which hath all its Angles Oblique, as the Triangle BDC.

24. An oblique-angled Triangle is either acute angled, or obtuse angled.

25. An acute angled Triangle is that which hath all its Angles acute, as the Triangle BDE.

26. An obtuse angled Triangle is that which hath one Angle obtuse, as the Triangle CDB, obtuse angled at B.

27. All equiangled (or like) Triangles have their Sides about the equal Angles proportional, (*Euch. Lib. 6. Pr. 4.*)

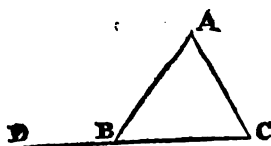


Let ABC and DEF be two Triangles equiangled, so as the Angles at B and D, at A and E, and also at C and F, be equal one to another, each to its correspondent; then I say it will be,

As AB : BC :: DE : DF.

As AB : AC :: DE : EF.

As AC : CB :: EF : DF.



28. If any one Side of a plain Triangle be continued, the outward Angle (made by that Continuation) is equal to the two inward and opposite Angles in the same Tri-

angle, (*Euch. Lib. 1. Pr. 32.*) As in the Triangle ABC, the Side CB, being continued to D. I say, the Angle ABD is equal to both the Angles, at A and C.

29. The three Angles of every plain Triangle, are together equal to two right Angles, (*Euch. Lib. 1. Pr. 32.*)

And from hence will follow these Corollaries.

1. That there can be but one right, or one obtuse Angle in any plain Triangle.

2. And

Chap. 2. Plain Trigonometry.

5

2. And if one Angle be either right or obtuse, the other two shall be acute.

3. That the third Angle of any plain Triangle, is the Complement of the other two, to two right Angles.

4. That if two Triangles be equiangled in any two of their Angles, they are wholly equiangled.

C H A P. II.

Concerning Sines, Tangents, and Secants, by which the Angles of Right-lined, and Sides and Angles of Spherical Triangles, are measured.

1. **T**H E Mensuration of Triangles is by knowing any three of the six Parts (*Vide 1. of Chap. 1.*) whether Sides or Angles, or both, to find out any of the other parts of the Triangle whether Sides or Angles.

2. And this is performed by the Rule of Proportion (commonly, and deservedly called the Golden Rule of Arithmetick) which teacheth of four Numbers proportioned one to another, any three of them being given, to find out the fourth.

3. So that for the Measuring (which is also called the Resolving) of Triangles, there must be certain Proportions of all the parts of a Triangle one to another, and those Proportions explain'd in Numbers. But the certain knowledge of the Proportions that all the parts of a Triangle have one to another, is not yet known; for that every crooked Line in a Triangle (as the Measures of the Angles in the Right-lined, and the Measures of Sides and Angles in a Spherical Triangle are) must be reduced to a Right Line, a Proportion for which was never yet

drant.) For the right Sines of Arches more or less than a Quadrant are the same (by the 9th above.)

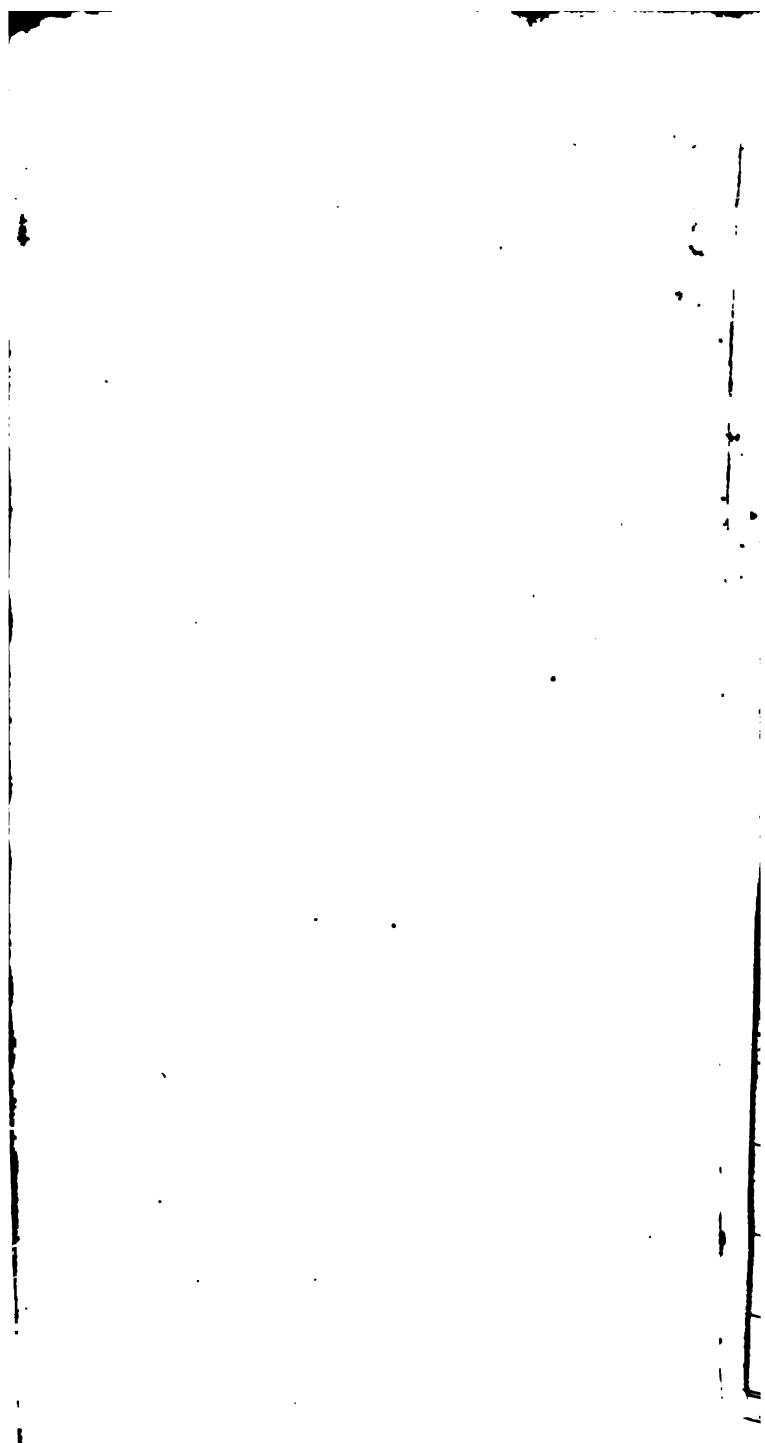
Note, The Tables of Sines, Tangents, and Secants made from the Circle, agreeable to the Lines above described, are to be understood of natural Sines, Tangents, and Secants; whose use in the Solution of Triangles was performed by Multiplication and Division: but the Tables now in use (sometimes called the Canon of Triangles) are those which were invented by my Lord *Neper*, Baron of *Marchiston* in *Scotland*, about the year 1621. which Canon consists of Artificial or Logarithmical Sines, and Tangents; and performeth the same by Addition and Subtraction, which the natural Canon did by Multiplication and Division.

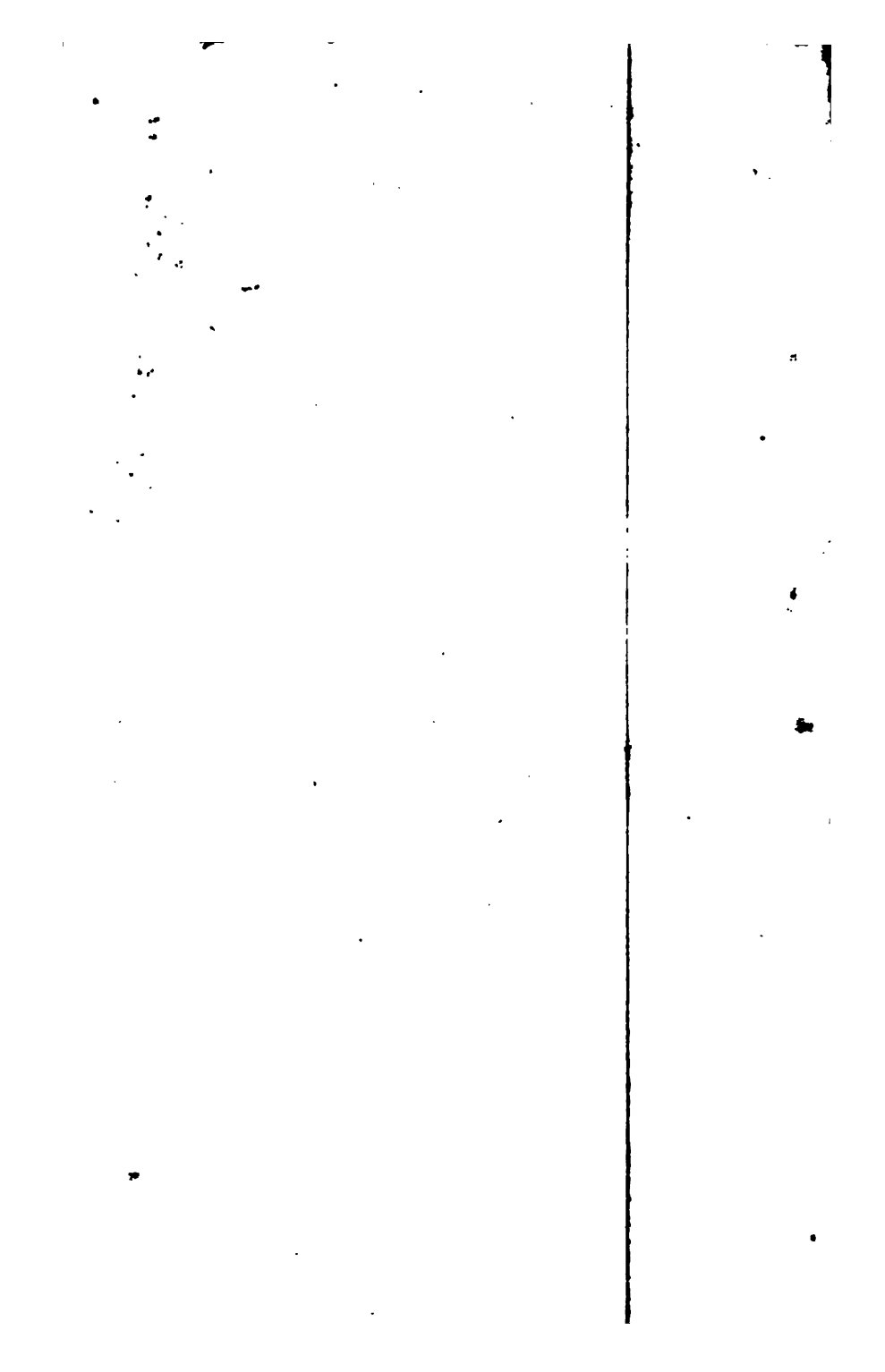
Having given a short Description, or Definition of Sines, Tangents, &c. I shall here shew the Fabrick, or Geometrical Construction of those and other Scales, commonly used in projecting the Sphere in Plano, and in Trigonometry, Navigation, Dialling, and other parts of practical Mathematicks, as they are deduced from a Circle.

1. Upon a Sheet of fine Pastboard, or such like Matter, describe a Semicircle of any Radius, as OR; upon the Center O, and Diameter SV, describe the Semicircle SRV, and divide the two Quadrants ORV, and OSR, each into 90 Degrees; and number them from S and V to R, by 10, 20, 30, &c. to 90, as in the Figure.

2. If you lay a Ruler from 10 in one Quadrant to 10 in the other, it will cut the Line OR in 10; and being laid from 20 to 20, it will cut OR in 20; and so from 30 and 30, and 40 and 40, &c. till you come to 90 at R: so shall OR be a Line of right Sines.

3. Upon the point S, erect ST perpendicular to SV; then laying a Ruler to the Center O, and to every 10 degrees in the Quadrant, draw Lines till they cut the
the





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the Perpendicular, so shall ST be a Line of Tangents.

4. Those Lines drawn from the Center to the Tangent Line are called Secants, which you may transfer into the Line RE, by setting one foot of the Compasses in the Center O; then strike an Arch from the Tangents to the Line RE, so shall RE be a Line of Secants.

5. The Line SV (which is equal to twice OR) is a Line of versed Sines, and is no other but the Line of right Sines doubled, as you may plainly see; but they are numbered from S to V by 10, 20, 30, 40, &c. to 180 Degrees.

6. The Line SR is a Line of Chords, the Degrees being transferred from the Quadrant by setting one foot of the Compasses in S, and drawing Arches from the Arch of the Quadrant to the streight Line SR.

7. The Rumb Line is nothing else but a Line of Chords divided into eight Parts, by allowing $11^{\circ} 15'$ to each part.

8. There is mention made of half Tangents, (they being useful in projecting the Sphere in Plano) which is nothing but a Line of Tangents double numbred. Thus 5 Degrees being numbred with 10, and 10 with 20, and 15 with 30, &c. so 45 Degrees of the Tangents is 90 Degrees of Semi-tangents, equal to 90 Degrees of the Sines, and to 60 Degrees of the Chords.

Having thus laid the Foundation, I shall next proceed to shew the Resolution of all plain or right-lin'd Triangles in as plain and familiar a Method as possibly I can.

C H A P. III.

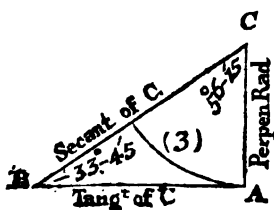
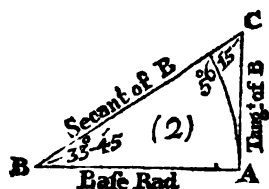
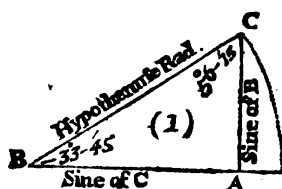
Of the Dimensions of right angled Plain Triangles.

IN the Solution of right angled plain Triangles, there are seven Cases, the resolving of which depends upon this following

A X I O M.

In every plain right angled Triangle, any of the three Sides may be put for Radius; and then will the other Sides be as Sines, Tangents, or Secants.

Thus in the three Triangles ABC.



1. If you put BC subtending the Right Angle for Radius, the Sides CA, and BA including the Right Angle, are Sines of the opposite Angles B and C. (by the 4th Def. of Chap. 1. and the 9th and 10th of Chap. 2.)

2. If you put the greater of the Sides including the Right Angle BA for Radius; then the lesser Side AC will be the Tangent of the Angle B, (by the 15 Def. of Chap. 2.) and the Hypotenuse BC is the Secant of the said Angle B, (by the 16 Def. of Chap. 2.)

3. If you put the lesser of the two Sides including the Right Angle CA for Radius; the greater Side including the Right Angle BA is the Tangent

Chap. 3. Plain Trigonometry. 11

gent of the greater acute Angle (by 15 Def. Chap. 2.) and the Hypothenufe BC is the Secant of the same (by Def. 16. Chap. 2.)

Now from this Axiom followeth this Confectary:

That in all right angled plain Triangles, the Angles being given, the Reasons of the Sides are also given three ways. And consequently,

If one Side be given, besides the three Angles, every of the other Sides are given by a threefold Proportion, according as you put this, that, or the other Side for Radius.

These things being premised, I will now proceed to the Solution of the several Cases.

C A S E I.

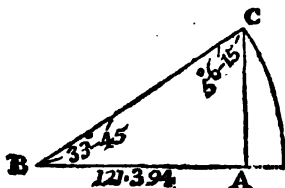
The two acute Angles B and C, and the Base B A, being given, to find the Perpendicular C A.

1. By making the Hypothenufe BC Radius.

As s. of the $\angle$ at the Perpend.	$56^{\circ} 15'$	9.9198464
Is to the Log. of the Base AB	121.394	2.0841992
So is the s. of the $\angle$ at the Base	$33^{\circ} 45'$	9.7447390

11.8289382

To the Log. of AC the Perpend.	81.113	1.9090918
--------------------------------	--------	-----------



And

And thus, by adding the second and third Terms in the Proportion together, the Sum is 11.8289382; from which subtract the first Term 9.9198464, and the Remainder is 1.9090918, the Logarithm of 81.113, which is the Perpendicular AC required.

But where Radius is not in the Proportion (as in this Example it is not) it may more readily be done by Addition only; for if instead of the first Term you set its Arithmetical Complement, that is only to write down what each Figure wants of 9, thus the Arithmetical Complement of 9.9198464 (the first Term) is 0.0801536, (which is the same thing as subtracting it from 10) then add all the three Terms together, the sum (abating Radius) shall answer the Question.

Let the same Example be again repeated.

As the s. of the $\angle C$ $56^{\circ} 15'$ Ar. Comp. 0.0801536

Is to the Base BA 121.394 Log. 2.0841992

So is the s. of the $\angle B$ $33^{\circ} 45'$ 9.7447396

To the Perpend. AC 81.113

1.9090918

Here I add all the three Terms together, and when I come to the Characteristicks, the sum is 11; and (because Radius is 10) I cast away 10, and set down only the 1.

Note, That s. stands for Sine; c. s. for Co-sine; t. for Tangent; c. t. for Co-tangent; sc. for Secant; c. sc. for Co-secant: And $^{\circ}$ $'$ $''$ set over Figures, stand for degrees, minutes, seconds; and $\angle$ stands for Angle.

Chap. 3. Plain Trigonometry.

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2. By making the Base AB Radius.

As Radius 45°

Is to the Base BA 121.394

So is the t. of $\angle B 33^\circ 45'$

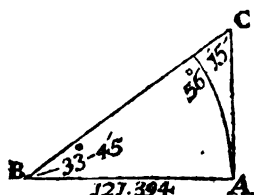
10.

2.0841992

9.8248926

To the Perpend. AC 81.113

1.9090918



Here I add the two last Terms together, and out of the Sum I mentally subtract 10 for Radius, because Radius is in the first place.

3. By making the Perpendicular AC Radius.

As t. of the $\angle$ at the Perp. C $56^\circ 15'$

Is to Radius 45°

So is the Base AB 121.394

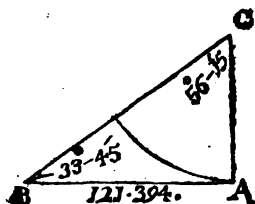
10.1751074

10.

2.0841992

To the Perpendicular AC 81.113

1.9090918



Here

Here I first mentally add Radius or 10, to the last Term, and from the Sum subtract the First, the Remainder is the Log. of the Answer, the same as before.

To perform the same Instrumentally.

1. By *Scale and Compasses*. Always extend the Compasses from the first Term to the Term that is of the same kind, whether it be the second, or third; that extent will reach from the remaining Term to the Answer. Thus, in the first Proportion, extend from $56^{\circ} 15'$ to $33^{\circ} 45'$ in the Line of Sines; that extent will reach in the line of Numbers from 121.39 to 81.11 the Answer. In the second Proportion, extend from 45° to $33^{\circ} 45'$ in the Line of Tangents; that extent will reach from 121.39 to 81.11 in the Line of Numbers. In some Cases it may be needful to use Cross-work, that is, to extend from the first Term in the Line of Sines to the second in the Line of Numbers, or from the first Term in Tangents to the second in Numbers, &c. but I think in most Cases it is better to work by the Directions above, except it be when the extent is too large for the Compasses.

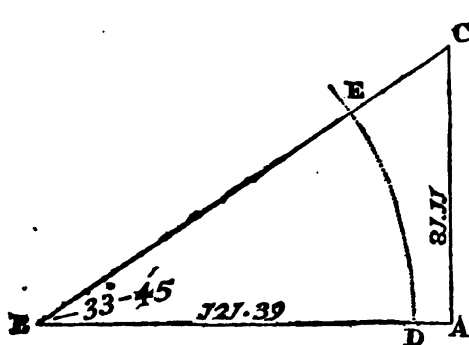
2. By the *Sliding Rule*. We will suppose the Line of Sines upon the Rule to be marked with SS, and the Line of Sines upon the Slider marked with S. Then the first Proportion will be thus work'd. Set $33^{\circ} 45'$ on S, to $56^{\circ} 15'$ on SS; then against 121.39 on A, is 81.11 on B. (A signifies the double Line of Numbers upon the Rule, and B the double Numbers upon the Slider.) The second Proportion you may work thus, Set $33^{\circ} 45'$ in the Tangents, to Radius; then against 121.39 on A, is 81.11 on B. Or if you turn the Slider so as Tangents and double Numbers may slide one by another, then you may set Radius (*viz.* 45° of Tangents) to 121.39 in the Line of Numbers; then against $33^{\circ} 45'$ in Tangents is 81.11 in the
Line

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Line of Numbers. The third Proportion, you may work just as you work this last.

To do the same Geometrically.



Line of Chords with your Compasses, and set one Foot in B, strike the Arch DE, and from the same Line of Chords take $33^{\circ} 45'$, and set from D to E; and draw the Line BC till it cut the Perpendicular in C: then if you measure AC by the same Scale you took BA from, you will find it 81.11; and if you measure BC, you will find it 146. So will the next Case be also resolved.

CASE

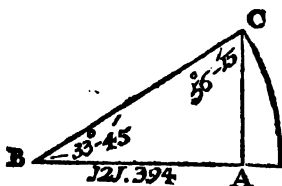
C A S E II.

The two acute Angles B and C being given, with the Base BA, to find the Hypothenufe BC.

1. By making the Hypothenufe Radius.

As s. of the $\angle C$ $56^{\circ} 15'$	9.9198464
Is to the Base BA 121.394	2.0841992
So is Radius 90°	10.

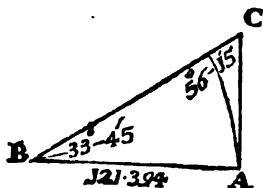
To the Hypothenufe BC 146.	2.1643528
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2. By making the Base Radius.

As Radius	10.
To the Base BA 121.394	2.0841992
So is the sc. of the $\angle B$ $33^{\circ} 45'$	10.0801536

To the Hypothenufe BC 146.	2.1643528
----------------------------	-----------



3. By

Chap. 3. Plain Trigonometry.

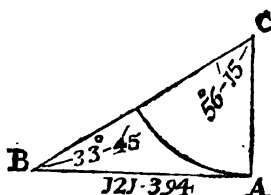
17

3. By making the Perpendicular Radius.

As the t. of the $\angle C$ $56^{\circ} 15'$ Ar. Com. 9.8248926
 To the Base BA 121.394 2.0841992
 So is the se. of the $\angle C$ 10.2952610

To the Hypoth. BC 146

2.1643528



Note, The Secant of an Arch or Angle is the Arithmetical Complement of the Co-sine of that Arch, the Radius 10. being added: thus the Secant of $56^{\circ} 15'$ is the Arithmetical Complement of the Sine of $33^{\circ} 45'$, and the Secant of $33^{\circ} 45'$ is the Arithmetical Complement of the Sine of $56^{\circ} 15'$, the Radius being added.

By Scale and Compasses.

To work the first Proportion, extend the Compasses from the Sine of $56^{\circ} 15'$ to Radius or 90° , that extent will reach from 121.39 to 146 in the Line of Numbers.

By the Sliding-Rule.

Set $56^{\circ} 15'$ upon S, to 90° upon SS; then against 121.39 upon B, is 146 upon A. But if you turn the Slider so as Sines and double Numbers may slide one by another, you may set $56^{\circ} 15'$ of Sines to 121.39 in

18. Plain Trigonometry. Part I:

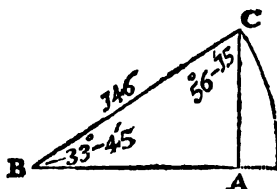
in the Line of Numbers: then against 90° of Sines you'll have 146 in the Line of Numbers; and you will also find $33^\circ 45'$ of Sines to be against 81.11 of Numbers. So that you may observe, that if 90° of Sines be set to the greatest Side or Hypotenuse, you will find every Angle against its opposite side.

C A S E III.

The two acute Angles B and C, with the Hypotenuse BC, being given, to find the Base BA.

1. By making the Hypotenuse Radius.

As Radius 90°	10.
Is to the Hypotenuse BC 146	2.1643528
So is s. of the $\angle C 56^\circ 15'$	9.9198464
To the Base 121.394	2.0841992



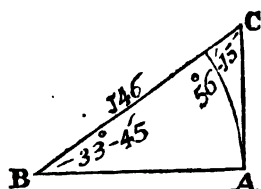
2. By

2. By making the Base Radius.

As fe. of the $\angle B$ $33^\circ 45'$	10.0801536
Is to the Hypothenufe BC 146	2.1643528
So is Radius 90°	10.

To the Base BA 121.394

2.0841992

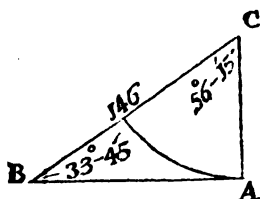


3. By making the Perpendicular Radius.

As fe. of the $\angle C$ $56^\circ 15'$	Ar. Com.	9.7447390
Is to the Hypothenufe BC 146		2.1643528
So is t. of the $\angle C$ $56^\circ 15'$		10.1751074

To the Base BA 121.394

2.0841992



By Scale and Compasses.

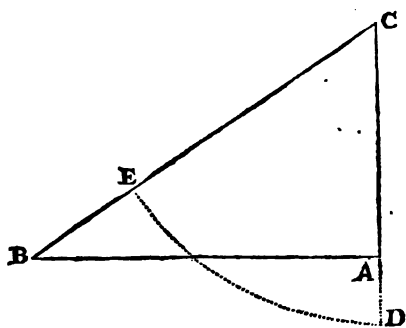
To work the first Proportion. Extend the Compasses from 90° to $56^\circ 15'$ in the Sines, that extent will

will reach from 146 to 121.394 in the Line of Numbers.

By the Sliding-Rule.

Set $56^{\circ} 15'$ upon S, to 90° upon SS; then against 146 upon A, is 121.394 upon B.

B Geometrical Protraction.



Draw the Line BC, and from a Scale of equal Parts take 146, and set from B to C; and upon C, with 60 degrees of Chords, strike the Arch DE; and set $56^{\circ} 15'$ from E to D, and draw CA, and

from B draw BA perpendicular to AC: then if you measure BA upon your Scale, you will find it to contain 121.39.

CASE

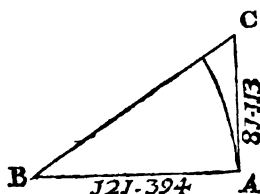
C A S E IV.

*The Base BA, and Perpendicular CA being given,
to find the two acute Angles B and C.*

1. By making the Base Radius;

As the Base BA	121.394	2.0841992
Is to Radius	45°	10.
So is the Perpend. AC	81.113	1.9090918
To the t. of the $\angle B$	$33^\circ 45'$	9.8248926

Whose Complement is $56^\circ 15'$ the $\angle C$.



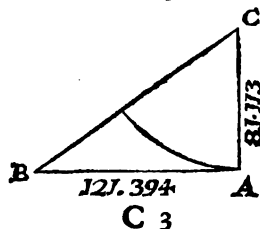
2. By making the Perpendicular Radius.

As the Perpend. CA	81.113	1.9090918
Is to Radius	45°	10.
So is the Base BA	121.394	2.0841992

To t. of the $\angle C$ $56^\circ 15'$

10.1751074

Whose Complement is $33^\circ 45'$ the $\angle B$.



By

By Scale and Compasses.

To work the first Proportion, extend the Compasses from 121.394 (the Base) to 81.113 (the Perpend.) in the Line of Numbers, that extent will reach from 45° to $33^\circ 45'$ in the Tangents. For the second Proportion, extend from 81.113 to 121.394 in the Line of Numbers, that extent will reach from 45° to $56^\circ 15'$ in the Tangents.

By the Sliding-Rule.

Let the Tangents and Numbers slide together, then set 121.394 in the Line of Numbers to 45° of Tangents; and against 81.113 in the Line of Numbers is $33^\circ 45'$, and also its Complement $56^\circ 15'$ in the Line of Tangents.

This Case is perform'd Geometrically, as the sixth Case following.

C A S E V.

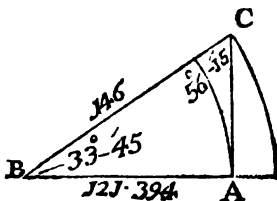
The Base BA, and the Hypothenufe BC being given, to find the two acute Angles B and C.

1. By making the Hypothenufe Radius.

As the Hypothenufe BC 146.	2.1643528
Is to Radius 90°	10.
So is the Base BA 121.394	2.0841991

To the s. of the $\angle C$ $56^\circ 15'$	9.9198463
--	-----------

Whose Complement $33^\circ 45'$ is the $\angle B$.



2. By

2. By making the Base Radius.

As the Base BA 121.394	2.0841992
Is to Radius 90°	10.
So is the Hypothenufe 146.	2.1643528
To the se. of the $\angle B$ 33° 45'	10.0801536

By Scale and Compasses.

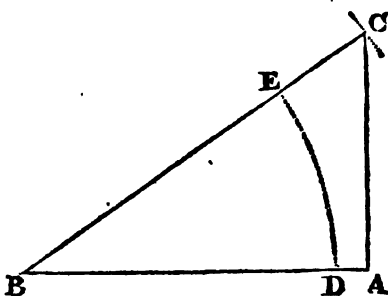
For the first Proportion, extend the Compasses from 146. to 121.394 in the Line of Numbers, that extent will reach from 90° to 56° 15' in the Sines.

By the Sliding Rule.

Set 146 upon A to 121.394 upon B, then against Radius on SS, is 56° 15' on S.

By Geometrical Protraction.

Draw the Base BA, and from a Scale of equal Parts, take 121.39, and set from B to A, and upon A raise the Perpendicular AC: then from the same Scale of equal Parts take 146, and set one foot of B the Compasses in B, with the other cross the Line AC in C, and draw the Line BC. Then with 60° of Chords strike the Arch DE, then take DE in your Compasses, and measure it in the Line of Chords; and you will find it to contain 33° 45', which is the measure of the Angle B, whose Complement is the Angle C.



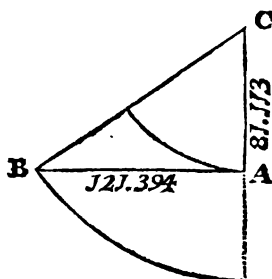
C A S E VI.

*The Base BA, and Perpendicular CA being given,
to find the Hypothenuse BC.*

THIS and the following Case do require two Operations, the first to find the acute Angle, and the second (from thence) to find the third Side.

1. Operation, the Perpend. AC Radius.

As the Perpend. CA	81.113	1.9090918	} Case 4. Pr. 2
Is to the Base BA	121.394	2.0841992	
So is Radius 45°		10.	
To the Tangent of the Ang.		10.1751074	}
C $56^\circ 15'$			



2. Operation, the Hypothenuse BC Radius.

As the s. of the $\angle C$	$56^\circ 15'$	9.9198464	} Case 2. Pr. 1.
Is to the Base BA	121.394	2.0841992	
So is Radius 90°		10.	
To the Hypothenuse	146	2.1643528	

By

By Scale and Compasses.

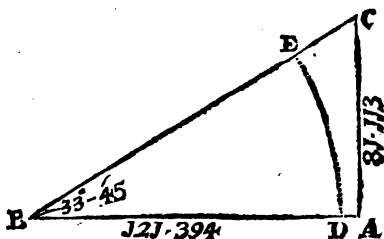
Extend the Compasses from 81.113 to 121.39 in the Line of Numbers, that extent will reach from 45° to $56^{\circ} 15'$ in the Line of Tangents; then extend from $56^{\circ} 15'$ to 90° in the Sines, that extent will reach from 121.394 to 146, the Hypothenufe required.

By the Sliding-Rule.

Set 81.113 in the Line of Numbers to 45° in the Tangents; then against 121.39 in Numbers is $56^{\circ} 15'$ in the Tangents; then set $56^{\circ} 15'$ on S, to 90 on SS; then against 121.39 on B is 146 on A.

By Geometrical Protraction.

Draw the Line BA, and from a Scale of equal Parts take 121.394 with your Compasses, and set from B to A, and upon A raise the Perpendicular AC; and from the same Scale take 81.113.



113, and set from A to C: then draw the Hypothenufe BC, and take BC in your Compasses, and measure it upon your Scale, and you will find it 146. If you take 60° of Chords in your Compasses, and set one foot in B, and with the other strike the Arch DE, then if DE be measured in the Scale of Chords, you will find it $33^{\circ} 45'$, the Angle B, whose Complement $56^{\circ} 15'$ is the Angle C.

CASE

C A S E VII.

*The Base BA, and the Hypotenuse BC being given,
to find the Perpendicular.*

1. Operation, Hypotenuse Radius.

As the Hypotenuse BC 146.

Is to Radius 90°

So is the Base BA 121.394

2.1643528

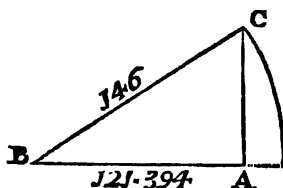
10.

2.0841992

To the Sine of $\angle C 56^\circ 15'$

Whose Complement is $\angle B 33^\circ 45'$

9.9198464



2. Operation, Hypotenuse Radius.

As Radius 90°

Is to the Hypotenuse BC 146

So is the s. of $\angle B 33^\circ 45'$

10.

2.1643528

9.7447390

To the Perpendicular AC 81.113

1.9090918

This Case may be otherwise resolved, thus.

Find the sum and difference of the Base, and Hypotenuse; then multiply the Sum and Difference together, and out of the Product extract the square Root, and that shall be the Perpendicular. By Logarithms
you

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you may add the Logarithms of the said Sum and Difference together, half the Sum thereof shall be the Log. of the Perpendicular, thus.

Hypothenuſe	146		
The Baſe	121.394		
The Sum	267.394	Log.	2.4271516
The Difference	24.606	Log.	1.3910308
Their Sum			3.8181824
half Sum			1.9090912

This is the Log. of 81.113 the Perpendicular, which is the ſame as before.

By Scale and Compaſſes.

For the firſt Operation, extend the Compaſſes from 146. to 121.394 in the Line of Numbers, that extent will reach from 90° to $56^{\circ} 15'$ in the Sines. Then for the ſecond Operation, extend from 90° to $33^{\circ} 45'$ in the Sines, that extent will reach from 146 to 81.113 in the Line of Numbers.

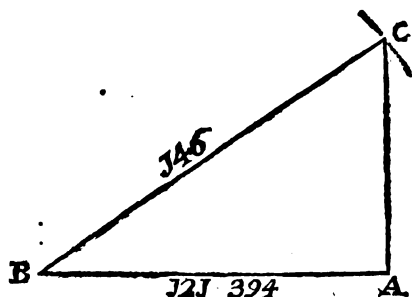
By the Sliding-Rule.

For the firſt, ſet 146 on A to 121.394 on B, then againſt 90° on SS, you will find $56^{\circ} 15'$ on S. Then for the ſecond, ſet 90° on SS to $33^{\circ} 45'$ on S, and againſt 146 on A, is 81.113 on B.

By Geometrical Protraction.

Draw the Line BA, and from your Scale take 121.394, and ſet from B to A, and upon A raiſe a Perpendicular :

pendicular: then from your Scale take 146, and set one foot of the Compasses in B, cross the Perpendicular in C, and measure AC upon your Scale, and you will find it to be 81.113.



C H A P. IV.

Of the Dimension of Oblique angled Plain Triangles.

IN the Solution of Oblique angled plain Triangles, there are but Five Cases, the resolving of which depends upon the three following Axioms. The first of which serves for the resolving of the first and second Cases. The second for resolving the third and fourth Cases. And the third is appropriated only to the fifth and last Case.

I shall first explain and demonstrate the said Axioms, and then proceed to the Resolution of the Cases thereby.

A X I O M I.

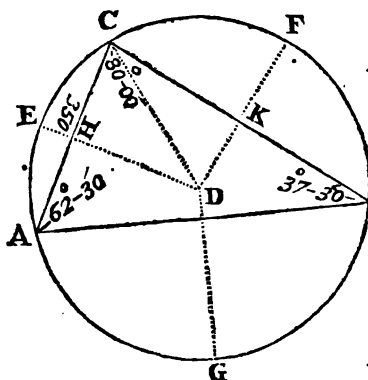
In every plain Triangle (as well right as oblique angled) the Sides are in proportion one to the other, as are the Sines of the Angles opposite to those Sides, & *contra*.

Con

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29

Construction. Let the Triangle ABC be inscribed in a Circle, and from the Centre D let there be drawn the several Radii DE, FD, DG, perpendicular to the respective Sides of the Triangle; so will they bisect as well the several Arches AEC, CFB, and BGA, as their Subtenses AC, CB, and BA: and let there be also drawn the Radius DC.



Demonstration. Now because the Angle at the Center EDC, is equal to the Angle in the Periphery ABC; and CDF at the Center, equal to CAB (by *Euc. 3. Lib. 20 Pr.*) therefore shall the halves of the Sides be as Sines: and what proportion the Side CA hath to the Side CB, the same shall the Sine CH have to the Sine CK: for what proportion the whole hath to the whole, the same proportion hath the half to the half, which was, &c.

And from this Axiom follow these Consequences:

1. If the Angles of a Triangle be given, the reason of the Sides is also given. And consequently,
If one Side be given besides the Angles, both the other Sides are also given.
2. If two Sides of the Triangle be given, with an Angle opposite to one of them, the Angle opposite to the other of them is also given.

CASE

C A S E I.

The Angles and one Side being given, to find the other two Sides.

In the Triangle ABC (in the Figure above) there are given the Angles CAB $62^{\circ} 30'$, and CBA $37^{\circ} 30'$, and the Side AC 350; and it is required to find the Side BC.

As the s. of the $\angle$ CBA $37^{\circ} 30'$	Ar. Com.	0.2155529
Is to the Side AC 350		2.5440680
So is the s. of the $\angle$ CAB $62^{\circ} 30'$		9.9479289
To the Side BC		509.976
		2.7075498

For the Side AB.

As the s. of the $\angle$ CBA $37^{\circ} 30'$	Ar. Com.	0.2155529
Is to the Side AC 350		2.5440680
So is the s. of the $\angle$ ACB $80^{\circ} 00'$		9.9933515
To the Side AB		566.203
		2.7529724

By Scale and Compasses.

For the first Proportion, extend the Compasses from $37^{\circ} 30'$ to $62^{\circ} 30'$ in the Sines; that extent will reach from 350 to 509.976 in the Line of Numbers. For the second Proportion, extend from $37^{\circ} 30'$ to $80^{\circ} 00'$ in the Sines; that extent will reach from 350 to 566.203 in the Numbers.

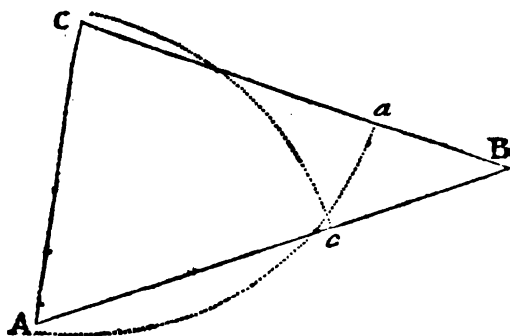
By

By the Sliding Rule.

For the first Proportion : Set $37^{\circ} 30'$ on S to $62^{\circ} 30'$ on SS, then against 350 on B is 509.976 on A.
 For the second Proportion : Set $37^{\circ} 30'$ on S to 80° on SS ; then against 350 on B is 566.203 on A.

By Geometrical Protraction.

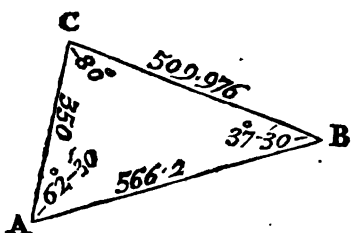
Draw the Line AC, and from your Scale take 350 with your Compasses, and set from A to C ; then upon A with 60° of Chords strike the Arch Cc, and take $62^{\circ} 30'$ from the Line of Chords, and set from C to c, and draw the Line AB thro the point c : then again with 60° of Chords, one foot of the Compasses in C, strike the Arch Aa, and take 80° out of the Line of Chords, and set from A to a ; and thro the point a draw CB, which will cut the other Line in the point B, the other Angle of the Triangle. And if you measure the Line AB in your Scale, you will find it 566.2, and you will find CB near 510.



CASE

C A S E II.

Two Sides, with an Angle opposite to one of them, being given, to find the Angle opposite to the other of them.



Let the two Sides AC and BC, and the Angle CAB, opposite to the Side CB, be given, and let the Angle B (opposite to the Side AC) be required.

As the Side BC 509.976	Ar. Com.	7.2924502
Is to the s. $\angle A$ $62^\circ 30'$		9.9479289
So is the Side AC 350		2.5440680

To the s. of the $\angle B$ $37^\circ 30'$	9.7844471
--	-----------

Or if the two Sides AB and BC, and the Angle A opposite to the Side BC, had been given, and the Angle C had been required; then,

As the Side BC 509.976	Ar. Com.	7.2924502
Is to the s. $\angle A$ $62^\circ 30'$		9.9479289
So is the Side AB 566.203		2.7529724

To the s. of the $\angle C$ $80^\circ 00'$	9.9933515
--	-----------

Note, In this Case there may a doubt arise: for if two Sides be given, whereof one of them is the greatest Side, together with the Angle opposite to the lesser of the two given Sides, and the Angle opposite to the greater of the given Sides

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Sides be demanded (as in the last Question) it will be doubtful whether the Angle found be Acute or Obtuse; for that the same Sine found in the Canon doth answer to both, so it may be doubted whether the Angle found in the last Question was 80 or 100 (*viz.* its Complement to 180 .) Nor can this doubt be cleared better than by a true delineation of the Triangle, or else by finding the third Angle.

By Scale and Compasses.

For the first Question, extend the Compasses from 509.976 to 350 in the Line of Numbers; that extent will reach from $62^{\circ} 30'$ to $37^{\circ} 30'$ in the Line of Sines. For the second Question, extend from 509.976 to 566.203 in the Line of Numbers, that extent will reach from $62^{\circ} 30'$ to $80^{\circ} 00'$ in the Sines.

By the Sliding-Rule.

For the first Question, set 509.976 on A to 350 on B, then against $62^{\circ} 30'$ on SS, is $37^{\circ} 30'$ on S. For the second Question, set 509.976 on B to 566.203 on A; then against $62^{\circ} 30'$ on S, is $80^{\circ} 00'$ on SS.

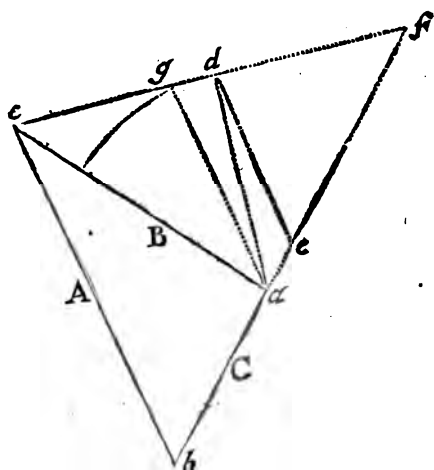
By Geometrical Protraction.

Draw the Line AC, and from your Scale take 350 , and set from A to C: then with 60° of Chords, strike the Arch D *d*, and from the Line of Chords take $62^{\circ} 30'$, and set from D to *d*, and through the point *d* draw the Line AB; then take 509.976 from your Scale of equal Parts, with your Compasses, and set one foot in C, with the other cross the Line AB in the point B, and draw CB. Then to measure the Angles, with 60° of Chords, set one foot of the Compasses in B, strike the Arch E *e*, and measure E *e* in the Line of Chords, which you will find to contain $37^{\circ} 30'$: After the same manner you must measure the

D

Angle

Let A , B and C represent the three Sides, and a , b , &c. the three Angles of the given Triangle; produce C , one of the given Legs of the Angle (a), given, till af become equal to B , the other Leg given; then bisect bf in e , join ce , and bisect cf in d , and



draw ad , which will be perpendicular to cf , (by 19 Def. Chap. 1.) and draw de , which will be parallel to cb (Eucl. 6 Lib. 2 Pr.) then will the Angle $cad = dae$, that is, to half caf : which external Angle $caf = c + b$, that is, to the Sum of the opposite Angles required.

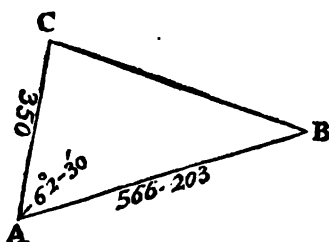
Then draw ga Parallel to cb ; so will the Angle gac be equal to the alternate one c . And if from half the sum of the opposite Angles, you take the lesser Angle, that is, if from cad , you take gac , there will remain the Angle gad , equal to half the Difference of the opposite Angles. And so also, if from bae , half the Sum of the Legs, you take C , the lesser Leg, there will remain ae equal to half the difference of the Legs. And then since the Triangle cad is Right-angled, if ad be made Radius, cd will be the Tangent of the Angle cad (i. e. the Tangent of half the sum of the opposite Angles;) and in the little Triangle gad , gd will be the Tangent of the Angle gad (i. e. the Tangent of half the difference of the opposite Angles) but the Segments of the Legs of any Triangle cut by Lines parallel to the Base being proportional, $cb : ca :: cd : gd$: That is in words, half the sum of the Legs, is to

half their Difference, as the Tangent of half the sum of the opposite Angles is to the Tangent of half their Difference. But Wholes are as their Halves: Therefore the Sum of the Legs is to their Difference, as the Tangent of half the Sum of the opposite Angles, is to the Tangent of half their Difference. Which was, &c.

From this Axiom the following Cases will easily be solved.

C A S E III.

Two Sides, with the Angle comprehended by them, being given, to find the other two Angles.



In the Triangle ABC, there is given the Side AB 566.203, and the Side AC 350, and the Angle comprehended $62^{\circ}30'$; the Angles ACB and ABC are required.

$$\begin{array}{rcl} AB & = & 566.203 \\ AC & = & 350 \end{array} \qquad \begin{array}{r} 180^{\circ} \ 00' \\ \text{Subt.} \ 62 \ 30 \end{array}$$

$$\begin{array}{rcl} \text{The sum} & = & 916.203 \\ \text{Rem.} & 117 \ 30 & = \text{Sum } \angle\text{'s B and C.} \end{array}$$

$$\begin{array}{rcl} \text{Differ.} & = & 216.203 \\ & & 58 \ 45 = \text{half Sum.} \end{array}$$

$$\begin{array}{rcl} \text{As } 916.203 \text{ the sum of the Sides} & \text{Ar. Com.} & 7.0380083 \\ \text{Is to } 216.203 \text{ the Diff. of the Sides} & & 2.3348617 \\ \text{So is t. } 58^{\circ} \ 45' \text{ the half Sum of the opposite Angles} & & \left. \begin{array}{l} \\ \end{array} \right\} 10.2169438 \\ \text{To the t. of } 21^{\circ} \ 15', \text{ half the diff. of the opposite Angles} & & \left. \begin{array}{l} \\ \end{array} \right\} 9.5898138 \end{array}$$

IF

Chap. 4. Plain Trigonometry.

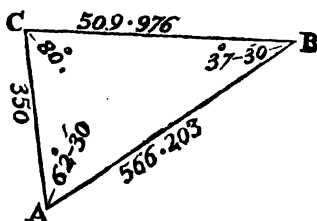
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If you add $21^{\circ} 15'$ to $58^{\circ} 45'$, the sum is $80^{\circ} 00'$, the Angle A C B; and if $21^{\circ} 15'$ be subtracted from $58^{\circ} 45'$, there will remain $37^{\circ} 30'$ the Angle A B C.

C A S E IV.

Two Sides, and the Angle comprehended between them, being given, to find the third Side.

In the Triangle A B C, there is given the side A B 566.203, and B C 509.976, and the comprehended Angle B $37^{\circ} 30'$; to find the third side A C.



The side A B =	566.203	$180^{\circ} 00'$
The side B C =	509.976	$37^{\circ} 30'$
The Sum =	1076.179	$142^{\circ} 30'$
Difference =	56.227	$71^{\circ} 15'$

As 1076.179 the Sum of the Sides Ar. Com. 6.9681154
Is to the Difference 56.227 1.7499449

So is t. $71^{\circ} 15'$ half the sum of the opposite Angles } 10.4692187

To t. $8^{\circ} 45'$ half the difference of the opposite Angles } 9.1872790

If $8^{\circ} 45'$ be added to $71^{\circ} 15'$, the sum will be $80^{\circ} 00'$ the greater Angle C; and being subtracted, the remainder is $62^{\circ} 30'$, the lesser Angle A.

D 3

Then

Then by the first Case of this Chapter.

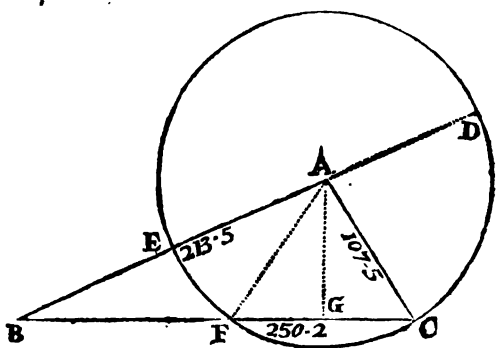
As s. $62^{\circ} 30'$ the $\angle A$	Ar. Com.	0.0520711
Is to 509.976 the side CB		2.7075498
So is s. $37^{\circ} 30'$ the $\angle B$		9.7844471
To 350, the side AC required		2.5440680

A X I O M III.

In all plain Triangles. As the greatest Side is in proportion to the Sum of the other two Sides, so is the Difference of those Sides, to a Segment, or part of the greatest Side; which being subtracted from the greater Side, a Perpendicular shall fall in the middle of the Remainder.

In the Triangle ABC, the greatest side is BC, the least side AC.

Upon A as a Center, at the distance of the shortest side AC, describe the Circle CDEF, and continue the side BA to D; so shall BD be the sum of the sides BA and AC, for AC and AD are equal, (by Construction) and BE is the Difference between the Sides AB and AC, and AC and AE are equal.



From

From hence follows this Demonstration.

As CB is to BD, so is EB to BF. Now the Rectangle of the Means is equal to the Rectangle of the Extremes, that is, $CB \times BF$ is equal to $BD \times EB$. But the Rectangle of BC in BF is equal to the Rectangle of BD in EB, (by *Eucl. Lib. 3. Pr. 36. Consect. 1.*) therefore the Proportion is true. And because the Triangle FAC hath the two Legs AF and AC equal by Construction, therefore the Perpendicular AG bisecteth the Base FC. (by *Def. 19. Chap. 1.*) Which was, &c.

By the help of the former Axiom the following Case is resolved.

C A S E V.

The three Sides of an oblique angled Triangle being given, to find the Angles.

IN the former oblique angled Triangle ABC, let AB 213.5, AC 107.5, and BC 250.2 be given; and let the Angles be required.

The side AB = 213.5

The side AC = 107.5

—————
Their sum = 321 = BD

—————
Their Differ. = 106 = BE Then say,

As the greatest side BC 250.2	Ar. Com.	7.6017127
Is to the sum of the other two 321 BD		2.5065050
So is the Differ. of the other two 106 BE		2.0253059

To the Difference of the Segments of the
Base 135.995 = BF } 2.1335236

Then if this Difference 135.995 be subtracted from
the greatest side BC 250.2, there will remain 114.205
D 4 equal

equal to FC, the half thereof 57.1025 is equal to CG the lesser Segment of the Base; and if 57.1025 be added to the said Difference 135.995, the Sum is 193.0975, equal to BG the greater Segment of the Base. By this means the oblique angled Triangle is reduced into two right angled Triangles, viz. ABG and AGC, both right angled at G, in either of which there is given the Hypothenufe and Base, so that the Angles may be found by Case V. of right angled plain Triangles, thus.

As the Hypothenufe AB 213.5	2.3293979
Is to the Radius	10.
So is the Base BG 193.0975	2.2857762

To the s. of the $\angle$ BAG $64^{\circ} 44' 51''$	9.9563783
---	-----------

The Complement of $64^{\circ} 44' 51''$ is $25^{\circ} 15' 09''$, and so much is the Angle ABG. Again,

As the Hypothenufe AC 107.5	2.0314085
Is to Radius	10.
So is the Base GC 57.1025	1.7566552

To the s. of the $\angle$ GAC $32^{\circ} 5' 8''$	9.7252467
---	-----------

Whose Complement is $57^{\circ} 54' 52''$, the Angle ACG. If you add the $\angle$ BAG $64^{\circ} 41' 51''$, and the $\angle$ GAC $32^{\circ} 5' 8''$ together, the Sum will be $96^{\circ} 49' 59''$; and so much is the whole $\angle$ BAC.

Another way to resolve this Case, and that at one Operation.

FROM half the Sum of the Sides, subtract each particular Side, and set down the Remainders. Then,

As

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As the Rectangle of half the Sum of the Sides, and the Difference between that half Sum, and the side opposite to the Angle required,

Is to the Rectangle of the other two Remainders ;

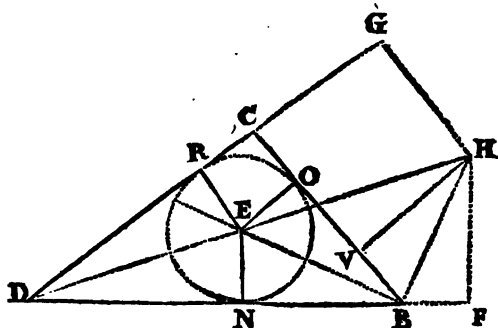
So is the square of Radius,

To the square of the Tangent of half the Angle sought.

I shall first lay down the *Demonstration of the Axiom*, and then proceed to explain it by Numbers.

Demonstration. In the Triangle BCD, let the Angle D be required.

First, let the Sides BD and DC be produced to F and G, so as DF may be equal to half the Sum of the Sides, DB, DC, and CB, and equal to DG ; and let Angles at F and G be Right. Upon the Center E describe the Circle NOR, making RG and NF equal to BC : then is DR or DN the Difference between DG the



half Sum of the Sides, and the side CB ; and the Difference between DG the half Sum, and the side DC, is the right Line GC or NB ; to which let OB be equal, and the Difference between DF the half Sum, and the Side DB, is the Line BF equal to CO, CR, or BV : and the Triangles ENB and BFH are like, because their Angles at N and F are Right ; and NBO and OBF are equal to two Right Angles, and NBE and FBH are together

ther equal to half two Right Angles: Therefore FBH and NEB are equal; and NBE is also equal to FHB. Then are the Triangles NEB and FBH similar. Then,

By simil. Δ s	1	EN : NB :: BF : FH
1 ..	2	EN $\times$ FH = NB $\times$ BF
By simil. Δ s	3	FD : ND :: HF : EN
3 $\times$ EN	4	HF : EN :: HF $\times$ EN : ENq. whose Root is the Radius of the Circle NOR.
	5	ND : Radius :: NE : tD.
2, 3, 4	6	FD : ND :: NB $\times$ BF : ENq
6 $\times$ ND	7	FD $\times$ ND : NDq :: NB $\times$ BF : ENq
Again,	8	ND : EN :: Radius : t. EDN
8 squared	9	NDq : NEq :: Rad.q. : tq. EDN.
7, 9	10	FD $\times$ ND : NB $\times$ BF :: Rad.q. : tq. EDN. That is,

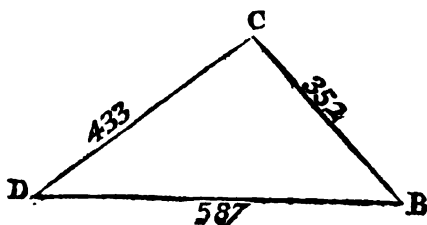
As the Rectangle of FD the half Sum of the Sides, and DN the Difference between the half sum FD and the side CB opposite to the Angle sought,

Is to the Rectangle of BN and FB the Differences of the other Sides and half Sum:

So is the square of Radius,

To the square of the Tangent of the Angle EDN, or half the Angle CDB, as was to be proved.

And now to explain the same in Numbers.



In the Triangle BCD, let the side BD 587, the side CD 433, and the side BC 352, be given; to find the Angle D.

BD.

$$\begin{array}{l|l} BD=587 & 99 \\ CD=433 & 253 \\ BC=352 & 334 \end{array} \left. \vphantom{\begin{array}{l} BD \\ CD \\ BC \end{array}} \right\} \text{The Differences.}$$

$$\text{Sum} = 1372$$

$$\text{half Sum } 686$$

Now set down the Arithmetical Complements of the Logarithms of the half Sum 686, and of the Difference between the half Sum and the side BC, opposite to the Angle D, viz. 334, and under them the Logarithms of the other two Differences 253 and 99, half their Sum will be the Tangent of half the Angle D required.

$$\text{The half Sum } 686 \quad \text{Ar. Comp. } 7.1636759$$

$$\begin{array}{l} \text{The Difference of the half} \\ \text{Sum and BC } 334 \end{array} \left. \vphantom{\begin{array}{l} \text{The Difference} \\ \text{Sum and BC} \end{array}} \right\} \text{Ar. Com. } 7.4762535$$

$$\begin{array}{l} \text{The Difference of the half} \\ \text{Sum and} \end{array} \left\{ \begin{array}{ll} CD \ 253 & 2.4031205 \\ BD \ 99 & 1.9956352 \end{array} \right.$$

$$\text{Sum } 19.0386851$$

$$\text{Half Sum } 9.5193425$$

Which half Sum of the Logarithms is the Tangent of $18^\circ 17' 44''$, the double whereof is $36^\circ 35' 28''$, the quantity of the Angle D required.

Again, let the Angle B be required.

$$\text{The half Sum } 686 \quad \text{Ar. Com. } 7.1636759$$

$$\begin{array}{l} \text{The Difference of the half} \\ \text{Sum and CD } 253. \end{array} \left. \vphantom{\begin{array}{l} \text{The Difference} \\ \text{Sum and CD} \end{array}} \right\} \text{Ar. Com. } 7.5968795$$

$$\text{The Difference of the half Sum and} \left\{ \begin{array}{ll} BC & 2.5237465 \\ BD & 1.9956352 \end{array} \right.$$

$$\text{Sum } 19.2799371$$

$$\text{Half Sum } 9.6399685$$

This

This half Sum is the Tangent of $23^{\circ} 34' 50''$
the double thereof is the $\angle B$ $47^{\circ} 09' 40''$

Let the Angle C be also required.

The half Sum	686	Ar. Com.	7.1636759
The Difference of the half	Sum and BD 99.	}	Ar. Com. 8.0043648
The Difference of the half	Sum and	{	DC 253 2.4031205
		{	BC 334 2.5237465
Sum			20.0949077
Half Sum			10.0474538

This half Sum is the Tangent of $48^{\circ} 07' 26''$
the double whereof is the $\angle C$ req. $96^{\circ} 14' 52''$
The Angle B being $47^{\circ} 09' 40''$
And the Angle D being $36^{\circ} 35' 28''$

The sum is just two right Angles $180^{\circ} 00' 00''$

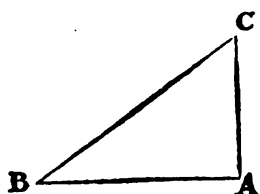
You may also find an Angle by having the three Sides, by Sines, thus.

As the Rectangle of the containing Sides or Legs,
Is to the Square of Radius;
So is the Rectangle of the Differences of the said
Legs from the half sum of the three Sides,
To the square of the Sine of half the Angle sought.

Let the Angle D in the foregoing Triangle be required.

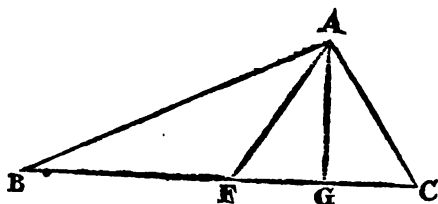
The side DC	433	Ar. Com.	7.3635121
The side DB	587	Ar. Com.	7.2313619
The Difference of the half	Sum and	{	CD 253 2.4031205
		{	BD 99 1.9956353
Sum			18.9936298
Half Sum			9.4968149

Which half Sum is the Sine of $18^{\circ} 17' 44''$, the double thereof is $36^{\circ} 35' 28''$, the Angle D, the same as before.



A Synopsis of the Doctrine of Plain Triangles.

Right Angled.			
Cases	Given	Requir.	Proportions.
I.	AB & B	AC	1. $s C : BA :: s B : AC.$ 2. $R : BA :: t B : AC.$ 3. $t C : BA :: R : AC.$
II.	AB & C	BC	1. $s C : BA :: R : BC$ 2. $R : BA :: se B : BC.$ 3. $t C : BA :: se C : BC.$
III.	BC & B	BA	1. $R : BC :: s C : BA.$ 2. $se B : BC :: R : BA.$ 3. $se C : BC :: t C : BA.$
IV.	AB & AC	B & C	1. $BA : R :: AC : t B.$ whose Complement is C. 2. $CA : R :: BA : t C.$ whose Complement is B.
V.	BC & AC	B & C	1. $BC : R :: BA : s C.$ whose Complement is B. 2. $BA : R :: BC : se B.$ whose Complement is C.
VI.	AB & AC	BC	1. $CA : BA :: R : t C.$ Then again, 2. $s C : BA :: R : BC.$
VII.	AB & BC	AC	1. $BC : R :: BA : s C.$ whose Complement is B. Then again, 2. $R : BC :: s B : AC.$



Oblique-Angled.			
Cases	Given	Requir.	● Proportions.
I.	AC A & B	AB	$s B : AC :: s C : AB.$
II.	AC CB & B	A	$CA : s B :: CB : s A.$
III.	AC CB & C	A & B	$CB + CA : CB - AC ::$ $t \frac{1}{2} A + B : t \frac{1}{2} \text{ the difference ;}$ which half diff. $\left\{ \begin{array}{l} \text{added to} \\ \text{subtr. from} \end{array} \right\}$ the half Sum, gives the $\left\{ \begin{array}{l} \text{greater} \\ \text{lesser} \end{array} \right\}$ Angle.
IV.	AC CB & C	AB	Find the Angles A and B by the last Case, then by the first Case you may find the Side AB.
V.	AB AC & CB	A B & C	$BC : AB + AC :: AB - AC :$ $BF.$ Then $BC - BF = FC$ and $\frac{1}{2} FC$ is CG. Thus is the Oblique Angled Triangle resolved into 2 Right-angled ones, in either of which the Hypotenuse is known, and also the Base.
In this Synopsis + signifies two Lines or Angles added together, and — signifies Subtraction, or the difference of two Lines or Angles.			

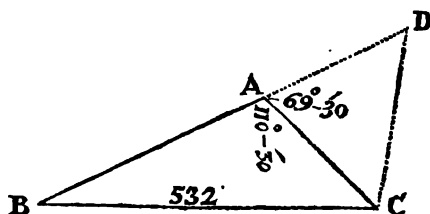
And thus have you the seven Cases of Right-angled, and five of Oblique-angled plain Triangles sufficiently demonstrated and explained. I shall next proceed to some Problems Trigonometrical.

C H A P. V.

Problems Trigonometrical.

Problem I. *One Side of an Oblique angled plain Triangle, the Angle opposite to that Side, and the Sum of the other two Sides being given; to find the other two Sides, and the Angles severally.*

IN the Triangle ABC, let the Side BC 532, and the Sum of the other two Sides BA and AC 637, and the Angle BAC $110^{\circ} 30'$ be given.



Extend the Side BA to D, making AD equal to AC, and draw the Line DC; so shall you have two other Oblique angled Triangles BDC and ADC. In the Triangle ABC is given the Angle BAC $110^{\circ} 30'$, and consequently in the Triangle ACD you have given CAD $69^{\circ} 30'$, it being the Complement of the other to 180° , (by Def. 16. Chap. 1.) also the Triangle ADC is Equicrural, by Construction; and therefore the Angles at the Base ADC and ACD are equal (by Def. 19. Chap. 1.) viz. each of them is equal to half the given Angle $55^{\circ} 15'$. Now in the Triangle BCD there is given,

1. BC

1. BC 532
2. BD 637
3. The Angle BDC $55^{\circ} 15'$

From whence you may find the Angle DCB (by Case 2. Chap. 4.)

As the Side BC 532	Ar. Com.	7.2740884
To the s. $\angle$ BDC $55^{\circ} 15'$		9.9146852
So is the Side BD 637		2.8041394
To the s. $\angle$ BCD $100^{\circ} 19'$		9.9929130

From which subtract the Angle ACD $55^{\circ} 15'$, and the Remainder is $45^{\circ} 04'$ for the $\angle$ ACB; and the Angle ABC is $24^{\circ} 26'$, and is found by subtracting the Sum of BCD $100^{\circ} 19'$ and D $55^{\circ} 15'$ from 180° .

The Sides AB and AC may be found thus (by Case I. Chap. 4.)

As the s. $\angle$ BAC $110^{\circ} 30'$	} Ar. Com.	0.0284124
(or $69^{\circ} 30'$)		
Is to the Side BC 532		2.7259116
So is s. $\angle$ ACB $45^{\circ} 04'$		9.8499897
To the Side AB 402.08		2.6043137
Again,		
As the s. $\angle$ BAC $110^{\circ} 30'$	Ar. Com.	0.0284124
Is to the Side BC 532		2.7259116
So is the s. $\angle$ ABC $24^{\circ} 26'$		9.6166164
To the Side AC 234.93		2.3709404

By Scale and Compasses.

For the first Proportion, extend from 532 to 637 in the Line of Numbers, that extent will reach from $55^{\circ} 15'$ to $79^{\circ} 40'$ in the Sines; but because the Angle sought is Obtuse (as appears by the Figure) therefore the Complement of $79^{\circ} 40'$ to 180° is $100^{\circ} 19'$

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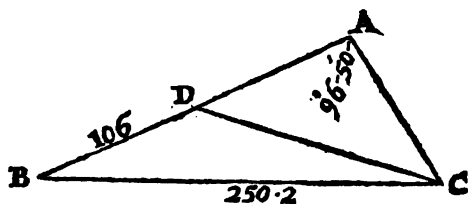
19' the Angle required. For the second Proportion, extend from $69^{\circ} 30'$ (the Complement of $110^{\circ} 30'$ to 180°) to $45^{\circ} 04'$ in the Sines, that extent will reach from 532 to 402.08. For the third Proportion, extend from $69^{\circ} 30'$ to $24^{\circ} 26'$ in the Sines; that extent will reach from 532 to 234.93 in the Line of Numbers.

By the Sliding-Rule.

For the first Proportion, set 532 on B to 637 on A; then against $55^{\circ} 15'$ on S, you will find $79^{\circ} 40'$ on SS. For the second Proportion, set $69^{\circ} 30'$ on SS, to $45^{\circ} 04'$ on S, then against 532 on A, is 402.08 on B. For the third Proportion, set $69^{\circ} 30'$ on SS, to $24^{\circ} 26'$ on S, then against 532 on A is 234.93 on B.

Problem II. *One side of an oblique angled Plain Triangle, the Angle opposite thereto, and the Difference of the other two Sides being given, to find the other Angles, and the two Sides severally.*

In the Triangle ABC, let BC 250.2, and the Difference of the other two Sides 106, and the Angle



BAC (opposite to BC) $96^{\circ} 50'$ be given.

Make AD equal to AC, and draw CD, constituting the Triangle ACD Equicrural; and the Angle DAC being $96^{\circ} 50'$, the Complement thereof to 180° is $83^{\circ} 10'$ for the two Angles ADC and ACD; which being equal one to the other, therefore each of them is half $83^{\circ} 10'$, that is $41^{\circ} 35'$; and by drawing the Line CD, there is also another Triangle made, wherein is given the Side BC 250.2, and BD 106 equal to the

the Difference of the two Sides AB and AC; and there is also given the Angle BDC $138^{\circ} 25'$, equal to the Complement of ADC, $41^{\circ} 35'$, to 180° . From these things given, you may find the Angles ACB and ABC, and also the Sides AB and AC, severally.

As the Side BC 250.2	Ar. Com. 7.6017127
Is to the s. $\angle$ BDC $138^{\circ} 25'$ (or its } Compl. $41^{\circ} 35'$)	9.8219775
So is the Side BD 106	2.0253059
To the s. $\angle$ BCD $16^{\circ} 19' 52''$	9.4489961

To which if ACD $41^{\circ} 35'$ be added, the Sum is $57^{\circ} 54' 52''$ for the whole Angle ACB. And if the Angle A be added to it, and the Sum subtracted from 180° , there will remain $25^{\circ} 15' 08''$ for the Angle ABC. Find the sides AB and AC as in the last Question, thus.

As the s. $\angle$ BAC $96^{\circ} 50'$ (or } its Comp. $83^{\circ} 10'$)	Ar. Com. 0.0030960
Is to the Side BC 250.2	2.3982873
So is the s. $\angle$ ACB $57^{\circ} 54' 52''$	9.9280146
To the Side AB 213.5	2.3293979

	Again,
As the s. of the $\angle$ BAC $96^{\circ} 50'$	Ar. Com. 0.0030960
Is to the Side BC 250.2	2.3982873
So is the s. $\angle$ ABC $25^{\circ} 15' 8''$	9.6300247
To the Side AC 107.5	2.0314080

By Scale and Compasses.

For the first Proportion, extend from 250.2 to 106 in the Line of Numbers; that extent will reach from $41^{\circ} 35'$ to $16^{\circ} 20'$ in the Line of Sines. For the second Proportion, extend from $83^{\circ} 10'$ to $57^{\circ} 55'$ in the

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the Sines; that extent will reach from 250.2 to 213.5 in the Line of Numbers. For the third Proportion extend from $83^{\circ} 10'$ to $25^{\circ} 15'$ in the Sines; that extent will reach from 250.2 to 107.5 in the Line of Numbers.

By the Sliding-Rule.

For the first Proportion: Set 250.2 on A to 106 on B, then against $41^{\circ} 35'$ on SS, is $16^{\circ} 20'$ on S. For the second Proportion: Set $83^{\circ} 10'$ on SS, to $57^{\circ} 55'$ on S, then against 250.2 on A, is 213.5 on B. For the third Proportion: Set $83^{\circ} 10'$ on SS, to $25^{\circ} 15'$ on S, then against 250.2 on A, is 107.5 on B.

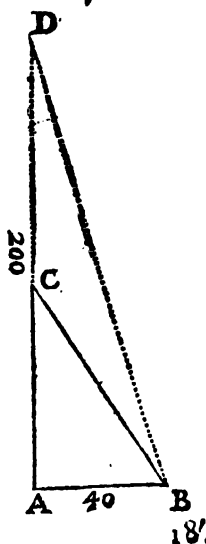
Problem III. *One side of a right angled Plain Triangle, and the other two Sides in one Sum, being given, to find the Angles and the Sides severally.*

IN the following Figure let the Base AB 40, and the Sum of the Perpendicular and Hypothenufe, AD 200, be given, to find the Perpendicular AC and Hypothenufe BC severally.

In the Triangle ABD is given the Base and Perpendicular, whereby you may find the Angles ABD and ADB, thus.

As the Base AB 40	1.6020600
To Radius 45	10.
So is the Perpendicular	
AD 200	2.3010300
To the t. ABD $78^{\circ} 41' 24''$	10.6987700

Whose Complement is the Angle D $11^{\circ} 18' 36''$; and because CD is equal to CB, therefore is the Angle CBD also $11^{\circ} 18' 36''$; then if 11°



18' 36" be subtracted from the whole Angle ABD
 $78^{\circ} 41' 24''$, there will remain the Angle ABC 67°
 $22' 48''$. Then to find the Sides, say,

As Radius to the Base AB 40

1.6020600

So is the t. ABC $67^{\circ} 22' 48''$

10.3802083

To the Perpendicular AC 96

1.9822083

Then subtract 96 from 200, and there will remain
 104 for the Hypothenufe CB.

By Scale and Compasses.

For the first Proportion, extend from 40 to 200 in
 the Line of Numbers, that extent will reach from
 45° to $78^{\circ} 41'$. For the second Proportion, extend
 from 45° to $67^{\circ} 23'$ in the Tangents, that extent will
 reach from 40 to 96 in the Line of Numbers.

By the Sliding-Rule.

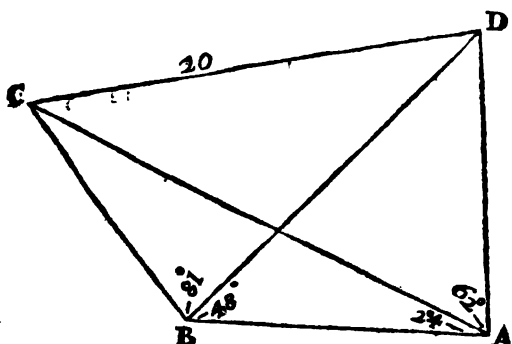
For the first Proportion, set 40 on B to 200 on A,
 then against Rad. on Tangents, is $78^{\circ} 41'$. For the
 second Proportion, set $67^{\circ} 23'$ of Tangents to Radius,
 then against 40 on B, is 96 on A.

Problem

Problem IV. *There are two Ports represented by C and D, which are 20 Miles asunder; and there are two Ships at Sea represented by A and B, and each of them makes Observation to the said Ports, and finds the Angles CAD $62^{\circ} 00'$ and CAB $24^{\circ} 00'$, and the Angles CBD $81^{\circ} 00'$ and DBA $48^{\circ} 00'$. To find the Distance of each Ship from the Ports, and of the Ships from each other, is what is required.*

THE $\angle ABC$ is 129° , to which add $BAC 24^{\circ}$, the sum is 153° , whose Complement to 180° is 27° for the $\angle ACB$.

And the $\angle BAD$ is 86° , to which add ABD , and the sum is 134° , whose Complement to 180° is $46^{\circ} 00'$ for the $\angle ADB$.



Now because there are not Data sufficient in any one of the Triangles, therefore for the present we will suppose the side AB 15.

Then in the $\triangle ABC$,

As the s. $\angle ACB 27^{\circ} 00'$ Ar. Com. 0.3429532

To AB 15 1.1760912

So is the s. $\angle ABC 129^{\circ} 00'$ (or its Com. 51°) 9.8905026

To the side AC 25.6772

1.4095470

E 3

ln

In the $\triangle ABD$.

As the s. $\angle ADB$ $46^{\circ} 00'$	Ar. Com. 0.1430659
Is to the side AB 15	1.1760912
So is the s. $\angle ABD$ $48^{\circ} 00'$	9.8710735
To the side DA 15.49637	1.1902306

In the $\triangle ACD$ (by *Case 3. Chap. 4.*)

The side AC=25.6772	180°
The side AD=15.49637	$62^{\circ} = \angle CAD$ sub.
The Sum 41.17357	118 diff. = ADC + ACD
The Difference 10.18083	59 the half.

As the Sum of the Sides AC and AD	} Ar. Com. 8.3853814
Is to the Difference of the same	
So is t. of half the opposite Angles 59° 00'	
To the t. of half their difference 22° 22' 05"	9.6143910

Which added to the half Sum makes $81^{\circ} 22' 05''$ for the Angle ADC, and subtracted from the half Sum, there remains $36^{\circ} 37' 55''$ for the Angle ACD.

Then again in the $\triangle ACD$.

As the s. of the $\angle ADC$ $81^{\circ} 22' 05''$	Ar. Com. 0.0049474
Is to the Side AC 25.6772	1.4095470
So is the s. of the $\angle DAC$ $62^{\circ} 00'$	9.9459349
To the side CD 22.93133	1.3604293

But CD the Distance of the Ports is but 20 Miles ; therefore to find the other Distances truly, you must argue thus ;

As

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As CD 22.93133 the assum'd side Ar. Com. 8.6395707

Is to DC 20 the true side 1.3010300

So is AC 25.6772 the assum'd side 1.4095470

To AC 22.39486 the true side 1.3501477

In the $\triangle ABC$.

As the s. of the $\angle ABC$ $129^{\circ} 00'$ Ar. Com. 0.1094974

Is to the side AC 22.39486 1.3501477

So is the s. of the $\angle ACB$ $27^{\circ} 00'$ 9.6570468

To the side AB 13.0825 1.1166919

In the $\triangle ADC$.

As the s. of the $\angle DAC$ $62^{\circ} 00'$ Ar. Com. 0.0540651

Is to the side CD 20 1.3010300

So is the s. of the $\angle ACD$ $36^{\circ} 37' 55''$ 9.7757359

To the side AD 13.5155 1.1308310

In the $\triangle ABD$.

As the s. of the $\angle ABD$ $48^{\circ} 00'$ Ar. Com. 0.1289265

Is to the side AD 13.5155 1.1308310

So is the s. of the $\angle BAD$ $86^{\circ} 00'$ 9.9989408

To the side BD 18.1425 1.2586983

In the $\triangle BCD$.

As s. DBC $81^{\circ} 00'$ Ar. Com. 0.0055801

Is to the side CD 20 1.3010300

So is the s. $\angle BDC$ $35^{\circ} 22' 05''$ 9.7625485

To the side CB 11.7208 1.0689586

Problem V. *There are three Ports represented by A, B, and C, which a Ship at D discovers, and is desirous to know how far she is distant from each of them; in order thereunto she makes Observation, and finds the Angle that A and C make with D to be $25^{\circ} 00'$, and the Angle that A and B make with D to be $19^{\circ} 00'$. Now between A the middlemost, and B the nearest Port, is 8 Miles; between B and C is 12 Miles, and between C and A is 7.2 Miles. Now I desire to know how far the Ship at D is distant from every one of the Ports at A, B, and C.*

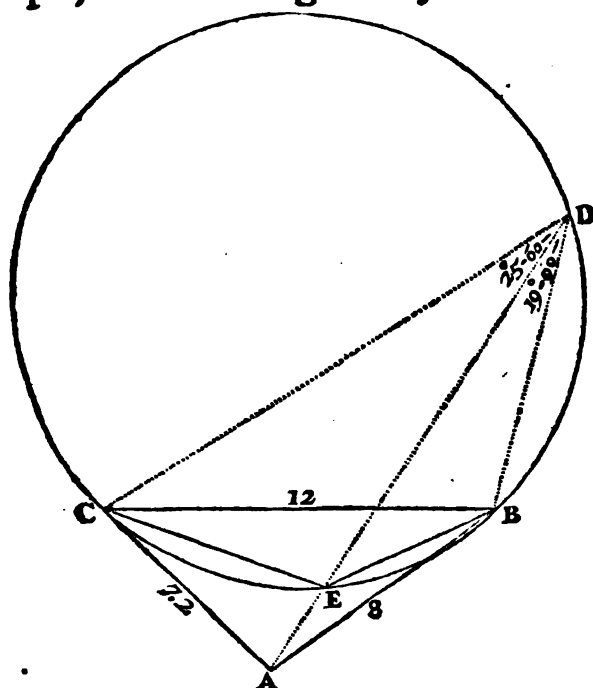
FIRST, draw the Lines DC, DA, and DB, and then through the three Points C, B, and D describe the Circle, and draw the Lines CE, and BE; then will the Angle CBE be equal to the Angle CDE; and also the Angles ECB and EDB are equal (by *Encl. 3 Lib. 21. Pr.*) Hence all the Angles, and one side, in the oblique angled Triangle ECB, are known; from which the other two sides may be found, thus:

$$\begin{array}{r}
 180 \\
 \hline
 \text{EBC} = 25^{\circ} \\
 \text{ECB} = 19^{\circ} \\
 \hline
 \text{Sum} \quad 44 \text{ sub. from } 180 \\
 \hline
 \text{CEB} = 136 \text{ Remains}
 \end{array}$$

$$\begin{array}{l}
 \text{As s. CEB } 136^{\circ} \\
 \text{To CB } 12 \\
 \text{So is s. EBC } 25^{\circ}
 \end{array}$$

$$\text{To CE } 7.30059$$

$$\begin{array}{r}
 0.1582287 \\
 1.0791812 \\
 9.6259483 \\
 \hline
 0.8633582 \\
 \text{Again,}
 \end{array}$$



Again,
 As s. CEB 136°
 To CB 12
 So is s. ECB 19°
 To EB 5.62408

0.1582287
1.0791812
9.5126419
0.7500518

Next find the three Angles of the Triangle ABC
 (as in Page 42) thus.

For the Angle ABC

Sides	$\left\{ \begin{array}{l} 12 \\ 8 \\ 7.2 \end{array} \right\}$	$\left \begin{array}{l} 1.6 \\ 5.6 \\ 6.4 \end{array} \right\}$	Differences
-------	--	--	-------------

Sum 27.2

Half 13.6

1276

The $\frac{a}{2}$

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Plain Trigonometry.

Part I.

The half Sum 13.6	Ar. Com.	8.8664611
The Difference of the half Sum and AC	}	Ar. Com. 9.1938200
The Difference of the half Sum and		
	AB	0.7481880
	CB	0.2041200
The Sum		19.0125891

Tangent of $17^{\circ} 47' 19''$ 9.5062945

ABC	35 34 38	
Subtr.	25 00 00	=EBC

Rem. 10 34 38 = ABE

Again for the Angle ACB.

The half Sum 13.6	Ar. Com.	8.8664611
The Difference of the half Sum and AB	}	Ar. Com. 9.2518120
The Difference of the half sum and		
	BC	0.2041200
	CA	0.8061800

The Sum 19.1285731

Half Sum 9.5642865

Which half Sum is the t. of $20^{\circ} 08' 13''$

which double is the $\angle$ ACB	40 16 26
Subtr.	19 00 00

Rem. 21 16 26 = ACE

Then in the $\triangle AEB$ find the $\angle$'s AEB and EAB
(by Case 3. Chap. 4)

The side AB = 8

The side EB = 5.62408

The Sum = 13.62408

The Differ. = 2.37592

The

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The $\angle ABE = 10^{\circ} 34' 38''$

The Compl. $\overline{169\ 25\ 22}$

The Half $= 84\ 42\ 41 = \frac{1}{2} AEB + EAB$

As the sum of the Sides 13.62408 Ar. Com. 8.8656929

To their Difference 2.37592 0.3758319

So is the t. of half the Sum of the

oppo^s. $\angle$'s $84^{\circ} 42' 41''$ $\overline{11.0335455}$

To the t. of half their

Difference $\left. \begin{array}{l} \\ \end{array} \right\} 62\ 02\ 26$ $\overline{10.2750708}$

The Sum $\overline{146\ 45\ 07} = AEB$

The Diff. $\overline{22\ 40\ 15} = EAB$

From $104^{\circ} 08' 56'' = CAB$

Sub. $\overline{22\ 40\ 15} = EAB$

Rem. $\overline{81\ 28\ 41} = CAE$

Then,

As the s. of the $\angle ADC\ 25^{\circ} 00'$ Ar. Com. 0.3740517

Is to the side AC 7.2 0.8573325

So is the s. of the $\angle DAC\ 81^{\circ} 28' 41''$ $\overline{9.9951784}$

To the side CD 16.84855 1.2265626

Again in the $\triangle ADB$.

As the s. of the $\angle ADB\ 19^{\circ} 00'$ Ar. Com. 0.4873581

Is to the side AB 8 0.9030900

So is the s. of the $\angle DAB\ 22^{\circ} 40' 15''$ $\overline{9.5859527}$

To the side BD 9.471109 0.9764008

Add $\left. \begin{array}{l} 22^{\circ} 40' 15'' \\ 19\ 00\ 00 \end{array} \right\} = DAB$
 $\left. \begin{array}{l} \\ \end{array} \right\} = ADB$

Sum $\overline{41\ 40\ 15}$

Compl. $\overline{138\ 19\ 45} = ABD$

Then

Then

As the s. of the $\angle ADB$ $19^{\circ} 00'$

0.4873581

Is to the side AB 8

0.9030900

So is the s. of the $\angle ABD$ $138^{\circ} 19' 45''$

9.8227238

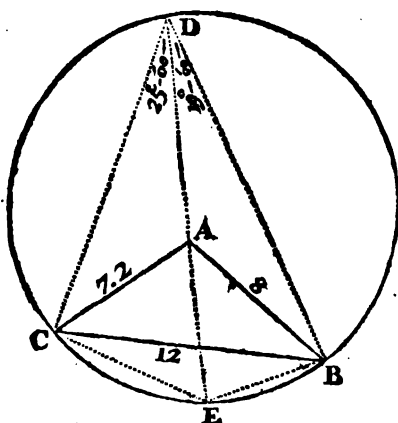
To the side AD 16.33698

1.2131719

Hence

The Distance from } to the Port B is 9.471109
 the Ship at D, } to the Port A is 16.33698 } Miles
 } to the Port C is 16.84855

This Problem may be varied, so that the Port, or Angle A, may be turn'd towards D, as in the following Scheme.



Let the Sides of the Triangle ABC , and the Angles at D , be the same as before; and let the Distances DC , DA , and DB , be required.

The Triangle ECB will be the same as before, and consequently the Sides CE and EB will be the same. Also the Angles of the given Triangle ABC

we will suppose to be found; the next thing to be found will be the Angles AEB and EAB in the Triangle ABE .

The Angle ABE is known, it being the sum of the two

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two Angles ABC, $35^{\circ} 34' 38''$, and CBE (=CDE)
 $25^{\circ} 00'$. viz. $60^{\circ} 34' 38''$

The Side AB=8

The Side EB=5.62408

Compl. $119\ 25\ 22$

The Sum 13.62408

The half $59\ 42\ 41$

The Differ. 2.37592

Then, As the sum of the } Ar. Com. 8.8656931
 Sides 13.62408 }
 Is to the Diff. of the Sides 2.37592 0.3758318
 So is the t. of half the Sum of the op- } 10.2335230
 posite $\angle$'s $59^{\circ} 42' 41''$ }

To the t. of half their Differ. $16^{\circ} 37' 27''$ 9.4750479

Which being added to $59^{\circ} 42' 41''$, the sum will be $76^{\circ} 20' 08''$ for the Angle AEB; and subtracted from $59^{\circ} 42' 41''$, the remainder is $43^{\circ} 5' 14''$ for the Angle EAB, whose Complement to 180 is $136^{\circ} 54' 46''$, the Angle BAD; to which add ADB 19° , and the Sum is $155^{\circ} 54' 46''$, whose Complement to 180 is $24^{\circ} 5' 14''$, the Angle ABD. And if EAB $43^{\circ} 5' 14''$ be subtracted from CAB $104^{\circ} 08' 56''$, there will remain $61^{\circ} 03' 42''$ for the Angle EAC, whose Complement to 180 is $118^{\circ} 56' 18''$ for the Angle CAD. Now having all these Angles, you may find the sides DC, DA, and DB, as followeth.

In the $\triangle ADB$.

As the s. $\angle$ ADB $19^{\circ} 00'$ Ar. Com. 0.4873581

Is to the side AB 8 0.9030900

So is the s. $\angle$ BAD $136^{\circ} 54' 46''$ } 9.8344912
 (or its Com. $43^{\circ} 5' 14''$) }

To the side BD 16.7857 1.2249393

Again,

Again,

As the s. $\angle$ ADB $19^{\circ} 00'$ Ar. Com. 0.4873581
Is to the side AB 8 0.9030900

So is the s. of the $\angle$ ABD $24^{\circ} 05' 44''$ 9.6107952

To the side DA 10.0286 1.0012433

In the $\triangle ACD$.

As the s. $\angle$ ADC $25^{\circ} 00'$ Ar. Com. 0.3740517
Is to the Side AC 7.2 0.8573325

So is the s. of the $\angle$ CAD $118^{\circ} 56' 18''$ } 9.9420780
(or $61^{\circ} 03' 42''$) }

To the Side DC 14.9095 1.1734622

Thus have you the three Distances DB 16.7857,
DA 10.0286, and DC 14.9095.

The DOCTRINE of Spherical Triangles.

C H A P. VI.

Of such Affections, Definitions, and Theorems, as are most necessary for the right understanding of the Nature and Dimensions of Spherical Triangles.

1. **T**HE three Sides of every Spherical Triangle are the Arches of three Great Circles of the Sphere, every one of them being less than a Semi-circle, or consisting of fewer Degrees than 180, which is the measure of a Semi-circle (by 8 Def. of Chap. 1.)

2. A

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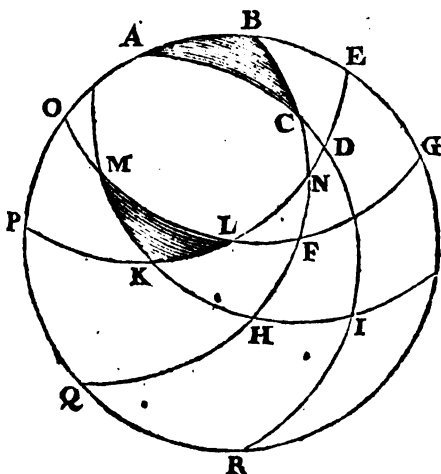
2. A Great Circle of the Sphere is such a one as divideth the whole Sphere or Globe into two equal Parts or Hemispheres, and so is every where distant from the Poles thereof 90 degrees.

3. If one Great Circle of the Sphere do, pass thro the Poles of another Great Circle, those two Great Circles intersect each other at Right Angles; and the contrary.

4. The Measure of a Spherical Angle is the Arch of a Great Circle described from the Angle, and intercepted between the two Sides, they being continued out till they be Quadrants.

In the Spherical Triangle ABC (in the following Scheme) the measure of the Spherical Angle at A is not the Arch BC, but the Arch ED intercepted between the two Sides AB and AC, continued till they be Quadrants, that is, to the Points D and E, because the Arch BC is not described upon the angular point A, but the Arch ED is; therefore the Arch BC cannot be the measure of the Angle A, (see Def 4. Ch. 1.)

5. The Sides of a Spherical Triangle being continued till they meet together, do make two Semi-circles, and at their intersection do comprehend an Angle equal and opposite to the first Angle.



Thus

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Thus in the Triangle ABC, the Sides AB and BC being continued till they meet in Q, do thereby constitute the other Triangle ACQ, which may be called supplemental to the Triangle ABC; for not only the Angle at Q is equal to its opposite at B, but also the other Sides and Angles are Complements of the Sides and Angles in the given Triangle ABC: for the Side CQ is the Complement of CB to a Semicircle; and the Angle ACQ is the Complement of ACB to a Semicircle, and so of the rest, (see *Def. 16. Chap. 1.*)

6. The Sides of a Spherical Triangle may be changed into Angles, and the contrary; the Complement of the greatest Side or greatest Angle to a Semi-circle being taken for the greatest Side or greatest Angle.

As in the Triangle ABC, obtuse angled at B, let DE be the measure of the Angle at A, and let FG be the measure of the acute Angle at B, (being the Complement of the obtuse Angle at B) and let HI be the measure of the Angle at C. Now

$$\left. \begin{matrix} KL \\ LM \\ KM \end{matrix} \right\} \text{ is equal to } \left\{ \begin{matrix} DE \\ FG \\ HI \end{matrix} \right\} \text{ because } \left\{ \begin{matrix} KD \\ LG \\ KI \end{matrix} \right\} \text{ and } \left\{ \begin{matrix} LE \\ FM \\ MH \end{matrix} \right\}$$

are Quadrants, and the common Complement is $\left\{ \begin{matrix} LD \\ LF \\ KH \end{matrix} \right\}$

therefore the sides of the Triangle KLM are equal to the Angles of the Triangle ABC, taking for the greatest Angle ABC, the Complement thereof FBG.

It may also be demonstrated, That the Sides of the Triangle ABC are equal to the Angles of the Triangle KLM (by the Converse of the former.) For

$$\text{The Side } \left\{ \begin{matrix} AB \\ BC \\ AC \end{matrix} \right\} \text{ is equal to } \left\{ \begin{matrix} OP \\ FH \\ DI \end{matrix} \right\} \text{ the measure of the Angle } \left\{ \begin{matrix} MLK \\ LMK \\ DKL \end{matrix} \right\}$$

which is the Complement of the Angle MKL.

$$\text{For that } \left\{ \begin{matrix} AD \\ AP \\ BF \end{matrix} \right\} \text{ and } \left\{ \begin{matrix} CI \\ OE \\ CH \end{matrix} \right\} \text{ are Quadrants, and their common Complement is } \left\{ \begin{matrix} CD \\ AO \\ CF \end{matrix} \right\}$$

There-

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Therefore the Sides may be turn'd into Angles, and the contrary ; which was to be proved.

7. A right angled spherical Triangle hath one right Angle, or more than one.

8. In the Figure annexed, supposing the Angles BAC ,

and BDC to be right Angles ;

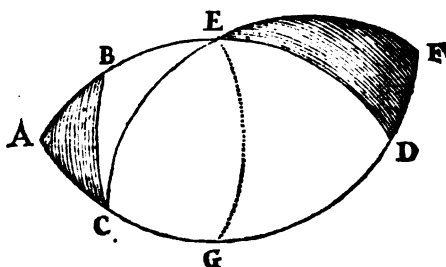
The Tri-
angle $\left\{ \begin{array}{l} BAC \\ BDC \\ CDE \end{array} \right\}$ hath the $\left\{ \begin{array}{l} \text{One right and two acute.} \\ \text{Two obtuse and one right.} \\ \text{One obtuse and two right.} \end{array} \right\}$ Angles

9. A right angled spherical Triangle, with two acute Angles, hath from the right Angle, a right angled Triangle opposite thereunto, with two obtuse Angles ; as the Triangles ABC and BDC .

10. The Sides of a right angled spherical Triangle, with two acute Angles, are all of them less than Quadrants ; as in the Triangle ABC , the Sides are each of them less than Quadrants.

11. In a right angled spherical Triangle with two obtuse Angles, the Sides that are opposite to the two obtuse Angles are greater than Quadrants ; but the third Side that subtends the right Angle is less. As in the Triangle BDC , the sides BD and CD subtending the obtuse Angles at B and C , are each of them greater than Quadrants, but the side BC is lesser.

12. The Sides subtending the right Angles of a spherical Triangle having divers right Angles, are Quadrants : As in the Triangle AEG , if the two great Circles AE and AG , do cut the great Circle EG at right Angles in the Points E and G , A is the Pole of the great Circle EG , (by the 3^d hereof) and AE and AG are Quadrants (by the 2^d hereof) but if the Angle



F

A

A be also a right Angle, then EG is also a Quadrant (by the 12. of Chap. 1.)

13. If the third Angle of a spherical Triangle, having two right Angles, be acute, the third Side is less than a Quadrant ; but if obtuse, then it is greater than a Quadrant.

14. An oblique angled Triangle consisteth simply of acute or obtuse Angles, or of both.

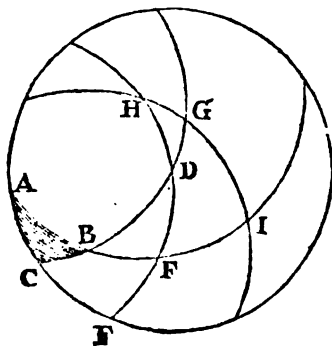
15. A spherical Triangle, with two obtuse Angles and one acute, is opposite to a spherical Triangle simply acute angled, & *contra*.

As, if the Angles at A and D be supposed acute, then the Triangle BDC with two obtuse Angles at B and C, and one acute at D, is opposite to the simply acute angled Triangle ABC.

16. A spherical Triangle with two acute Angles and one obtuse, is opposite to a spherical Triangle simply obtuse, & *contra*.—As, if the Angle at A and D be supposed obtuse, then the Triangle ABC, with two acute Angles at B and C, and one obtuse at A, is opposite to the simply obtuse angled Triangle BDC.

17. The three Angles of every spherical Triangle are greater than two right Angles.

This is manifest in spherical Triangles having more right, or obtuse Angles than one. But in acute angled Triangles, it may be thus demonstrated :

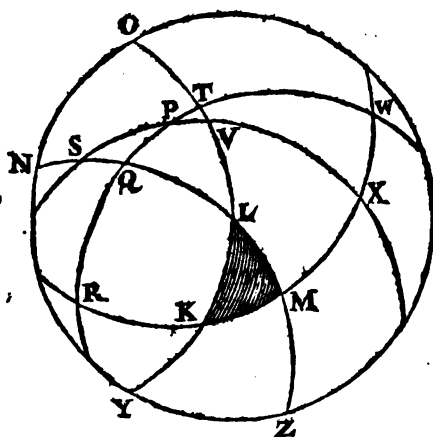


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In the right angled spherical Triangle ABC, right angled at C, and acute angled at A and B,

The measure of the acute Angle $\left. \begin{matrix} \text{BAC} \\ \text{ABC} \\ \text{or} \\ \text{DBE} \end{matrix} \right\} \text{Arch} \left. \begin{matrix} \text{EF} \\ \text{GI} \end{matrix} \right\}$ by the 4th hereof.

But the Arches EF and BD together are equal to a Quadrant; therefore the Arches FE and GI added together are more than a Quadrant; and consequently the Angles answering to these Arches, namely the Angles BAC and ABC together, are more than a Quadrant. But the Angle ACB is a right Angle, therefore the three Angles together are greater than two right Angles.



Again, in the acute angled spherical Triangle KLM,

The Measure of the acute Angle $\left\{ \begin{matrix} \text{KLM} \\ \text{MKL} \\ \text{LMK} \end{matrix} \right\}$ is the Arch $\left\{ \begin{matrix} \text{NO} \\ \text{VX} \\ \text{QR} \end{matrix} \right\}$

But these three Arches NO, VX, and QR, together, are more than two Quadrants. For PQ and PV (being the Complements of the two Arches QR and

VX) added together are less than the Arch NO, the Measure of the third Angle; consequently the three Arches, which are the Measures of the three Angles, are more than two Quadrants.

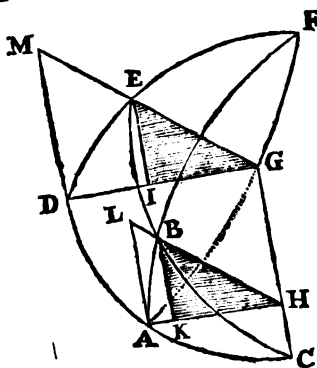
Thus having set down the most principal Definitions, and Affections of spherical Triangles, I shall next give the usual Axioms, with their Demonstrations; and then I shall lay down the universal Proposition of the Honourable the Lord *Neper*, by which all the Cases of right angled Triangles, and the first ten Cases in oblique angled Triangles are solved.

C H A P. VII.

Of the four Fundamental Axioms, for the Solution of spherical Triangles; with their Demonstrations.

AXIOM I.

IN right angled spherical Triangles, having the the same acute Angles at the Base; the Sines of the Hypothenuses, and of the Perpendiculars, are proportional one to the other, & *contra*.



If the Quadrant of a Circle CBEG be inclined to another Quadrant CADG, and two other perpendicular Quadrants cut both of them, *viz.* FBAG and FEDG, and the latter cuts them in their Extremities in D and E: having let fall Perpendiculars from the common

Sections E and B, thro the Planes of the perpendicular

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cular Quadrants, and the inclined Quadrant, (*viz.* on the one side EG and BH, as right Sines of the Segments EC and BC; on the other EI and BK, as right Sines of the Segments ED and BA) you'll have two Triangles EIG and BKH, right angled at I and K, equiangular at G and H (by reason of the same inclination of the Plane CBEGC) and consequently similar.

But these things will be much more natural if you cut four Quadrants of fine Past-board, (as I have done for my own Satisfaction and others.) You may strike a Circle, and divide it into four Quadrants, by drawing two Diameters at right Angles to each other, and cutting the Strokes half thro: But cut two of them quite asunder, that is, cut one stroke from the Center quite thro; cut in each of those two a slit, that is, slit one of them about half way from the Center towards the Limb; the other slit half way from the Limb towards the Center: then put them one over the other, and turn it up, one of the whole Quadrants at right Angles to the other, and the other two Quadrants at right Angles to the Planes of the Quadrants they fall upon, (as near as you can guess) and fasten them with Glew or Shoemakers Paste. So shall you have the spherical Triangle ABC (in this Past-board Instrument) in its natural Order, with the Complements of the Sides; and you may easily draw Lines within it, to represent the Sines of the Sides; which being drawn, you will find the Sines of the Hypotenuses and Perpendiculars, with the Lines IG, and KH, to form the similar Triangles EGI and BHK above-mentioned.

The two Tangent Lines DM and AL will be only imaginary Lines raised from the bottom edges of the two perpendicular Quadrants, which with the Sines of the Bases DG, (or Radius) and AH, and the Lines HL and GM drawn from the extremities of the Sines of the Bases at H and G, and by the extremities of the

Hypothenufes of the fpherical Triangles at B and E, and imagined to be extended to L and M above, where the Tangents meet with them; thefe Lines fo drawn, or imagined to be drawn, will form two other fimilar Triangles represented in the Scheme by AHL and DGM.

There are in this Figure conftituted by the feveral Circles two fpherical Triangles ABC, and CDE, which at this time we fhall principally make ufe of; in which,

The	{	Hypothenu-	{	EC & BC	and the	{	EG & EH		
		fes are						DE & AB	Sines the
		Perpendicu-							
lars are	DC & AC								
Bafes are									

Now the Angle ACB at the Bafe is common to both Triangles; and the Triangles made by the Sines of the Hypothenufes and Perpendiculars being equiangular, the Sides about the equal Angles will be proportional (by *Eucl. Lib. 6. Pr. 4.*) That is,

EG : EI :: BH : BK. Or, EI : EG :: BK : BH.

But EG being the Sine of the Hypothenufe EC, and EI the Sine of the Perpendicular ED, in the fpherical Triangle EDC, right angled at D; and BH is the Sine of the Hypothenufe BC; and BK the Sine of the Perpendicular BA, in the Triangle ABC, right angled at A: Therefore the Sines of the Hypothenufes, and of the Perpendiculars are proportional; which was to be demonftrated.

AXIOM II.

In right angled fpherical Triangles, having the fame acute Angles at the Bafe; the Sines of the Bafes, and the Tangents of the Perpendiculars, are proportional, & *contra*.

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In the two Triangles DGM and AHL, in the former Diagram, DG being the Sine of the Base DC of the spherical Triangle EDC, AH is the Sine of the Base AC of the spherical Triangle ABC, and DM and AL are Tangents of the Perpendiculars of the same spherical Triangles; (but this will more plainly appear by the Past-board Instrument, as it is above explained.) Now these Triangles being equiangular, the Sides will be proportional to each other; that is,

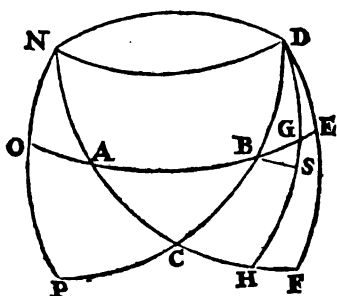
$$DG : DM :: AH : AL \text{ Or, } DM : DG :: AL : AH.$$

But DG and AH are Sines of the Bases of the spherical Triangles EDC, and ABC; and DM and AL are Tangents of the same spherical Triangles. Therefore the Sines of the Bases, and Tangents of the Perpendiculars are proportional, and the contrary: which was, &c.

A X I O M III.

In all spherical Triangles, the Sines of their Sides are directly proportional to the Sines of their opposite Angles, & *contra*.

Let ABC be a spherical Triangle right angled at C; and let the Sides thereof be continued till they make the Quadrants AE, AF, and CD. Also from the Pole of the Quadrant AF, *viz.* from D, let there be two other Quadrants drawn to H and F; so shall there be three new Triangles, BDE and GDE right angled, and BDG oblique angled. Now I say, in the right angled spherical Triangle ABC,



$$\text{ACB} : \text{AB} :: \text{ABC} : \text{AC}$$

$$\text{Or, } \text{ACB} : \text{ABC} :: \text{AB} : \text{AC, } \&c.$$

Likewise, in the Oblique angled spherical Triangle BDG, I say,

$$\text{BDG} : \text{BG} :: \text{BGD} : \text{BD} \quad \text{and so is } \text{DBG} : \text{DG, } \&c.$$

First, As to the Right-angled Triangle ABC,

Wherein $\left\{ \begin{array}{l} \text{ACB} \\ \text{BAC} \\ \text{ABC} \end{array} \right\}$ and the $\left\{ \begin{array}{l} \text{AE or ND} \\ \text{EF} \\ \text{OP} \end{array} \right\}$ are of the same quantity, by the 4th of Chap. 6.

Now, it is all one if I say,

$$\text{ACB} : \text{AB} :: \text{BAC} : \text{BC.} \quad \text{Or, } \text{AE} : \text{AB} :: \text{EF} : \text{BC.}$$

Axiom 1.

In like manner, it is all one if I say,

$$\text{ACB} : \text{AB} :: \text{ABC} : \text{AC.} \quad \text{Or, } \text{OB} : \text{AB} :: \text{OP} : \text{AC.}$$

Axiom 1.

But by the Rules already demonstrated,

$$\text{ACB} : \text{AB} :: \text{ABC} : \text{AC, and so is } \text{BAC} : \text{to BC.}$$

Therefore,

$$\text{As the Sine of } \text{ABC} : \text{AC} :: \text{the Sine of } \text{BAC} : \text{BC.}$$

Secondly, As to the Oblique angled Triangle BDG, because (by the Demonstration of Right angled above)

BD

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BD : DEB :: DE : DBE. And, DG : DEG :: DE : DGE.

Also,

DG : DB :: DBE (or DBG) : DGB, *&c.*

And if from B, a Perpendicular be let fall to S;

Because then,

BD : BSD :: BS : BDS. And,

BG : BSG :: BS : BGS, or to BGD.

Therefore also,

BG : BD :: BDS (or BDG) : DGB, *&c.*

For if thus,

As $\left\{ \begin{smallmatrix} 4 \\ 2 \\ 2 \end{smallmatrix} \right\}$ to $\left\{ \begin{smallmatrix} 12 \\ 12 \\ 4 \end{smallmatrix} \right\}$ So is $\left\{ \begin{smallmatrix} 1 \\ 1 \\ 3 \end{smallmatrix} \right\}$ to $\left\{ \begin{smallmatrix} 3 \\ 6 \\ 6 \end{smallmatrix} \right\}$ And
Then
Which was to be
demonstrated.

A X I O M IV.

This Axiom is made use of in resolving the XI. and XII. Cases of Oblique angled spherical Triangles, where three Sides are given to find an Angle, or three Angles to find a Side. And is thus,

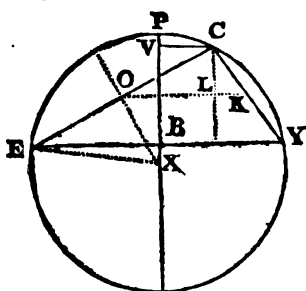
As the Rectangle of the Sines of the Sides comprehending the required Angle, is to the Square of Radius; so is the Rectangle of the Sines of the half Sum, and half difference of the Base, and difference of the Legs, or Sides comprehending the Angle required, to the Square of the Sine of half the Angle required.

Before I proceed to the Demonstration, I will lay down the following Lemma, to facilitate the Work.

L E M M A.

The versed Sines of two Arches given, the right Sines of the half Sum and half difference of those Arches are mean proportionals between the whole Sine, and half the difference of the versed Sines.

Let



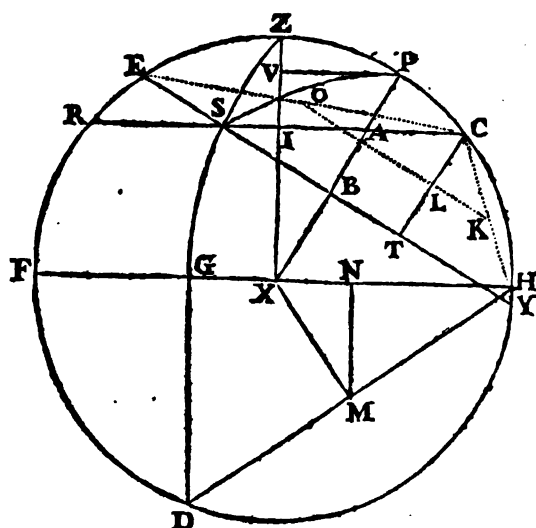
Let the given Arches be PE and PC , and their versed Sines PB , and PV , their difference is BV ; the Sum of the given Arches EPC , and their difference CY . Also let the Sine of the half Sum be EO , or OC , and the Sine of half the difference

CK : I say, that OE and CK are mean Proportionals between XE and CL . For the Angles EXO , CYE , and CKL are equal, (by *Euel. Lib. 3. Prop. 20.*) and the Angles at O and L are right. Therefore the Triangles XEO , KCL , are alike, and $XE : EO :: KC : CL$. Which was, &c.

CONSECTARY.

If the Triangle ECY is inscribed in a Circle, and from any Angle, as ECY , the Perpendicular do fall on the opposite side EY , the half of the Legs OC , CK , are mean proportionals between half the Perpendicular CL , and Semidiameter of the Circle XE : therefore the Rectangles of OC , CK , and XE , CL are equal.

Let



Let the Sides of the Triangle ZPS be known, and the Angle required be SZP; then shall ZS the one Side be equal to ZC; and PC will be the Difference of the Sides. In like manner shall the Base PS be equal to PY, or PE; and CE the Sum of the Base and of the difference of the Sides; and CY the difference of them. All these things are known. Moreover draw PV, the Sine of the Side PZ, and CI is the Sine of the other Side ZS, let it be continued to S. In like manner let CT be drawn Perpendicular to the right Line EY; and OK bisecting the right Lines EC, SC, CT, and CY. Lastly, let the Arch HD be the measure of the Angle sought PZS, and the right Line MN perpendicular to the right Line GH, bisect the Lines HD and HG. I say then,

As the Rectangle Figure of the Sines of the Sides PV, CI,

To the Square of the Radius PX;

So is the Rectangle Figure of the Sines of the half Sum, and half difference of the Base, and of the difference of the Sides OC, CK,

To

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To the Square of the Sine of half the requir'd Angle HM.

For the Triangles XPV, XAI, SAB, and SCT, are Equiangular, because the Lines PX, and CT, and also CI, and PV are Parallels; therefore the Angles SAB, SCT, IAX, and VPX are equal; and the Angles V, I, B & T, are Right: and therefore it will be,

From Similar Δ s
And also

1 $\times$ 2

3. first part $\div$ cs

4. first part $\div$ 2

5. 1st part $\times$ HX

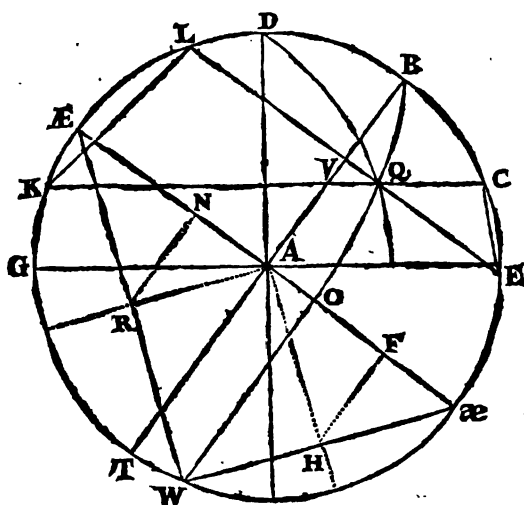
But

therefore, 5, 6, 7

1	CT : CS :: PV : PX.
2	CS : HG :: CI : HX.
3	CT $\times$ CS : CS $\times$ HG :: PV $\times$ CI : PX $\times$ HX.
4	CT : HG :: PV $\times$ CI : PXq.
5	CL : HN :: PV $\times$ CI : PXq.
6	CL : HN :: CL $\times$ HX : HN $\times$ HX = HMq.
7	CL $\times$ HX = OC $\times$ CK, by the Lemma above.
8	PV $\times$ CI : PXq :: OC $\times$ CK : HMq.

Which was to be demonstrated.

This



This Axiom may be otherwise demonstrated, thus:

The Sides of the Triangle DBQ being known, let the Angle B be required; the Triangles LQK, and CQE are like, because their correspondent Sides are parallel; the Angle Q is equal to GÆ, or the Side DB; the Angle K is equal to E, which is equal to half LDC, the Base and Difference of the Sides; KL is equal to the Base, less by LD the Difference of the Sides; KBE is the Sum of the Base and Sides; CE is the Difference between the Base DQ, and the Sum of the Sides DE; the Angle KCE is equal to half the Complement of KDE the Sum of the three Sides, and therefore the Sine of KCE is also the Sine of the half Sum of the three Sides KD, DB, and BE; and OÆ is the versed Sine of the Angle B, and QL the versed Sine of the same Arch in a lesser Circle; Oæ is the versed Sine of the Complement of B, and QE the versed Sine of the like Complement: And ÆR is equal to the Sine of half B; and Hæ is equal to the Cosine of half B. First in the Triangle QLK;

By

By plain Trigon.	1	$s.Q : s.K :: \frac{1}{2} KL : \frac{1}{2} QL$
And	2	$VL : A\bar{E} :: \frac{1}{2} QL : A\bar{N}$
From 1st and 2d, by mult. extremes and means, and re- ducing them to Proportion,	3	$s.Q \times VL : s.K \times \frac{1}{2} KL :: A\bar{E} : A\bar{N}$
By mult. the latter part of 3d by $A\bar{E}$,	4	$s.Q \times VL : s.K \times \frac{1}{2} KL :: A\bar{E}q : A\bar{N} \times A\bar{E}$
By Simil. Δs	5	$A\bar{E} : A\bar{R} :: A\bar{R} : A\bar{N}$
5 $\therefore$	6	$A\bar{E} \times A\bar{N} = A\bar{R}q$
From 4th, 6th	7	$s.Q \times VL : s.K \times \frac{1}{2} KL :: A\bar{E}q : A\bar{R}q$. Therefore,

As the Rectangle of the Sine of BD in VL (the Sine of BQ) the Sides comprehending the Angle B,

To the Square of $A\bar{E}$ Radius;

So is the Rectangle under the Sine of $\frac{1}{2} LC$, the Sum of the Base and Difference of the Sides, and the Sine of $\frac{1}{2} KL$, the Difference of the Base and Difference of the Sides,

To the Square of $A\bar{R}$ the Sine of half B, the Angle required.

Another Variety may be thus:

As the Rectangle under the Sines of the Sides comprehending the Angle required,

To the Square of Radius;

So is the Rectangle made of the Sine of the half Sum of the three Sides, and the Sine of half the Difference of the Base and the other Sides,

To the Square of the Cosine of half the required Angle.

In

In the Δ QCE	1	$sQ : sC :: \frac{1}{2} CE : \frac{1}{2} QE$
And	2	$VE : Az :: \frac{1}{2} QE : xF$
From 1 and 2, by mult. ex- tremes and means, and redu. to pro- portionals,	3	$sQ \times VE : sC \times \frac{1}{2} CE :: Az : xF$
By mult. the latter part of 3, by Az,	4	$sQ \times VE : sC \times \frac{1}{2} CE :: Azq : xFxAz$
From simil. Δ	5	$Az : xH :: xH : xF$
5 $\therefore$	6	$Az \times xF = xHq$
From 4, 6,	7	$sQ \times VE : sC \times \frac{1}{2} CE :: Azq : xHq$. That is,

As the Rectangle of the Sines of the comprehending Sides, sQ and VE ,
To the Square of Radius Az ;
So is the Rectangle of the Sines of the half Sum of the 3 sides KCE , and half Difference of the Base and other Sides $\frac{1}{2} CE$,
To the Square of xH , the Cosine of half the Angle B required.

This Axiom may be otherwise varied, thus.

As the Rectangle of the half Sum of the three Sides, and Sine of the Difference of the Base and half Sum of the Sides,
To the Square of Radius ;
So is the Rectangle made of the Sine of the half Sum of the Base and Difference of the Sides, and Sine of the Difference of the Base and Difference of the Sides,
To the Square of the Tangent of half the Angle required.

That

	1	Co-line : Sine :: Radius : Tan- gent.
That is,	2	$\alpha H : \text{ÆR} :: A\alpha : t \frac{1}{2} B$
The 2d Sq.	3	$\alpha Hq : \text{ÆR}q :: A\alpha q : t \frac{1}{2} Bq$
By the 7 step } Page 79.	4	$sQ \times VE :: sC \times \frac{1}{2} CE :: A\alpha q : \alpha Hq.$
By the 7 step } Page 78.	5	$sQ \times VL : sK \times \frac{1}{2} KL :: A\text{Æ}q : \text{ÆR}q.$
From 4, 5	6	$sC \times \frac{1}{2} CE : sK \times \frac{1}{2} KL :: \alpha Hq : \text{ÆR}q.$
From 2, 6	7	$sC \times \frac{1}{2} CE : sK \times \frac{1}{2} KL :: A\alpha q : t \frac{1}{2} Bq.$
And also	8	$sK \times \frac{1}{2} KL : sC \times CE :: A\alpha q : c t \frac{1}{2} Bq.$
		Hence it is,

As the Rectangle of the Sines of the half Sum of the three Sides, and of the Difference of the Base and half Sum of the Sides,

To the Square of Radius ;

So is the Rectangle made of the Sines of the half Sum of the Base and Difference of the Sides, and the Difference of the Base and Difference of the Sides,

To the Square of the Tangent of half the Angle sought.

Or it may be otherwise expressed, thus.

As the Rectangle of the Sine of the half Sum of the three Sides, and Sine of the Difference of the Base and half Sum,

To the Square of Radius ;

So is the Rectangle made of the Sines of the Difference of each containing Side and half Sum,

To the Square of the Tangent of half the Angle required.

Which is in effect the same as the former.

Thus

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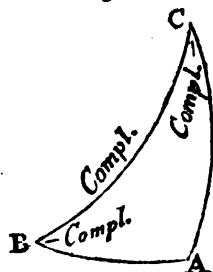
Thus have I sufficiently explained and demonstrated the Four Axioms, by which all the Cases in Spherical Triangles may be solved ; yet I shall recommend to you one universal Proposition, by which alone all the Cases (except when all the Sides are given to find an Angle, or all the Angles given to find a Side) are solved ; and which is as followeth.

The Universal Proposition, or Axiom, first invented by the Right Honourable John Lord Neper, Baron of Merchiston in Scotland.

S E C T. I.

IN a rectangled Spherical Triangle, there are (besides the right Angle) five other Parts ; whereof those three which are more remote from the right Angle, Lord *Neper* changed into their Complements.

As in this Triangle ABC, right angled at A ; for the three remote Parts, to wit, the Angles B and C, and the Side BC, he takes their Complements ; these three Complements, with the Sides AB and AC, do make five Parts, which he calleth Circular.



Viz. the { Side AB
Side AC
Complement of the Angle at B
Complement of the Angle at C
Complement of the Side BC.

But the right Angle at A is set aside from being any of the circular Parts.

S E C T. II.

Now in the resolution of a rectangled spherical Triangle there are two other Terms given (besides the right Angle) to find out a third.

S E C T. III.

These three Terms (namely the two that are given, and the third which is required) must be first looked upon according to their circular Parts.

S E C T. IV.

Of which one is named the middle (or mean) Part; the other two are called the extreme Parts; borrowing the Appellation from the situation of the Terms themselves: for, of three Terms, one must necessarily be in the middle, and the other two in the extremes; therefore the circular Part of the middle Term, is called the middle Part; and the circular Parts of the extreme Terms, are called the extreme Parts.

S E C T. V.

But the extreme Parts may be twofold, either Conjunct, or Disjunct: For those three Terms (besides the right Angle) do come in question according as the two Extremes are from either Part immediately joined to a third Mean, or disjoined from the same by a Side or Angle interposed on both sides; so are their circular Parts named Extremes Conjunct, or Extremes Disjunct.

S E C T. VI.

But of those three Terms which fall in question, we will subject all their varieties in their circular
Parts,

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Parts, according as every one of them ought (in respect of each other) to be called the middle Part, or the Extremes Conjunct or Disjunct; as in the following Analysis or Table is fully demonstrated.

If the	{	Side AB	be the middle	{	Side AC & Com. B	{	are Extremes
		Side AC			Side AB & Com. C		
		Compl. C			Side AC & Com. BC		
		Compl. B			Side AB & Com. BC		
		Com. BC			Com. B & Comp. C.		

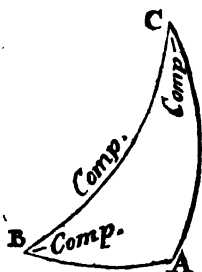
part, then the

And

{	Com. BC & Comp. C	}	Extremes
	Comp. BC & Com. B		
	Side AB & Com. B		
	Side AC & Com. C		
	Side AB & Side AC		

Disjunct.

But here it is to be noted, that the Sides AB and AC comprehending the right Angle A, are supposed to be joined together, because the right Angle is not reckoned amongst the circular Parts.



S E C T. VII.

Therefore in the Resolution of a right angled spherical Triangle, to know the mean and extreme Parts, you must observe, that,

1. If one of the three Terms (which besides the right Angle, comes in question) doth stand alone by it self, severed from the other two on both sides, (as the Side BC, from the Sides CA and BA, by the Angles B and C interposed) that shall be the middle Term, and so its circular Part shall be called the middle Part; and the other two circular Parts are the Extremes disjoined. But,

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2. If the three Terms do immediately adhere together, the middle Term doth easily shew the middle Part; and the extreme Terms the extreme Parts Conjunct.

These things being all rightly understood, the whole Trigonometry of Sphericals will be resolved by this one Proposition, which therefore we call Catholick or Universal.

S E C T. VIII.

The Universal Proposition.

The Sine of the middle Part, and Radius, are reciprocally proportional with the Tangents of the extreme Parts Conjunct, and with the Co-sines (or Sines Complements) of the Extremes Disjunct. That is,

As Radius

To the Tangent of one of the Extremes Conjunct;

So is the Tangent of the other Extreme Conjunct,

To the Sine of the middle Part; & *contra*.

Then also,

As Radius

To the Co-sine of one of the Extremes Disjunct;

So is the Co-sine of the other Extreme Disjunct,

To the Sine of the middle Part, & *contra*.

Corollary.

Wherefore, 1. If the middle Part be sought, the Radius shall be in the first place of the Proportion; but if one of the Extremes be sought, then the other Extreme shall be in the first place. Of the second and third places it mattereth nothing how they are disposed. Also,

2. If the Extremes in any Proportion be Disjunct from the middle Part, the Proportion will be performed

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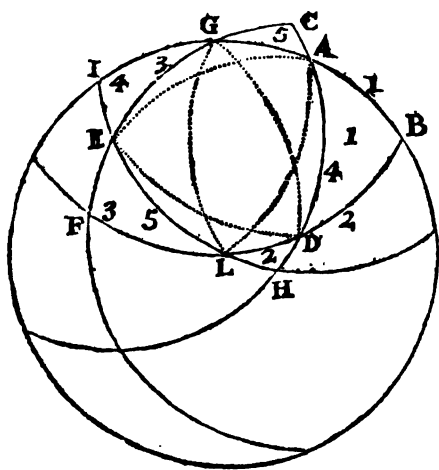
formed by Sines only: But if the Extremes be Con-
junct to the middle Part, it will be performed by Sines
and Tangents jointly.

Tho a particular Demonstration of this universal
Proposition may seem needless, because where the Ex-
tremes are Disjunct, the Proportion differs nothing
from the common ones; and in the Extremes Con-
junct, where it is commonly said,

As Radius
To the Tangent;
Here we say,
As the Co-tangent
Is to Radius:

And likewise inversely and contrarily, which is
plainly the same thing; because the Radius is a mean
proportional between the Tangent of an Arch, and
the Tangent Complement of the same Arch: Yet for
the benefit of the ingenious, I will give the following
Demonstration.

If five great Circles of the Sphere be so ordered,
that the first intersects the second, the second the
third, the third the fourth, the fourth the fifth, and



the fifth the first,
at right Angles;
the right angled
Triangles made
by their Inter-
sections, do all
consist of the
same circular
Parts.

Let G repre-
sent the Zenith,
A the North
Pole, and D the
Sun being in the
Horizon: So that
IGB is an Arch

G 3

of

of the Meridian of the Place, BDF an Arch of the Horizon, FEC an Arch of the Circle described about the Sun, CAH an Arch of the Meridian of the Sun, HLI an Arch of the Equinoctial. Then do these five Arches retain the Conditions required.

The first intersecting the second in B; the second the third in F; the third the fourth in C; the fourth the fifth in H; and the fifth the first in I; and these Intersections at B, F, C, H, I, are at right Angles: there I say the right angled Triangles made by the Intersections of these Circles, namely ABD, DHL, LFE, EIG, and GCA, do consist of the same circular Parts; as here appeareth.

The five circular Parts in the

Triangle	{	ABD, are AB, BD, com. BDA, com. AD, com. DAB.
		DHL, are com. HLD, com. LD, com. LDH, DH, HL.
		LFE, are com. ELF, LF, FE, com. FEL, com. EL.
		EIG, are IG, com. IGE, com. GE, com. GEL, EI.
		GCA, are com. GA, com. AGC, GC, CA, com. CAG.

Where you may observe, that to the Side AB in the first Triangle, is equal the Complement HLD in the second, or the Complement of ELF in the third, or IG in the Fourth, or the Complement of GA in the fifth, &c. The same uniformity of the circular Parts is also apparent in quadrantal Triangles.

As in the same Scheme G from D, D from E, E from A, A from L, and L from G, are distant by Arches each equal to a Quadrant. Here are therefore five quadrantal Triangles, as appears in the Scheme.

Of the five circular Parts in a spherical Triangle, right angled, or quadrantal.

THE Sine of the middle Part with Radius, is equal to the Tangents of the Extremes adjacent, or to the Co-sines of the Extremes disjunct.

PART I. Touching the first part of this Axiom, in right angled Triangles; the middle Part is either one of the Sides, or one of the oblique Angles, or the Hypothensuse.

CASE 1. Let the middle Part be a Side; as in the Triangle ABD, let AB be the middle Part, and BD, and the Complement of A, the Extremes adjacent: Then I say, that the Sine of AB the middle Part, with Radius, is equal to the Tangent of DB, with the Tangent of the Complement of DAB.

For (by *Axiom 2. Page 70, 71.*) as the Sine of AB is to Radius, so is the Tangent of DB to the Tangent of the Angle at A: Therefore also alternately, As the Sine of AB, to the Tangent of DB; so is Radius to the Tangent of A.

But Radius is a mean proportional between the Tangent of an Arch and the Tangent Complement of the Arch. So then it is, As Radius is to the Tangent of A, so is the Tangent Complement of A to Radius. Then it will be, As the Sine of AB to the Tangent of DB, so is the Tangent Complement of A to Radius.

CASE 2. Let the middle Part be an Angle, as in the Triangle DHL, let the Complement of HLD be the middle Part, and HL and Complement of LD the Extremes adjacent: Then I say, that the Sine Complement of HLD with Radius, is equal to the Tan-

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gent of HL, with the Tangent of the Complement of LD.

For the Complement of HLD is equal to AB, and the Complement of LD is DB; and HL is the Complement of DAB; and we proved before, that the Sine of AB with Radius, is equal to the Tangent of DB with the Co-tangent A: Therefore also the Co-sine of HLD with Radius, is equal to the Co-tangent of LD with the Tangent of HL.

C A S E 3. Let the middle Part be the Hypothenufe, as in the Triangle GCA, let the Complement of AG be the middle Part, and the Complement of AGC, and Complement of CAG, the Extremes adjacent.

Then also I say, that the Co-sine of AG with Radius is equal to the Co-tangent of AGC, with the Co-tangent of CAG.

For we have before proved, that the Sine of AB with Radius is equal to the Tangent of DB with the Co-tangent of A; but the Complement of AG is equal to AB, and the Complement of AGC is equal to DB, and the Complement of CAG equal to the Complement of DAB. Therefore also the Co-sine of AG with Radius, is equal to the Co-tangent of AGC, with the Co-tangent of CAG.

Therefore in a right angled Triangle, the Sine of the middle Part with Radius, is equal to the Tangents of the Extremes adjacent.

I say further, that,

P A R T II. The Sine of the middle part with Radius is equal to the Sines Complement of the opposite Extremes.

For here also the middle Part is either one of the Sides, or the Hypothenufe, or one of the oblique Angles.

C A S E

CASE 1. Let the middle part be a Side, as in the Triangle ABD, let DB be the middle part, and the Complement of AD, and Complement of A, the Extremes Disjunct. Then I say, that the Sine of BD with Radius, is equal to the Sine of AD with the Sine A.

For (by Axiom 1. Page 68.) As the Sine of AD to Radius, so is the Sine of DB to the Sine of A; therefore, the Sine of DB with Radius, is equal to the Sine of AD with the Sine of A.

CASE 2. Let the Hypotenuse be the middle Part, as in the Triangle DHL; let the Complement LD be the middle Part, and DH and HL, the Extremes Disjunct: Then I say, that the Co-sine of LD with Radius, is equal to the Co-sine of DH with the Co-sine of HL.

For the Complement of LD is equal to DB, and DH equal to the Complement of AD, and HL equal to the Complement of DAB: And it was proved before, that the Sine of DB with Radius, is equal to the Sine of AD with the Sine of DAB.

CASE 3. Let one of the oblique Angles be the middle Part, as in the Triangle ELG; let the Complement of IGE be the middle Part: then I say, that the Co-sine of IGE with Radius, is equal to the Sine of GEI with the Co-sine of EI. For the Complement of IGE is equal to DB; and GEI is equal to AD; and EI is equal to the Complement of DAB.

Therefore in a right angled Triangle, the Sines of the middle Part with Radius, are equal to the Sines Complement of the Extremes disjunct. Which was to be demonstrated.

But here Note, That when a Complement in any Proportion does chance to concur with a Complement
in

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Let AC (in the Triangle ABC) be $46^{\circ} 31'$, and C $60^{\circ} 00'$.

Then, As Radius to cs. C $60^{\circ} 00'$ 9.698970
So is t. AC $46^{\circ} 31'$ 10.023003

To t. of BC $27^{\circ} 48'$ 9.721973

By Scale and Compasses.

Extend the Compasses in the Line of Sines from 90° to 30° (the Comp. of C.) that extent will reach from $46^{\circ} 31'$ in the Line of Tangents, down to $27^{\circ} 48'$. Here the Line of Tangents is suppos'd to be continu'd forwards beyond 45° ; but if the Line of Tangents extend no further than 45° , and then be numbered downwards again, as most Lines of Tangents are, then that first extent must be applied downwards from 45° in the Line of Tangents, and the lower Leg of the Compasses kept there where it falls, till you pull in the other Leg to $46^{\circ} 31'$, numbred downwards from 45° , and that last extent will reach from 45° to $27^{\circ} 48'$, the Side sought.

By the Sliding-Rule.

Set Radius on SS to 30° on S, then against 45° on TT, observe what degree and minute on T; then move T downwards, till the same point stand against $46^{\circ} 31'$ on TT; and then against 45° on TT you have $27^{\circ} 48'$ on T. But this trouble and inconvenience may in some measure be removed, by having the Slider somewhat longer than they usually are, and the Tangent Line continued to the end thereof; for then you will have no more to do, but to set Radius on S to 30° on SS, then against $46^{\circ} 31'$ on T is $27^{\circ} 48'$ on TT.

The Side sought will be less than a Quadrant, if the Hypotenuse be less than a Quadrant, and the given Angle acute; or, if the Hypotenuse be greater than a Qua-

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a Quadrant, and the given Angle obtuse : But it will be greater than a Quadrant, if the Hypotenuse be less than a Quadrant, and the given Angle obtuse; or if the Hypotenuse be greater than a Quadrant, and the given Angle acute.

C A S E III.

The Hypotenuse AC, and the Angle A given; to find the other oblique Angle C, an Extreme Conjunct.

Ass. $CG : s. CI :: t. HG : t. IF$ (by Ax. 2.)

That is, As $cs. AC : Rad. :: ct. A : t. C$.

But by the universal Proposition, the Comp. of AC is the middle Part, and the Complement of A and C the Extremes Conjunct; therefore it will be,

As $ct. A : cs. AC :: Radius : ct. C$.

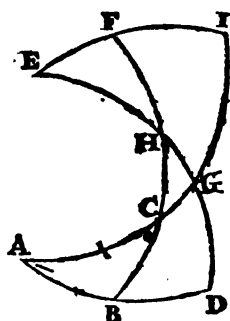
Let AC be $46^{\circ} 31'$ (as before) and A $40^{\circ} 00'$.

Then, as $ct. A 40^{\circ} 00'$

To $cs. AC 46^{\circ} 31'$

So is Radius

To $ct. C 60^{\circ} 00'$



10.076186

9.837679

10.

9.761493

By Scale and Compasses.

Extend the Compasses from $43^{\circ} 29'$ (the Comp. of AC) to $90'$ in the Line of Sines, that extent will reach from $50^{\circ} 00'$ (the Comp. of A) to 60 in the Line of Tangents.

By

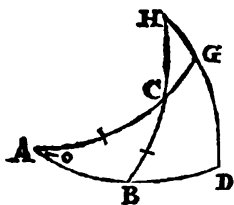
By the Sliding Rule.

Set $43^{\circ} 29'$ on S to 90 on SS, then against 50° on TT, is 60 on T.

The Angle found will be acute, if the Hypothenuſe be leſs than a Quadrant, and the given Angle acute; or if the Hypothenuſe be greater than a Quadrant, and the given Angle obtuſe: and the ſaid Angle will be obtuſe, if the Hypothenuſe be leſs than a Quadrant, and the given Angle obtuſe; or if the Hypothenuſe be greater than a Quadrant, and the given Angle acute.

C A S E IV.

The Hypothenuſe AC, and the Side BC given; to find the Angle A oppoſite to the Side given, an Extreme Diſjunct.



$$\text{As } s. AC : s. AG :: s. BC : s. DG$$

(by Ax. 1.)

$$\text{That is, As } s. AC : R :: s. BC : s. A.$$

But by the univerſal Propoſition, BC being the middle Part, and the Complements of AC, and the Angle A Extremes

Diſjunct, therefore it will be the ſame as before.

Let AC be $46^{\circ} 31'$, and BC $27^{\circ} 48'$; then,

As $s. AC \ 46^{\circ} 31'$

To Radius

So is $s. BC \ 27^{\circ} 48'$

To $s. A \ 40^{\circ} 00'$

9.86068

10.

9.66875

9.80807

By

By Scale and Compasses.

Extend the Compasses from $46^{\circ} 31'$ to Radius, or 90° in the Sines, that extent will reach from $27^{\circ} 48'$ to 40° .

By the Sliding Rule.

Set $46^{\circ} 31'$ on S to 90 on SS, then against $27^{\circ} 48'$ on S, is $40^{\circ} 00'$ on SS.

The Angle found will be acute, if the Side given be less than a Quadrant; obtuse, if greater.

C A S E . V.

The Hypotenuse AC, and the Side BC given; to find the adjacent Angle C, the middle Part.

As t. IG : s. EI :: t. FH : s. EF
(Ax. 2.)

That is, As t. AC : R :: t. BC : cs. C.

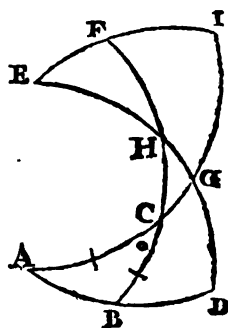
But by the universal Proposition, the Complement of the Angle C is the middle Part; and the Complement of the Hypotenuse AC, and the Side BC are Extremes Conjunct; therefore it will be,

R : ct. AC :: t. BC : cs. C.

Let AC be $46^{\circ} 31'$, and BC $27^{\circ} 48'$: then

As t. AC $46^{\circ} 31'$
To Radius
So is t. of BC $27^{\circ} 48'$

To cs. C $60^{\circ} 00'$



10.023003
10.
9.722008
9.699005
By

By Scale and Compasses.

Extend the Compasses in the Line of Tangents from 45° to $27^{\circ} 48'$, and apply that extent from $46^{\circ} 31'$ downwards; keep the lower Leg of the Compasses fixt till you extend the other to 45, this last extent will reach from 90 in the Sines to 30° , the Complement of C. But if you have a Line of Tangents with the Degrees continued beyond 45, you need but extend the Compasses from $46^{\circ} 31'$ to $27^{\circ} 48'$ in the Tangents, that extent will reach from 90 in the Sines to 30° .

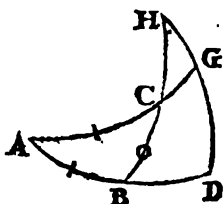
By the Sliding Rule.

Set $27^{\circ} 48'$ on T to 45 on TT, then against $46^{\circ} 31'$ on TT, observe the Degree and Minute on T, and bring that Point to 45 on TT; and against 90 on SS, you have 30° on S. But if your Slider have the Degrees continued beyond 45, you may set $46^{\circ} 31'$ on T to $27^{\circ} 48'$ on TT; then against 90 on S, is 30° on SS.

The Angle found will be acute, if both the Hypo-
thense, and the given Side be greater, or less than a
Quadrant; but obtuse, if one of them be greater, and
the other less.

CASE VI.

The Hypotenuse AC, and the Side AB given; to find the other Side BC, an Extreme Disjunct.



As s. BD: s. CG :: s. BH: s.
CH (by Ax. 1.)

That is, As cs. AB : cs. AC ::
R : cs. BC.

But by the universal Proposition, the Complement of AC is the middle Part, and AB and BC are Extremes Disjunct; therefore

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therefore it will be the same as above.

Let AC be $46^{\circ} 31'$, and AB $38^{\circ} 56'$; then

As cs. AB $38^{\circ} 56'$ 9.890911

To cs. AC $46^{\circ} 31'$ 9.837679

So is Radius 10.

To cs. BC $27^{\circ} 48'$ 9.946768

By Scale and Compasses.

Extend the Compasses from $51^{\circ} 04'$ (the Comp. AB) to $43^{\circ} 29'$ (the Comp. of AC) that extent will reach from 90 to $62^{\circ} 12'$ in the Line of Sines.

By the Sliding Rule.

Set $51^{\circ} 04'$ on S, to 90 on SS; then against $43^{\circ} 29'$ on S, is $62^{\circ} 12'$ on SS, whose Complement is $27^{\circ} 48'$, the Side BC sought.

The Side sought will be less than a Quadrant, if both the Hypothenuse and given Side be less than a Quadrant, but greater than a Quadrant, if either the Hypothenuse be greater, and the given Side less; or if the Hypothenuse be less, and the given Side greater.

C A S E V H.

The Side BC, and the Angle opposite thereunto A being given; to find the other Side AB, the middle part.

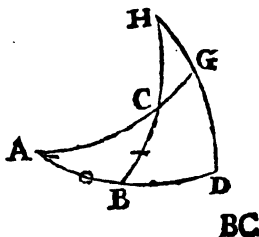
As t.DG : t.BC :: s. AD : s. AB
(by Ax. 2.)

That is, t. A : t. BC :: R. : s.

AB.

But by the universal Proposition, AB is the middle part, and the Complement of A, and Side

H



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BC are Extremes Conjunct; therefore it will be,
Rad. : ct. A :: t. BC : s. AB.

Let the Angle A be $40^{\circ} 00'$, and the Side BC $27^{\circ} 48'$; then

As Radius, to ct. A $40^{\circ} 00'$	10.076186
So is t. BC $27^{\circ} 48'$	9.722008

To s. AB $38^{\circ} 56'$	9.798194
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By Scale and Compasses.

Extend the Compasses from 40° to $27^{\circ} 48'$ in the Tangents, that extent will reach from 90 to $38^{\circ} 56'$ in the Line of Sines.

By the Sliding Rule.

Set 40° on TT, to $27^{\circ} 48'$ on T; then against 90 on SS is $38^{\circ} 56'$ on S.

The Side found will be less than a Quadrant, if the Angle opposite thereto, and not given, be acute, but greater if it be obtuse: In like manner it will be less, if the Hypotenuse be less than a Quadrant, and the Side given also less than a Quadrant: Or if the Hypotenuse be less than a Quadrant, and the given Side greater, the Side found will be greater than a Quadrant: Lastly, if both the Hypotenuse and Side given be greater than a Quadrant, the Side found will be less than a Quadrant, but greater if the Hypotenuse be greater than a Quadrant, and the given Side less.

CASE

C A S E VIII.

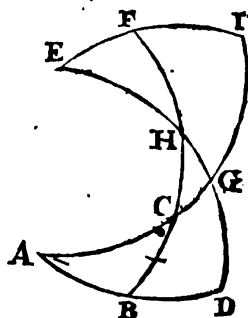
The Side BC, and the Angle opposite thereunto A, being given; to find the other oblique Angle at C, Extreme Disjunct.

Ass. $CH : s.CF :: s.GH : s.IF$

(Ax. 1.)

That is, $cs. BC : R. :: cs. A : s.C.$

But by the universal Proposition, the Comp. of A is the middle Part; and BC, and the Complement of the enquired Angle C, are Extremes Disjunct; therefore the Analogy will be the same as above.



Let the side BC be $27^{\circ} 48'$, and the Angle A $40^{\circ} 00'$.

As $cs. BC \ 27^{\circ} 48'$	9.946737
To Radius	10.
So is $cs. A \ 40^{\circ} 00'$	9.884254
To $s. C \ 60^{\circ} 00'$	9.937517

By Scale and Compasses.

Extend the Compasses from $62^{\circ} 12'$ (the Comp. of $27^{\circ} 48'$) to Rad. or 90 in the Sines, that extent will reach from 50° (the Comp. of 40°) to 60 in the same Line.

By the Sliding Rule.

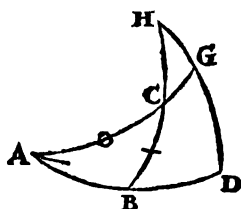
Set $62^{\circ} 12'$ on S, to 90 on SS, and against 50° on S is 60° on SS.

The Angle found will be acute, if the Side not given be less than a Quadrant; obtuse if greater: in like manner,

manner, if the Hypothenuſe be leſs than a Quadrant, and the given Angle acute. Or if the Hypothenuſe be greater than a Quadrant, and the given Angle obtuſe, the Angle found will be acute: But if the Hypothenuſe be leſs than a Quadrant, and the given Angle obtuſe, or if the Hypothenuſe be greater than a Quadrant, and the given Angle acute, the Angle found will be obtuſe.

C A S E IX.

The Side BC, and Angle opposite thereto A given; to find the Hypothenuſe AC, Extreme Diſjunct.



As the s. DG : s. BC :: s.

AG : s. AC (*Ax. 1.*)

That is, As s. A : s. BC ::
Rad. : s. AC.

But by the universal Proposition, BC is the middle Part, and the Complement of A, and the Complement of AC

the Extremes Diſjunct; therefore the Analogy will be the ſame as above.

Let BC be $27^{\circ} 48'$ (as before) and the Angle A $40^{\circ} 00'$; then,

As the s. A $40^{\circ} 00'$

To the s. BC $27^{\circ} 48'$

So is Radius

9.808067

9.668746

10.

To the s. AC $46^{\circ} 31'$

9.860679

By Scale and Compasses.

Extend the Compasses in the Line of Sines from 40° to $27^{\circ} 48'$, that extent will reach from 90 to $46^{\circ} 31'$.

By

By Scale and Compasses.

Extend the Compasses from 90 to $38^{\circ} 56'$ in the Sines, that extent will reach from 40° to $27^{\circ} 48'$ in the Tangents.

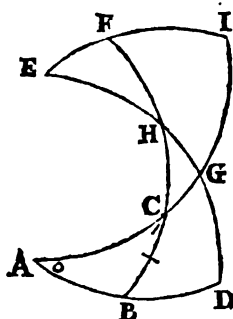
By the Sliding-Rule.

Set 90 on SS , to $38^{\circ} 56'$ on S ; then against 40 on TT , is $27^{\circ} 48'$ on T .

The Side found will be less than a Quadrant, if the given Angle opposite thereto be acute, but greater if obtuse.

C A S E XI.

The Side BC, and the adjacent Angle C given; to find the other oblique Angle A, middle Part.



As the $s. FC : s. HC :: s. IF :$
 $s. HG. (Ax. 1.)$

That is, As $Rad. : cs. BC ::$
 $s. C : cs. A.$

But by the universal Proposition, A is the middle Part, and the Side BC , and the Complement of the Angle C , are Extremes Disjunct; and the Analogy will be the same as above.

Let the Side BC be $27^{\circ} 48'$, and the Angle C $60^{\circ} 00'$: then,

As Radius

Is to $cs. BC$ $27^{\circ} 48'$

So is $s. C$ $60^{\circ} 00'$

To the $cs. A$ $40^{\circ} 00'$

10.

9.946737

9.937530

9.884267

By

By Scale and Compasses.

Extend the Compasses from 90 to 62° 12', that extent will reach from 60° to 50, the Comp. of A. in the Line of Sines.

By the Sliding Rule:

Set 90 on SS, to 62° 12' (the Compl. of 27° 48') on S; then against 60 on SS, is 50° on S.

The Angle found will be acute, if the Side given be less than a Quadrant, obtuse if greater.

C A S E XII.

The Side AB, and the adjacent Angle A given, to find the Hypotenuse AC, Extreme Conjunct.

As the s. HD : s. HG :: t.

BD : t. CG (Ax. 2.)

That is, As Rad. : cs. A :: ct.

AB : ct. AC.

But by the universal Proposition, the Complement of A is the middle Part; and the Side AB, and the Complement of

AC, are Extremes Conjunct: therefore it will be,

As t. AB : Rad. :: cs. A : ct. AC.

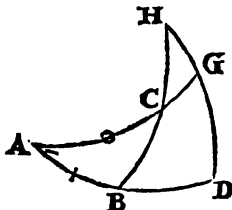
Let AB be 38° 56', and the Angle A 40° 00'; then,

As t. AB 38° 56'

To Radius

So is cs. A 40° 00'

To ct. AC 46° 31'



9.907336

10.

9.884254

9.976918

By Scale and Compasses.

Extend the Compasses from 90° to 50° (the Compl. of A) in the Sines; that extent will reach from 45 (in the Tangents) to $52^\circ \frac{1}{2}$: keep that foot of the Compasses fixt, and pull the other into $51^\circ 4'$ (the Compl. AB) this last extent will reach from 45° to $46^\circ 31'$.

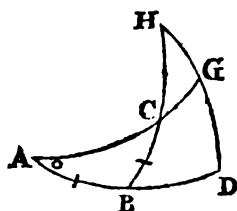
By the Sliding Rule.

Set 90° on S, to 50° on SS; then against $51^\circ 4'$ on T, is $43^\circ 29'$ on TT.

The Hypotenuse will be less than a Quadrant, if the given Side be less than a Quadrant, and the Angle given adjacent thereto be acute; as also if the given Side be greater than a Quadrant, and the given adjacent Angle obtuse. And the contrary.

C A S E XIII.

The two Sides AB, and BC given; to find either of the oblique Angles, as A, Extreme Conjunct.



As the s. AB : s. AD :: t.

BC : t. DG. (Ax. 2.)

That is, As s. AB : Rad. :: t. BC : t. A.

But, by the universal Proposition, the Side AB is the middle Part, and the Side BC, and Complement of A, the

Extremes Conjunct; then it will be,

As t. BC : s. AB :: Rad. : ct. A.

Let AB be $38^\circ 56'$, and BC $27^\circ 48'$: then,

As s. AB $38^\circ 56'$

To Radius

So t. BC $27^\circ 48'$

To t. A $40^\circ 00'$

9.798247

10.

9.722008

9.923761

By

By Scale and Compasses.

Extend the Compasses from $38^{\circ} 56'$ to 90 in the Sines; that extent will reach from $27^{\circ} 48'$ to 40° in the Tangents.

By the Sliding Rule.

Set $38^{\circ} 56'$ on S to 90 on SS, then against $27^{\circ} 48'$ on T, is 40 on TT.

The Angle found will be acute, if the Side opposite to the Angle sought be less than a Quadrant, but, obtuse, if greater.

C A S E XIV.

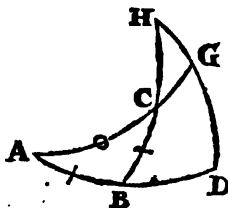
The two Sides AB, and BC given; to find the Hypothenuse AC, the middle Part.

As the s. BH : s. CH :: s. BD : s. CG. (Ax. 1.)

That is, As Rad. : cs. BC ::

cs. AB : cs. AC.

But by the universal Proposition, the Hypothenuse AC is the middle Part, and the Sides AB and BC are Extremes Disjunct; therefore the Analogy will be the same as above.



Let AB be $38^{\circ} 56'$, and BC $27^{\circ} 48'$; then,

As Radius

To cs. BC $27^{\circ} 48'$

So cs. AB $38^{\circ} 56'$

To cs. AC $46^{\circ} 31'$

10.

9.946737

9.890911

9.837648

By

By Scale and Compasses.

Extend the Compasses from 90 to $62^{\circ} 12'$ (the Compl. of BC) the extent will reach from $51^{\circ} 4'$ (the Compl. of AB) to $43^{\circ} 29'$ the Compl. of AC.

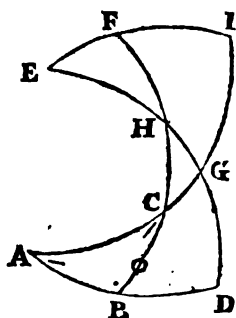
By the Sliding Rule.

Set 90 on SS, to $62^{\circ} 12'$ on S; and against $51^{\circ} 4'$ on SS, is $43^{\circ} 29'$ on S.

The Hypothensuse found will be less than a Quadrant, if both the Sides are less or greater; but otherwise it will be greater, if one be less, and the other greater.

C A S E X V.

The two Angles A and C given; to find either of the Sides, viz. BC, Extreme Disjunct.



As the s. IF : s. GH :: s. CF : s. CH (Ax. 1.)

That is, As s. C : cs. A :: Rad. : cs. BC.

But by the universal Proposition, the Compl. of the Angle A is the middle Part; and the Compl. of the Angle C, and the Side BC Extremes Disjunct: therefore the Analogy will be the same as above.

Let the Angle A be $40^{\circ} 00'$, and C $60^{\circ} 00'$; then,

As s. C $60^{\circ} 00'$

To cs. A $40^{\circ} 00'$

So is Radius

9.937530

9.884254

10.

To the cs. BC $27^{\circ} 48'$

9.946724

By

By Scale and Compasses.

Extend the Compasses from 60° to 50° (the Compl. of A) that extent will reach from 90 to $62^\circ 12'$, the Compl. of BC.

By the Sliding Rule.

Set 60° on SS, to 50° on S; then against 90 on SS, is $62^\circ 12'$ on S.

The Side found will be less than a Quadrant, if the given Angle opposite thereto be acute, but greater if obtuse.

C A S E XVI.

The two oblique Angles A and C given; to find the Hypothenufe AC, the middle Part.

As the t. IF : t. GH :: s. CI :
s. CG. (Ax. 2.)

That is, As t. C : ct. A :: Rad. :
cs. AC.

But by the universal Proposition, the Complement of the Hypothenufe AC is the middle Part, and the Complements of the Angles A and C, are Extremes Conjunct; therefore it will be,

As Rad. : ct. A :: ct. C : cs. AC.

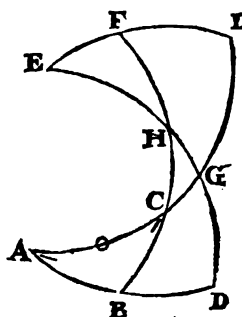
Let A be 40° and C 60° , as before; then,

As Radius

To ct. A $40^\circ 00'$

So is ct. C $60^\circ 00'$

To cs. AC $46^\circ 31'$



10.

10.076186

9.761439

9.837625

By

By Scale and Compasses.

Extend the Compasses from 60 to 50 in the Tangents, that extent will reach from 90 to $43^{\circ} 29'$ in the Sines.

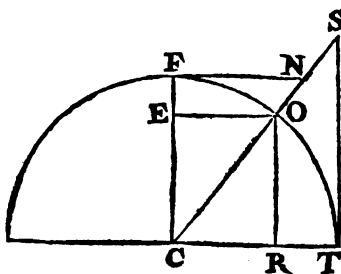
By the Sliding Rule.

Set 60 on T to 50 on TT; then against 90 on SS, is $43^{\circ} 29'$ on S, the Complement of AC.

The Hypothenufe found will be less than a Quadrant, if both the oblique Angles be acute or obtuse, but greater if one of them be acute, and the other obtuse.

To vary Proportions.

The chief Grounds for varying Proportions are built upon the following Theorems.



1. **T**HE Radius is a mean Proportional between the Tangent of an Arch, and the Tangent of its Complement; that is,
As the Tangent TS to Radius CT,
So is Radius CF to the Co-tangent FN.

2. The Radius is a mean Proportional between the Sine of an Arch, and the Secant of the same Arch's Complement: that is,
As the Sine OR to Radius OC, so is Radius CF to the Secant of the Arch's Complement CN.

3. The

3. The Rectangles of all Tangents, and their Complements, being respectively equal to the Square of Radius, are reciprocally Proportional. That is,

As the Tangent of an Arch or Angle, is to the Tangent of another Arch or Angle; so is the Tangent of the Complement of the latter Arch, to the Tangent of the Complement of the former.

And by varying the second Term into the place of the Third, we may compare the Tangent of one Arch to the Co-tangent of another, &c. That is,

As the Tangent of an Arch or Angle, is to the Co-tangent of another Arch; so is the Tangent of the latter Arch, to the Co-tangent of the former.

4. The Sines of the Arches, and the Secants of their Complements, are reciprocally proportional; that is,

As the Sine of an Arch, to the Sine of another Arch; so is the Co-secant of the latter Arch, to the Co-secant of the former.

And by changing the second and third Terms, a Sine may be compared with a Secant.

In any Proportion, if the two first Terms be,

As the Tangent of an Arch to Radius; to bring the Radius into the first place, it may be said, As the Radius is to the Co-tangent of that Arch, because there is the same Proportion between these two latter Terms, as between the two former. Now in all the Theorems, the two latter Terms consist either of the Parts, or of the Complements of the Parts of the two former: hence it will not be difficult to vary any Proportion propounded.

1. From hence it will follow, that a Proportion wholly in Tangents may be changed into their Complements, without altering the order of the Terms; and the Converse.

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If it were, As Tangent 10° to Tangent 20° ; so Tangent 52° to t. $69^\circ 15'$.

It would also be, As t. 80° to t. 70° ; so t. 38 to t. $20^\circ 45'$.

That if the two latter Terms of any Proportion being Tangents are only changed into their Complements, it infers a Transportation of the first Term into the second Place.

As in the Example above, As t. 20° to t. 10° ; so t. 38° to Tangent $20^\circ 45'$.

3. If the two former Terms of a Proportion being Tangents, are changed into their Complements, it likewise infers a change of the third Term into the place of the fourth.

And then if the fourth Term be sought, it will hold: As the second Term to the first, so is the third Term (as at first propounded) to the fourth.

In the first Example: As t. 70° : t. 80° :: t. 52° : t. $69^\circ 15'$.

4. A Proportion wholly in Secants, may be changed into a Proportion wholly in Sines, without altering the order of the Places, only by taking their Complements; and the Converse.

If it were, As se. 80° : se. 70° :: se. 60° : se. 10° .

It would also hold in Sines, As s. 10° : s. 20° :: s. 30° : s. 80° .

5. If the two latter Terms being Secants, should be changed into Sines, and the Converse, if they were Sines to be turned into Secants, it would be done only by taking their Complements; but then must the second and first Terms change Places one with another.

IF

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If the Proportion were, As s. 12° : s. 42° :: sc. 36° : sc. $75^{\circ} 26'$.

It would also hold, As s. 42° : s. 12° :: s. 54° : s. $14^{\circ} 34'$.

6. That if the two former Terms of a Proportion in Secants, should be changed into Sines, and the Converse; this would infer a changing of the fourth Term of that Proportion into the place of the third: But the third Term not being that which is sought, the Rule to do it, would be to imagine the two first Terms to change places, and then to take their Complements.

If the Proportion were, As sc. 36° : sc. $75^{\circ} 26'$:: s. 12° : s. 42° .

It would also hold, As s. $14^{\circ} 34'$: s. 54° :: s. 12° : s. 42° .

7. Two Terms, whether the former or latter, in any Proportion, being as a Sine to Tangent, may be varied.

For, as the Tangent of an Arch, to the Sine of another Arch; so is the Co-secant of the latter Arch, to the Co-tangent of the former.

And by transposing the order of the Terms,

As a Sine, to a Tangent; so is the Co-tangent of the latter Arch, to the Co-secant of the former.

8. Lastly, Observe that if four Terms or Numbers are Proportional, their Order may be so transposed, that each of those Terms may be the last in Proportion: and so of any four Proportional Terms, if three be given, the other that is wanting may be found.

Thus,

As First to Second, so Third to Fourth.

As Second to First, so Fourth to Third,

As Third to Fourth, so is First to Second.

As Fourth to Third, so is Second to First.

The

The Rules being well understood, it will be easy to vary each of the sixteen Cases of right angled spherical Triangles six several ways, as I shall plainly shew, in the Pages next following.

We will begin with the First Case, and shew all its Variations.

The Proportion you will find to be (as in Case I. Page 90.)

1. As Radius : s. AC :: s. A : s. BC.

But we may say,

2. As fe. c. AC : Rad. :: s. A : s. BC.

Because (by the 2d Theorem, Pag. 108.) As Radius to s. AC, so is fe. c. AC to Radius.

Again we may say,

3. As fe. c. A : Rad. :: s. AC : s. BC.

Because, As Radius to s. A ; so is fe. c. A to Radius.

Again,

4. It may be, As s. A : fe. c. AC :: Rad. : fe. c. BC.

Because, As Rad. : s. BC :: fe. c. of BC : Radius.
Then by inverting the Proportion in the second, and two last Terms of this,

Again it may be said,

5. As Radius : fe. c. A :: fe. c. AC : fe. c. BC.

By changing the Sines in the two last Terms in the third Proportion into Secant Complements, and transposing the two first Terms ;

Again we may say,

6. As s. AC : Rad. :: fe. c. A : fe. c. BC.

By changing the Sines in the two last Terms of the first Proportion into Secant Complements, and transposing the two first Terms.

Thus have you the Proportions varied six ways, which are all the ways it can be varied.

We will next shew how to vary the Proportions in the Second Case, which is in Sines and Tangents.

The

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The first Proportion by the universal Proposition is, As ct. AC : Rad. :: cs. C : t. BC.

But we may say,

2. As Rad. : cs. C :: t. AC : t. BC.

Because, as the ct. CA : Rad. :: Rad. : t. AC.

Again it may be,

3. As cs. C : ct. AC :: Rad. : ct. BC.

Because as Rad. : t. BC :: ct. BC : Radius. But because BC is required, therefore the Co-tangent of AC and the Co-sine of C in the first Proportion, must change places, and also the Co-tangent of BC and Radius in this.

Again we may say,

4. As se. C : Rad. :: t. AC : t. BC.

Because, from the second Proportion, as Radius to the Co-sine of C, so is the Secant of C to Radius.

Again it may be said,

5. As t. AC : se. C :: Rad. : ct. BC. For, From the two first Terms in the third Proportion, as the cs. C : ct. AC :: t. AC : se. C.

Again it may be said,

6. As Rad. : ct. AC :: se. C : ct. BC.

Because, as t. AC : Rad. :: Rad. : ct. AC in the fifth Proportion.

From what has been said in these two Examples, the Rules for varying Proportions being well understood, it will not be difficult to vary any Proportion which can offer it self. Therefore I shall say no more about it, only set down all the Variations of the other Cases.

1	As ct. A : cs. AC :: Rad. : ct. C
2	As R. : t. A :: cs. AC : ct. C
3	As fe. AC : R. :: t. A : ct. C
4	As t. A : fe. AC :: R. : t. C
5	As R. : ct. A :: fe. AC : t. C
6	As cs. AC : R. :: ct. A : t. C

Case 3.

1	As s. AC : R :: s. BC : s. A
2	As R. : fe.c. AC :: s. BC : s. A
3	As fe.c. BC : R. :: fe.c. AC : s. A
4	As fe.c. AC : fe. c. BC :: R. : fe.c. A
5	As s. BC : s. AC :: R. : fe.c. A
6	As R. : fe.c. BC :: s. AC : fe.c. A

Case 4.

1	As t. AC : R :: t. BC : cs. C
2	As R. : ct. AC :: t. BC : cs. C
3	As ct. BC : R. :: ct. AC : cs. C
4	As ct. AC : ct. BC :: R : fe. C
5	As R. : t. AC :: ct. BC : fe. C
6	As t. BC : R. :: t. AC : fe. C

Case 5.

1	As cs. AB : cs. AC :: R : cs. BC
2	As R. : fe. AB :: cs. AC : cs. BC
3	As fe. AC : R. :: fe. AB : cs. BC
4	As cs. AC : R. :: cs. AB : fe. BC
5	As R. : fe. AC :: cs. AB : fe. BC
6	As fe. AB : R. :: fe. AC : fe. BC

Case 6.

1	As R. : ct. A :: t. BC : s. AB
2	As t. A : R. :: t. BC : s. AB
3	As ct. BC : R. :: ct. A : s. AB
4	As t. BC : t. A :: R. : fe. c. AB
5	As R. : ct. BC :: t. A : fe. c. AB
6	As ct. A : R. :: ct. BC : fe. c. AB

Case 7.

1	$As\ ct. BC : R. :: cs. A : s. C$	} Case 8.
2	$As R. : s. BC :: cs. A : s. C$	
3	$As\ se. A : R. :: se. BC : s. C$	
4	$As\ se. BC : se. A :: R. c. se. C$	
5	$As R. : cs. BC :: se. A : se. c. C$	
6	$As\ cs. A : R. :: cs. BC : se. c. C$	

1	$As\ s. A : s. BC :: R. : s. AC$	} Case 9.
2	$As R. : se. c. A :: s. BC : s. AC$	
3	$As\ se. c. BC : R. :: se. c. A : s. AC$	
4	$As\ s. BC : s. A :: R. : se. c. AC$	
5	$As R. : se. c. BC :: s. A : se. c. AC$	
6	$As\ se. c. A : R. :: se. c. BC : se. c. AC$	

1	$As\ ct. A : R. :: s. AB : t. BC$	} Case 10.
2	$As R. : t. A :: s. AB : t. BC$	
3	$As\ se. c. AB : R. :: t. A : t. BC$	
4	$As\ s. AB : ct. A :: R. : ct. BC$	
5	$As\ t. A : se. c. AB :: R. : ct. BC$	
6	$As R. : ct. A :: se. c. AB : ct. BC$	

1	$As R. : s. C :: cs. BC : cs. A$	} Case 11.
2	$As\ se. c. C : R. :: cs. BC : cs. A$	
3	$As\ se. BC : R. :: s. C : cs. A$	
4	$As\ cs. BC : R. :: se. c. C : se. A$	
5	$As R. : se. BC :: se. c. C : se. A$	
6	$As\ s. C : R. :: se. BC : se. A$	

1	$As\ t. AB : R. :: cs. A : ct. AC$	} Case 12.
2	$As R. : ct. AB :: cs. A : ct. AC$	
3	$As\ se. A : R. :: ct. AB : ct. AC$	
4	$As\ cs. A : R. :: t. AB : t. AC$	
5	$As\ ct. AB : R. :: se. A : t. AC$	
6	$As R. : t. AB :: se. A : t. AC$	

1	As s. AB : R. :: t. BC : t. A	} Case 13
2	As t. BC : s. AB :: R. : ct. A	
3	As R. : ct. BC :: s. AB : ct. A	
4	As R. : fe.c. AB :: t. BC : t. A	
5	As ct. BC : R. :: fe.c. AB : t. A	
6	As fe. c. AB : ct. BC :: R. : ct. A	

1	As R. : cs. BC :: cs. AB : cs. AC	} Case 14.
2	As fe. BC : R. :: cs. AB : cs. AC	
3	As fe. AB : R. :: cs. BC : cs. AC	
4	As cs. AB : fe. BC :: R. : fe. AC	
5	As R. : fe. AB :: fe. BC : fe. AC	
6	As cs. BC : R. :: fe. AB : fe. AC	

1	As s. C : cs. A :: R. : cs. BC	} Case 15.
2	As R. : fe.c. C :: cs. A : cs. BC	
3	As fe. A : R. :: fe.c. C : cs. BC	
4	As cs. A : s. C :: R. : fe. BC	
5	As R. : fe. A :: s. C : fe. BC	
6	As fe.c. C : R. :: fe. A : fe. BC	

1	As R. : ct. C :: ct. A : cs. AC	} Case 16.
2	As t. C : R. :: ct. A : cs. AC	
3	As t. A : R. :: ct. C : cs. AC	
4	As ct. A : R. :: t. C : fe. AC	
5	As R. : t. A :: t. C : fe. AC	
6	As ct. C : R. :: t. A : fe. AC	

Thus have I gone thro all the Variations of the sixteen Cases of right angled spherical Triangles: I shall proceed to the Resolution of oblique angled spherical Triangles, in the next Chapter, with the Variations of their Proportions.

C H A P. IX.

The Solution of oblique angled Spherical Triangles.

IN oblique angled spherical Triangles; if the things given and required be opposite, they are solved by the third Axiom, as in the two following Cases,

C A S E I.

Two Sides, and an Angle opposite to one of them, being given, to find the Angle opposite to the other,

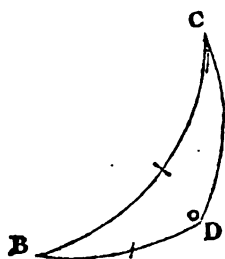
Let the Sides $BD\ 18^{\circ}\ 47'$, and $BC\ 42^{\circ}\ 51'$, and the Angle $C\ 23^{\circ}\ 30'$, be given; to find the Angle D , opposite to BC .

Ass. $BD\ 18^{\circ}\ 47'$ A.C. 0.4921572

To s. $C\ 23^{\circ}\ 30'$ 9.6006997

So is s. $CB\ 42^{\circ}\ 51'$ 9.8325609

To s. $D\ 57^{\circ}\ 22'$ 9.9254178



Here it may be doubtful whether this Angle D be obtuse or acute, that is, whether it be $57^{\circ}\ 22'$, or its Complement to 180° , that is $122^{\circ}\ 38'$, which in this Example it is supposed to be.

In some Cases the Affection of the Angle sought cannot be determined from what is given.

Such Cases are, when the given Angle is acute, and the opposite Side less than a Quadrant, and the adjacent or other Side greater than the opposite Side, and its Complement to a Semicircle also greater than the opposite;

I 3

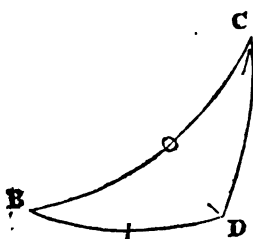
Also

Also when the given Angle is obtuse, and the opposite Side greater than a Quadrant, and also greater than the other Side, and greater than the Complement of the said other side to a Semicircle.

C A S E II.

Two Angles and a Side opposite to one of them being given, to find the Side opposite to the other.

Let the two Angles C $23^{\circ} 30'$, and D $122^{\circ} 38'$, and the Side BD $18^{\circ} 47'$ be given; to find the Side CB.



Ass. C $23^{\circ} 30'$	A.C.	0.3993003
To s. BD $18^{\circ} 47'$		9.3078428
So s. D $122^{\circ} 38'$		9.9253037
($57^{\circ} 22'$)		

To s. CB $42^{\circ} 51'$ 9.8324468

In some Cases the Affection of the Side sought cannot be determin'd from what is given.

Such Cases are, when the given Angle is acute, and the opposite Side less than a Quadrant, and the other Angle greater than the former Angle, and its Complement to a Semicircle also greater than the said Angle:

Also when the given Angle is obtuse, and the opposite Side greater than a Quadrant, the other Angle being less than this Angle, and its Complement to a Semicircle also less than this Angle. In all other Cases the determination is certain.

There be ten other Cases in oblique angled spherical Triangles, which may be resolved by the Catholick Proposition (by two Operations at the least) and that most fitly, by reducing the oblique Triangle into two right

right angled Triangles, by letting fall a Perpendicular.

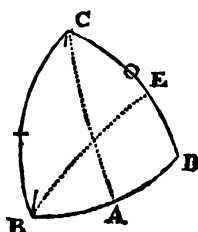
The Perpendicular must be let fall in such manner as that the rectangled Triangle may be resolved from the three things given in the oblique angled Triangle, and that by two Operations only, as this Rule teacheth.

General Rules for letting fall the Perpendicular.

1. **L**ET the Perpendicular fall from the extremity of a given Side; and let it subtend an adjacent Angle given. And besides,

2. If the three things given be conterminal, let it also fall from the extremity of the Side sought, and let it subtend the Angle sought.

In the Triangle BCD, let the Angles C and B, and the Side CB be given, and the Side CD required.

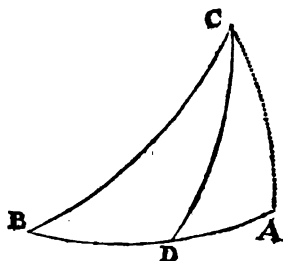
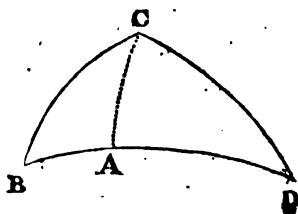
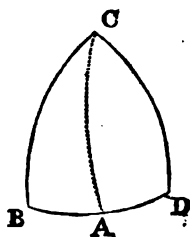


The Perpendicular CA is let fall from the extremity of the given Side BC, and subtendeth the adjacent Angle B; and the three Terms given being conterminal (in this Example) it also falleth from the extremity of the required Side CD, and subtendeth the Angle sought D.

But if the Perpendicular BE be constituted according to the first part of the Rule, then the Triangle may be resolved, but not at two Operations, for which one thing's sake the second part of the Rule comes to be observed, and that in those two Cases only; for in the other the first part of the Rule sufficeth.

The Perpendicular falls within the Triangle, when the Angles at the Base (or Side upon which the Perpendicular,

pendicular falls) are both of one Species, *viz.* either both obtuse, or both acute; but without the Triangle, if one be obtuse and the other acute; as in these Examples.



: The Perpendicular being rightly let fall, the first Operation will be in the first Triangle, *viz.* in that wherein three things are given: and that will be either for finding of the vertical Angle or the Base, and that by the universal Proposition.

First, If an Angle be sought, then first let the vertical Angle be found, unless the Perpendicular fall upon a known Side.

Secondly, If a Side be sought, let first the Base be found, unless the Perpendicular fall from a known Angle (for then the contrary must be observed every way.)

The second Operation consists in comparing the two Homogenial Terms of both Triangles between themselves, *viz.* of the Hypothenufes with the vertical Angles, or with the Bases; and also of the Angles at the Base with the same.

For

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For the Homogenial Terms of each Triangle, are proportional, and Homogenial in the Sines and Tangents of the circular Parts.

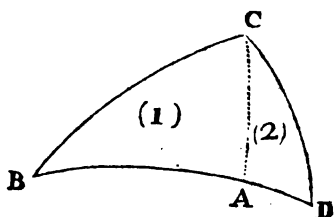
As in the oblique angled spherical Triangle BCD, reduced into the two right angled spherical Triangles, ABC, and ACD, by the Perpendicular CA: I say, that in the Sines and Tangents of the circular Parts, it will be,

As the Sine of the Base AB,

To the Co-tangent of the Angle at the Base B;

So is the Sine of the Base AD,

To the Co-tangent of the Angle at the Base, D.



For the Perpendicular AC being assumed, the Base AB in the first Triangle ABC, will be the middle Part; then the Complement of the Angle at B, and the Perpendicular AC, are the Extremes Conjunct: Therefore by the universal Proposition,

As the Tangent of AC is to Radius,

So is the Sine of AB, to the Co-tangent of B.

In like manner in the second Triangle ADC; the Base AD will be the middle Part; and the Complement of the Angle at D, and the Perpendicular AC, are the Extremes Conjunct: wherefore,

As the Tangent of AC, is to Radius;

So is the Sine of AD, to the Co-tangent of D.

Wherefore (by the 11 Pr. 5 *Lib. Euc.*) it will be,

As the Sine of AB, to the Co-tangent of B;

So is the Sine of AD, to the Co-tangent of D.

And there is the like reason of demonstrating in the other Parts; for the Perpendicular shall be always one (and the same) of the Extremes in both Triangles.

Where.

Wherefore, to determine a Proportion of two Homogenous Terms of both Triangles betwixt themselves, we must imagine the Perpendicular to be compared both with the Term given, and the Term sought in the second Triangle, that it may be understood which of those may be made the middle Part, and which is the one Extreme, and of what kind it is; whereupon Sines or Tangents may be fitted to them, according to the Universal Proposition, to which do answer in all things the Homogenous Parts of the first Triangle: then the Perpendicular being rejected, together with the Radius, we compare the middle Part, and one of the Extremes in the first Triangle, with the middle Part, and one of the Extremes of the second, as is done before, and as will be manifest in the eight following Cases.

Having thus laid the Foundation, I shall next proceed to solve those Eight Cases, both by letting fall a Perpendicular, and also without a Perpendicular, by help of the two following Axioms, invented and demonstrated by Mr. *Oughtred*, in his Trigonometry.

A X I O M I.

As the Sine of half the Sum of two Sides;
To the Sine of half their Difference;
So is the Co-tangent of half the contained Angle;
To the Tangent of half the Difference of the other
Angles.

Again :

As the Co-sine of half the Sum of the Sides;
To the Co-sine of half their Difference;
So is the Co-tangent of half the contained Angle;
To the Tangent of half the Sum of the other Angles.

AXIOM

A X I O M II.

As the Sine of half the Sum of two Angles,
To the Sine of half their Difference;
So is the Tangent of half the interjacent Side,
To the Tangent of half the Difference of the other
Sides.

Again:

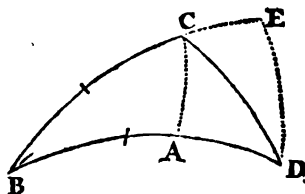
As the Co-sine of half the Sum of the Angles,
To the Co-sine of half their Difference;
So is the Tangent of half the interjacent Side,
To the Tangent of half the Sum of the other Sides.

C A S E III.

Two Sides BC, and BD, and the contained Angle B, being given, to find either of the other Angles.

Let the Side BC be 30°
 $00'$, the Side BD $42^\circ 09'$,
and the Angle B $36^\circ 08'$;
and the Angle D be re-
quired.

The Perpendicular CA
being let fall by the
second Rule, Page 119,
then in the Triangle ABC, there is given the
Hypotenuse BC, and the Angle B, to find the Seg-
ment of the Base BA, the first Operation; wherein
the Complement of B is the middle Part, and the
Complement of BC, and the Segment BA, Extremes
Conjunct; and the first Operation will be the same as
in Case 2. Chap. 8.



As the ct. BC $30^{\circ} 00'$	10.23856
To Radius	10.
So is cs. B $36^{\circ} 08'$	9.90722
To t. BA $25^{\circ} 00'$	9.66866

Subtra $\hat{c}$ t BA 25° from BD $42^{\circ} 09'$, there remains $17^{\circ} 09'$ AD.

Next we must compare the Angle B, the Segment BA, and the Perpendicular AC; in which we shall find BA the middle Part, and the Complement of B, and the Perpendicular, Extremes Conjun $\hat{c}$ t: and so likewise in the Triangle ACD, the Segment AD is the middle Part, and the Complement D, and AC, Extremes Conjun $\hat{c}$ t; therefore it will be,

As Sine AB 25	Ar. Com.	0.374052
To the Tangent Compl. of B $36^{\circ} 08'$		10.136615
So Sine AD $17^{\circ} 09'$		9.469636
To the Tangent Compl. of D $46^{\circ} 18'$		9.980303

To find the Angle BCD.

The Perpendicular DE in the former Figure being let fall, say

As ct. BC $42^{\circ} 09'$	10.043277
To Radius	10.
So cs. of B $36^{\circ} 08'$	9.907222
To t. BE $36^{\circ} 10'$	9.863945

From $36^{\circ} 10'$ subtra $\hat{c}$ t BC $30^{\circ} 00'$, and there remains CE $6^{\circ} 10'$.

Then in the Triangle BDE, compare the Angle B, the Base BE, and the Perpendicular DE, and you will find BE the middle part, and the Complement of B, and

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and the Perpendicular DE, Extremes Conjunct; and in the Triangle CDE, it will be CE the middle Part: therefore the second Operation will be,

As s. BE $36^{\circ} 10'$	Ar. Com. 0.229048
To ct. B $36^{\circ} 08'$	10.136615
So s. CE $6^{\circ} 10'$	9.031089
Tot. DCE $76^{\circ} 00'$	9.396752

The Complement of 76° to 180° is $104^{\circ} 00'$, the Angle BCD.

To find the Angles by Axiom 1. Page 122.

As s. of half the Sum of the Sides BC and	} 0.230001
BD $36^{\circ} 04' \frac{1}{2}$	
To s. of half their Difference $6^{\circ} 04' \frac{1}{2}$	9.024608
So ct. of half the Angle B $18^{\circ} 04'$	10.486507

To t. of half the Difference of the other	} 9.741116
Angles $28^{\circ} 51'$	

As cs. of half the Sum of the Sides 36°	} 0.092456
$04' \frac{1}{2}$	
To cs. of half their Difference $6^{\circ} 04' \frac{1}{2}$	9.997553
So ct. of half the Angle B $18^{\circ} 04'$	10.486507

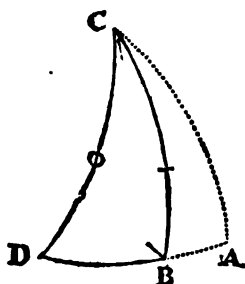
To the t. of half the Sum of the other	} 10.576516
Angles $75^{\circ} 09'$	
Add and Subtr. 28 51	

The Angle	{	BCD $104^{\circ} 00'$ the Sum.
		BDC $46^{\circ} 18'$ the Difference.

CASE

C A S E IV.

The two Angles B and C, and the Side included between them, BC, being given; to find the Side CD.



Let the Angle DCB be $36^{\circ} 08'$, the Angle CBD 104° , and the Side CB included $30^{\circ} 00'$.

Having let fall the Perpendicular from the extremity of the known Side BC, subtending the adjacent known Angle CBA, and the Angles being of different kinds, the Perpendicular falls without the Triangle; then in the Triangle ABC is given the Side BC $30^{\circ} 00'$, and the Angle CBA $76^{\circ} 00'$ (the Complement of 104°) to find the Angle BCA, which is done by Case 3. Chap. 8. thus:

As ct. of B $76^{\circ} 00'$	9.39677
To Radius	10.
So cs. of BC $30^{\circ} 00'$	9.93753
To ct. of BCA $16^{\circ} 04'$	10.54076

Then if the Side BC, the vertical Angle ACB, and the Perpendicular AC be compared, we shall find the Complement of the vertical Angle ACB, the middle Part; and the Complement of the Side BC, and the Perpendicular AC, Extremes Conjunct; and in the Triangle ADC, the Complement of the vertical Angle ACD is the middle Part, and the Complement of DC, and the Perpendicular AC, Extremes Conjunct. Wherefore the Operation of the Homogenial Parts in both Triangles, will be thus:

A₂

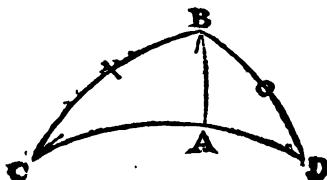
Chap. 9. Spherical Trigonometry. 127

As cs. of ACB $16^{\circ} 04'$ 0.017304
 To ct. of BC $30^{\circ} 00'$ 10.238560
 So is cs. of ACD $52^{\circ} 12'$ 9.787394

To ct. of DC $42^{\circ} 09'$ 10.043258

Again, let the Angles B and C, and the interjacent Side BC, be given (as before) and let the Side BD be required.

Having let fall the Perpendicular BA, by the second Part of the Rule, Page 119. we next find the vertical Angle CBA, by Case the 3d. Chap. 8. thus :



'As ct. C $36^{\circ} 08'$ 10.136615
 To Radius 10.
 So is cs. BC $30^{\circ} 00'$ 9.937531
 To ct. CBA $57^{\circ} 42'$ 9.800916

Subtract $57^{\circ} 42'$ from $104^{\circ} 00'$, and there remains the vertical Angle ABD $46^{\circ} 18'$. Then if the Side BC, the vertical Angle CBA, and the Perpendicular AB be compared; we shall find the Comp. of the vertical Angle CBA the middle Part, and BC and AB Extremes Conjoinct; and the other vertical Angle ABD is the middle Part in the second Triangle, and it will be the Co-sines of the vertical Angles, and Co-tangents of the Extremes BC, and BD: and the Proportion of the Homogenial Terms will be,

As
 π

As cs. CBA $57^{\circ} 42'$	Ar. Com. 0.272173
To ct. BC $30^{\circ} 00'$	10.238560
So is cs. ABD $46^{\circ} 18'$	9.839404
To ct. BD $24^{\circ} 04'$	10.350136

By having the Angles B and C, and interjacent Side BC, (as before) given; to find the other Sides, without a Perpendicular, by *Axiom 2. Page 123.*

As the s. of half the Sum of the Angles	
B and C $70^{\circ} 04'$	0.026830
To the s. of half their Difference $33^{\circ} 56'$	9.746811
So is t. of half the included Side BC $15^{\circ} 00'$	9.428052

To the t. of half the Difference of the	
other Sides $9^{\circ} 02'$	9.201693

Again,

As cs. of half the Sum of the Angles	
B and C $70^{\circ} 04'$	0.467339
To cs. of half their Difference $33^{\circ} 56'$	9.746811
So is t. of half the included Side $15^{\circ} 00'$	9.428052

To t. of half the Sum of the other	
Sides $33^{\circ} 07'$	9.814306
Add and subtract 9 02	

The Sum 42 09 the Side CD.

The Diff. 24 05 the Side BD.

Here the half Difference added to the half Sum, gives the greater Side; and the half Difference being subtracted from the half Sum, the remainder is the lesser Side sought.

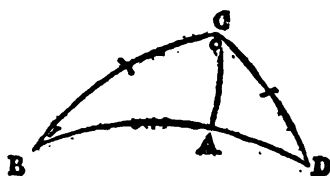
CASE

CASE V.

Two Sides, BC and CD, and the Angle B opposite to one of them, being given, to find the comprehended Angle C.

Let the Side BC be $30^{\circ} 00'$, and CD $24^{\circ} 04'$, and the Angle B $36^{\circ} 08'$.

The Perpendicular let fall from the extremity of the given Side BC, subtends the given Angle B, and falls within the Triangle, because the Angles at the Base are both acute.



In the Triangle ABC we have given (besides the right Angle) the Side BC and Angle B; to find the vertical Angle ACB: which may be done by *Case I. Chap. 8.* thus.

As ct. B $36^{\circ} 08'$	10.136615
To Radius	10.
So is cs. BC $30^{\circ} 00'$	9.937530
To ct. BCA $57^{\circ} 42'$	9.800915

Having thus found the Angle BCA in the first Triangle, we must next consider the homogeneal Terms in both Triangles; which will be the Side BC and the Angle BCA in the first, and the Side CD and Angle ACD in the second Triangle. Now the Side BC, the Angle BCA, and the Perpendicular CA in the first, being compared, we shall find the Comp. of BCA the middle Part, and the Comp. of BC and CA Extremes Conjun&: and likewise in the second Triangle the

K

Comp.

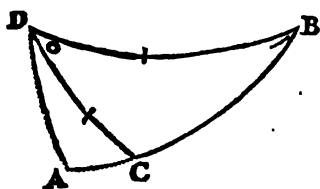
130 Spherical Trigonometry. Part I.

Comp. of ACD will be the middle, and the Comp. of CD and AC Extremes: therefore it will be,

As ct. BC $30^{\circ} 00'$	Ar. Com.	9.761439
To cs. BCA $57^{\circ} 42'$		9.727827
So is ct. CD $24^{\circ} 04'$		<u>10.350058</u>
To cs. ACD $46^{\circ} 18'$		9.839324

If this Angle ACD $46^{\circ} 18'$ be added to the former Angle BCA $57^{\circ} 42'$, the Sum is $104^{\circ} 00'$, the whole Angle BCD required.

Suppose the Sides BD $42^{\circ} 09'$, and CD $24^{\circ} 04'$, and the Angle B $36^{\circ} 08'$ were given; to find the comprehended Angle D.



The Perpendicular AD being let fall from the extremity of the given Side CD, subtends the given Angle B, but falls without the Triangle, because the Angles at the

Base, B and C, are of different Species.

In the Triangle ABD we have given the Side BD, and the Angle B, to find the vertical Angle BDE; thus.

As ct. B $35^{\circ} 08'$	10.136615
To Radius	10.
So is cs. BD $42^{\circ} 09'$	<u>9.870047</u>
To ct. BDA $61^{\circ} 34'$	9.733432

Having found the Angle BDA, we must next find the Angle CDA; wherefore we must compare CD, the Angle CDA, and the Perpendicular DA; and we shall find the Comp. of CDA the middle Part: and in the

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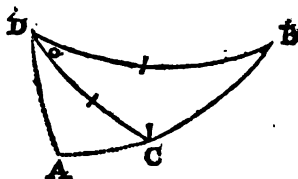
the Triangle ABD, the Comp. of the Angle BDA, is the middle Part; and the Sides DB and CD are the homogenial Extremes Conjunct: wherefore it will be,

As ct. BD $42^{\circ} 09'$	9.956723
To cs. BDA $61^{\circ} 34'$	9.677731
So is ct. CD $24^{\circ} 04'$	10.350058
To cs. CDA $15^{\circ} 13'$	9.984512

If this Angle CDA $15^{\circ} 13'$ be subtracted from the whole Angle BDA $61^{\circ} 34'$, there will remain $46^{\circ} 21'$ for the Angle BDC required.

Again, we will suppose the Sides BD $42^{\circ} 09'$, and CD $24^{\circ} 04'$, and the obtuse Angle at C $104^{\circ} 00'$; given; to find the comprehended Angle D:

The Perpendicular DA being let fall from the extremity of the given Side DC, subtends the given Angle ACD, (the Comp. of BCD.) First, find the verticle Angle ADC, thus:



As ct. ACD $46^{\circ} 00'$ (the Comp. of $\angle$ BCD 104°)	9.396771
To Radius	10.
So is cs. DC $24^{\circ} 04'$	9.960505
To the ct. ADC $15^{\circ} 16'$	10.563734

Then,	
As ct. DC $24^{\circ} 04'$	9.549942
To cs. ADC $15^{\circ} 16'$	9.984397
So is ct. BD $42^{\circ} 09'$	10.043277
To cs. ADB $61^{\circ} 34'$	9.677616

K 2

From

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From $ADB\ 61^{\circ}\ 34'$ subtract $ADC\ 15^{\circ}\ 16'$, and there remains $46^{\circ}\ 18'$, the Angle BDC required.

Having the Sides $BD\ 42^{\circ}\ 09'$ and $DC\ 24^{\circ}\ 04'$, and the Angle $C\ 104^{\circ}\ 00'$, given, to find the contained Angle D , without the help of a Perpendicular.

By the first Case of this Chapter, find the Angle B , thus.

As s. Side $BD\ 42^{\circ}\ 09'$	Ar. Com.	0.173229
To s. of the opposite Angle $BCD\ 104^{\circ}$	}	9.986904
(or 76°)		
So is s. Side $DC\ 24^{\circ}\ 04'$		9.610446
To s. of the opposite Angle $B\ 36^{\circ}\ 08'$		9.770379

Thus have we two Angles and their opposite Sides, to find the other Angle; by the Inverse of either of the Proportions in the first Axiom, page 122. the former will be,

As s. of half the difference of the Sides	}	0.803679
$9^{\circ}\ 02'\frac{1}{2}$		
To t. of half the difference of the	}	9.827897
Angles $33^{\circ}\ 56'$		
So s. of half the Sum of the Sides	}	9.737370
$06'\frac{1}{4}$		
To ct. of half the Angle required	}	10.368946
$23^{\circ}\ 09'$		

So if $23^{\circ}\ 09'$ be doubled, it makes $46^{\circ}\ 18'$, the Angle D , the same as before.

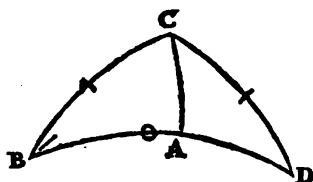
If the Sum of the given Sides be more than a Semi-circle, in that Case resolve the opposite Triangle.

CASE VI.

Two Sides BC and CD, and the Angle B, opposite to one of them, being given, to find the third Side BD.

Let the Side BC be $30^{\circ} 00'$, CD $24^{\circ} 04'$, and the Angle B $36^{\circ} 08'$.

The Perpendicular AC being let fall, the given Triangle is thereby reduced into two rectangled Triangles, ABC and ADC; in the first is given (besides the right Angle) the Side BC and Angle B; to find the Segment of the Base, BA (by Case 2. Chap. 8.) thus.



As ct. BC $30^{\circ} 00'$	10.238561
To Radius	10.
So is cs. B $36^{\circ} 08'$	9.907222
To t. BA $25^{\circ} 00'$	9.668661

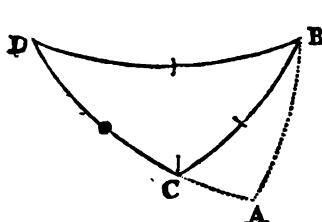
Then if BA, BC, and the Perpendicular AC in the first Triangle, be compared, we shall find the Comp. of BC the middle Part, and AB and AC Extremes Disjunct; and so likewise will the Compl. of CD be the middle Part in the second Triangle: therefore it will be,

As cs. BC $30^{\circ} 00'$	Ar. Com.	0.062469
To cs. BA $25^{\circ} 00'$		9.957276
So is cs. CD $24^{\circ} 04'$		9.960505
To cs. AD $17^{\circ} 09'$		9.980250

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Add AB $25^{\circ} 00'$ to AD $17^{\circ} 09'$, the Sum is $42^{\circ} 09'$; the whole Side BD required.

But suppose the Sides BC and BD, and the Angle C, given; to find CD.



Let BC be $30^{\circ} 00'$, BD $42^{\circ} 09'$, and C 104° .

The Perpendicular BA being let fall, in the Triangle ABC we have given the Side BC, and Angle BCA (being the Compl.

of BCD) to find the Base AC; thus.

As ct. BC $30^{\circ} 00'$	10.238561
To Radius	10.
So is cs. ACB $76^{\circ} 00'$	9.383675
To t. AC $7^{\circ} 57'$	9.145114

Then in the Triangle ABC, if BC, AC, and the Perpendicular AB, be compared, we shall find the Compl. of BC the middle Part, and AB and AC Extremes Disjunct; and in the Triangle ABD, the Compl. of BD will be the middle, and AB and AD Extremes; therefore it will be,

As cs. BC $30^{\circ} 00'$	Ar. Com. 0.062469
To cs. AC $7^{\circ} 57'$	9.995806
So is cs. BD $42^{\circ} 09'$	9.870047
To cs. AD $32^{\circ} 01'$	9.928322

Then if AC $7^{\circ} 57'$ be subtracted from AD $32^{\circ} 01'$, there will remain CD $24^{\circ} 04'$.

Having the Sides BC and BD, and the Angle C (as before) given, to find the other Side CD, without letting fall a Perpendicular. First,

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First, we must find the Angle D opposite to the other given Side BC, by *Case 1.* of this Chapter ; thus.

As s. BD $42^{\circ} 09'$	Ar. Com.	0.173229
To s. BCD 104° (or 76° its Comp.)		9.986904
So s. BC $30^{\circ} 00'$		9.698970
To s. D $46^{\circ} 18'$		9.859103

Then by the Inverse of either of the Proportions in the second Axiom, find the Side CD, thus.

As the s. of half the Difference of the	}	0.316483
Angles $28^{\circ} 51'$		
To the s. of half their Sum $75^{\circ} 09'$		9.985247
So is t. of half the Difference of the	}	9.027055
Sides $6^{\circ} 04'\frac{1}{2}$		
To the t. of half the Side required $12^{\circ} 02'$	}	9.328785

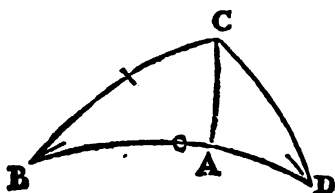
Then if $12^{\circ} 02'$ be doubled, it makes $24^{\circ} 04'$, the Side CD required.

C A S E VII.

Two Angles B and D, and the Side BC opposite to one of them, being given ; to find the Side BD included between them.

Let the Angle B be $36^{\circ} 08'$, and D $46^{\circ} 18'$, and the Side BC $30^{\circ} 00'$.

First, find the Segment of the Base AB, thus.



136 Spherical Trigonometry. Part 1.

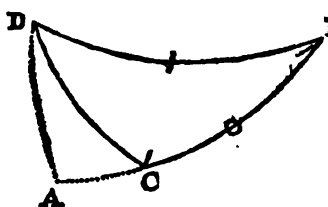
As ct. BC $30^{\circ} 00'$	10.23856
To Radius	10.
So is cs. B $36^{\circ} 08'$	9.90722
To t. AB $25^{\circ} 00'$	9.65866

Then the Angle B, the Base BA, and Perpendicular AC, being compared, AB is found to be the middle Part, and AC and the Compl. B Extremes Con-junct; and in the other Triangle, AD is the middle Part: therefore it will be,

As ct. B $36^{\circ} 08'$	9.863385
To s. of AB $25^{\circ} 00'$	9.625948
So is ct. D $46^{\circ} 18'$	9.980285
To s. AD $17^{\circ} 09'$	9.469618

Add AB $25^{\circ} 00'$ to AD $17^{\circ} 09'$, the Sum is $42^{\circ} 09'$, the Side BD required.

Let us suppose the Angles B and C, and the Side BD given; and let BC be required.



Let B be $36^{\circ} 08'$, C 104° , and BD $42^{\circ} 09'$.

In the Triangle ABD we have given (besides the right Angle A) the Hypothenuse BD, and the Angle B; to find the Base AB; thus.

As ct. BD $42^{\circ} 09'$	9.956723
To Radius	10.
So is cs. B $36^{\circ} 08'$	9.90722
To t. BA $36^{\circ} 10'$	9.863945
	Then

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Then if the Angle B, the Base AB, and Perpendicular AD, be compared, BA will be the middle Part, and the Compl. of B, and AD, Extremes Conjunct; and AC is the middle Part in the Triangle ACD: therefore it will be,

As ct. B $36^{\circ} 08'$	9.863385
To s. BA $36^{\circ} 10'$	9.770952
So is ct. DCA 76° (the Compl. of BCD 104°)	9.396771
To s. AC $6^{\circ} 10'$	9.031108

If AC $6^{\circ} 10'$ be subtracted from BA $36^{\circ} 10'$, there remains BC $30^{\circ} 00'$.

Let the Angles B and C, and the Side BD be given; to find BC (as before) but without a Perpendicular.

First, we must find the other opposite Side CD, thus.

As s. BCD 104° (or its Compl. 76°)	Ar. Com. } 0.013096
To s. BD $42^{\circ} 09'$	9.826771
So is s. B $36^{\circ} 08'$	9.770606
To s. CD $24^{\circ} 04'$	9.610473

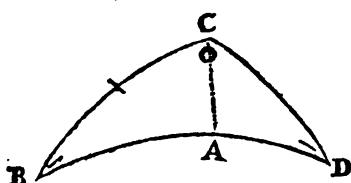
Then by the Inverse of either of the Proportions in the second Axiom, find BC thus.

As cs. of half the Difference of the	two Angles $33^{\circ} 56'$	} 0.081085
To cs. of half their Sum $70^{\circ} 04'$		9.532660
So t. of half the Sum of the two Sides	$33^{\circ} 06\frac{1}{2}'$	} 9.814312

To t. of half the third Side $15^{\circ} 00'$ 9.428057
 Then if $15^{\circ} 00'$ be doubled, it makes $30^{\circ} 00'$, the
 Side BC sought. C A S E

C A S E VIII.

Two Angles B and D, and the Side BC opposite to one of them, being given, to find the third Angle C.



Let B be $36^{\circ} 08'$, and
D $46^{\circ} 18'$, and the Side
BC $30^{\circ} 00'$.

First, find the ver-
tical Angle ACB, thus:

As ct. B $36^{\circ} 08'$	10.136615
To Radius	10.
So cs. BC $30^{\circ} 00'$	9.937530
To ct. ACB $57^{\circ} 42'$	<u>9.800915</u>

Then in the Triangle ABC, if the Angles B and ACB, and the Perpendicular AC be compared, the Comp. of B will be the middle Part, and the Comp. of ACB, and AC, Extremes Disjunct; and in the other Triangle ACD, the Angle D will be the middle Part: therefore it will be,

As cs. B $36^{\circ} 08'$	Ar. Com. 0.092778
To s. ACB $57^{\circ} 42'$	9.926991
So is cs. D $46^{\circ} 18'$	9.839404
To s. ACD $46^{\circ} 18'$	<u>9.859173</u>

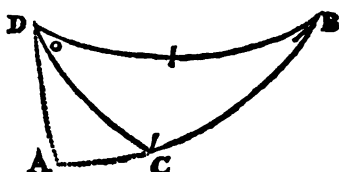
Then if ACB $57^{\circ} 42'$ be added to ACD $46^{\circ} 18'$, the Sum will be $104^{\circ} 00'$, the whole Angle C.

Again,

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Again, suppose the Angles B and C, and the Side BD, given; to find the third Angle D.

In the Triangle ABD, there is given the Angle B, and Hypotenuse BD; to find the Angle ADB; thus.



As ct. B $36^{\circ} 08'$	10.136615
To Radius	10.
So cs. BD $42^{\circ} 09'$	9.870047
To ct. ADB $61^{\circ} 34'$	9.733432

Then in the Triangle ABD, if the Angles B and ADB, and the Perpendicular AD be compared, B will be the middle Part, and AD and the Comp. of ADB will be Extremes Disjunct; and in the Triangle ADC, C will be the middle Part, and AD, and Compl. of ADC Extremes Disjunct; wherefore it will be,

As cs. B $36^{\circ} 08'$	Ar. Com. 0.092778
To s. ADB $61^{\circ} 34'$	9.944172
So is cs. ACD 76° (the Com. of BCD 104°)	9.383675
To s. ADC $15^{\circ} 16'$	9.420625

If ADC $15^{\circ} 16'$ be subtracted from ADB $61^{\circ} 34'$, there will remain $46^{\circ} 18'$, the Angle D required.

Let B and C, and the Side BD be given (as before) to find the third Angle D, without a Perpendicular.

First, find CD opposite to the other Angle B, thus.

As

As s. C $104^{\circ} 00'$ (or 76° its Comp.)	A. C.	0.013096
To s. BD $42^{\circ} 09'$		9.826771
So s. B $36^{\circ} 08'$		9.770606
To s. CD $24^{\circ} 04'$		9.610473

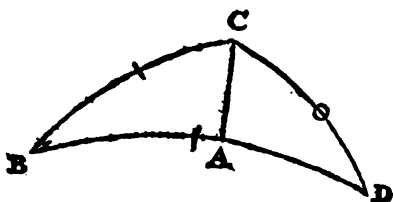
Then by either of the Proportions in the first Axiom inverted,

As cs. of half the Difference of the Sides?		
BD and CD $9^{\circ} 02' \frac{1}{2}$	}	0.005430
To cs. of half their Sum $33^{\circ} 06' \frac{1}{2}$		9.923057
So t. of half the Sum of the Angles B	}	10.440508
and C $70^{\circ} 04'$		
To ct. of half the contained Angle?		
$23^{\circ} 09'$	}	10.368995

Then if $23^{\circ} 09'$ be doubled, it will make $46^{\circ} 18'$, the Angle D required.

C A S E IX.

Two Sides BC and BD, and the Angle B comprehended between them, being given, to find the third Side CD.



Base AB; thus.

Let BC be $300^{\circ} 00'$,
BD $42^{\circ} 09'$, and the
Angle B $36^{\circ} 08'$.

In the Triangle
ABC, there is given
the Angle B, and
the Hypotenuse
BC; to find the

As

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As ct. BC $30^{\circ} 00'$	10.23856
To Radius	10.
So cs. B $36^{\circ} 08'$	9.90722
To t. AB $25. 00'$	9.66866

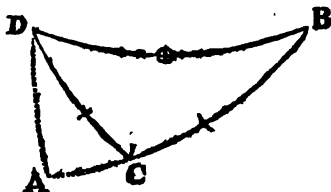
Subtract AB $25^{\circ} 00'$ from BD $42. 09'$, and there remains AD $17^{\circ} 09'$.

Then AB and BC, and the Perpendicular AC being compared, we shall find the Complement BC the middle Part, and AB and AC Extremes Disjunct; and the Comp. of CD will be the middle Part in the other Triangle : then it will be,

As cs. AB $25^{\circ} 00'$	Ar. Com. 0.042725
To cs. BC $30^{\circ} 00'$	9.937531
So cs. AD $17^{\circ} 09'$	9.980247
To cs. CD $24^{\circ} 04'$	9.960503

Again, suppose BC $30^{\circ} 00'$, and CD $24^{\circ} 04'$, and the comprehended Angle C, given; to find the Side BD.

In the Triangle ACD, is given the Hypotenuse CD, and Angle ACD; to find the Base AC, thus.



As ct. CD $24. 04'$	10.350058
To Radius	10.
So cs. ACD $76^{\circ} 00'$	9.383675
To t. AC $6^{\circ} 10'$	9.033617

Add AC $6. 10'$ to BC $30^{\circ} 00'$, the Sum is $36. 10'$, the whole Base AB : then it will be,

As

As cs. AC $6^{\circ} 10'$
 To cs. DC $24^{\circ} 04'$
 So is cs. AB $36^{\circ} 10'$

Ar. Com. 0.002521
 9.960505
 9.907037

To cs. BD $42^{\circ} 09'$

9.870063

Mr. Collins solves this Case after the following Method.

1. If both Sides are equal, then it will be,

As the Radius to the Sine of the common Side,
 So is the Sine of half the Angle to the Sine of
 half the Side sought.

2. In all other Cases, one or both of the including
 Sides being less than Quadrants, it will hold,

As the Radius to the Co-sine of the Angle in-
 cluded,
 So is the Tangent of the lesser Side to the Tan-
 gent of a fourth Arch.

If the Angle included be less than 90° , subtract the
 fourth Arch from the other Side; but if it be more,
 from the other Side's Complement to 180° . The re-
 mainder is called the Residual Arch. Then,

As the Co-sine of the 4th Arch to the Co-sine
 of the Arch remaining,
 So is the Co-sine of the lesser Side to the Co-sine
 of the Side sought.

When the contained Angle is acute, and the residual
 Arch more than 90° , or when the said Angle is ob-
 tuse, and the residual Arch less than a Quadrant, the
 Side sought is greater than a Quadrant; in all other
 Cases less.

To

To resolve this Case without a Perpendicular.

I shall here make use of Mr. Collins's Proportion; which is thus.

As the Cube of Radius is to the Rectangle of the comprehending Sides,

So is the Square of the Sine of half the contained Angle,

To half the Difference of the versed Sines of the third Side, and of the Arch of Difference between the two including Sides.

Which is thus; double the Log. Sine of half the Angle given, and thereto add the Log. Sines of the containing Sides, and from the Left-hand of the Sum dash out 3 for the Cube of Radius; so rests the Log. of half the Difference of those two versed Sines. Which half Difference doubled, and added to the versed Sine of the Difference of the Legs or containing Sides, gives the versed Sine of the Side sought.

We will suppose (in the former Triangle) the Side BC $30^{\circ} 00'$, CD $24^{\circ} 04'$, and the Angle C $104^{\circ} 00'$, given; to find the Side BD.

The Log. Sine of BC $30^{\circ} 00'$ 9.698970

The Log. Sine of DC $24^{\circ} 04'$ 9.610446

The Log. Sine of 52° (half 104°) doubled 19.793064

The natural Sine against 39.102480

Is 1266318; which doubled is 2532636

The natural versed Sine of $5^{\circ} 56'$ (the } 0053572
diff. of the Sides) }

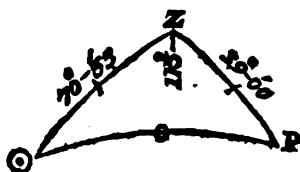
The Sum is the natural versed Sine of } 2586208
 $42^{\circ} 09'$, the Side BD }

If

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If the Logarithm of the Number 2 be added to the Logarithm of the middle Terms, we shall have the Log. of the whole Difference in the last place; having found it, take the Number that stands against it in the natural Sines, and add to it the natural versed Sine of the Difference of the Legs, and the Sum is the natural versed Sine of the Side sought.

Example. In the Triangle $\odot ZP$, let there be given the Side $\odot Z$, the Comp. of the Sun's Altitude $70^\circ 53'$, and the Side ZP the Comp. of the Latitude $39^\circ 00'$, with the Angle included $\odot ZP$ 115° the Sun's Azimuth from the North; to find the Side $\odot P$, the Sun's



distance from the elevated Pole.

The Log. Sine of $70^\circ 53'$	9.9753646
Log. Sine of $39^\circ 00'$	9.7988718
Log. Sine of $57^\circ 30'$ doubled	19.8520584
Log. of 2	0.3010300

The natural Sine against	39.9273248
Is 8458830	

Add 1508746 the natural versed Sine of $31^\circ 53'$, the
Diff. of the Legs.

Sum 9967576 the natural versed Sine of $89^\circ 49'$, the
Side $\odot P$ sought.

Note. If the Sum of the four Log. Sines be more than Radius (after 3 is dash'd out) look for it in the Log. Secants, and take the natural Secant that stands against it, and to that add the natural versed Sine of the Difference of the Legs; the Arithmetical Complement of the Sum is the natural versed Sine of the Complement of the Side sought, to 180° .

Example. The Side $\odot Z$ $70^\circ 53'$, and the Side ZP $38^\circ 28'$, and the Angle $\odot ZP$ 145° , being given, to find $\odot P$.

The

The Log. Sine of $70^{\circ} 53'$	9.9753646
The Log. Sine of $38^{\circ} 28'$	9.7938317
The Log. Sine of $72^{\circ} 30'$ doubled	19.9588390
The Log. of 2.	0.3010300

The natural Secant against 10.0290653

Is 10692463

Add 1558280 the natural versed Sine of $32^{\circ} 25'$, the
Diff. of the Legs.

12250743

The Arithmetical Complement 7749257 is the versed Sine of 77° ; whose Complement to 180° is 103° , the Side $\odot P$ sought. So the Sun's Declination is 13° South.

This Proportion is of great use for calculating the Distances of Places on the Earth by their Latitudes and Longitudes, and of the Distances of Stars by their Declinations and Right Ascensions, or Latitudes and Longitudes; but it is of very excellent use for calculating the Sun's Altitude at all Hours, for it finds two Altitudes at one Operation, the double Rectangle being fix'd for that Declination, thus:

Add the Logarithms of the Number 2, and of the Co-sines of the Declination and Latitude together, the Sum may be called the fix'd Logarithm; double the Logarithm Sine of half the Hour from Noon, and add it to the fix'd Logarithm, the Sum (rejecting 3 towards the Left-hand, for the Cube of Radius) is the Logarithm of the Difference: Take the natural Sine that stands against it, and subtract it from the natural Sine of the Meridian Altitude, and there remain the natural Sines of the Altitudes sought.

If this Difference cannot be subtracted from the Sine of the Meridian Altitude, it argues the Sun hath no Altitude above the Horizon; in this Case subtract that from this, and there will remain the natural Sine of

L

the

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the Sun's Altitude for the like Hour from Midnight in
Summer.

Example. Let it be required to calculate the Sun's
Altitude, when he hath $23^{\circ} 30'$ both North and South
Declination for the Latitude of *London*, at 2 and 5 a
Clock in the Afternoon, or (which is all one) for the
Hours 10 and 7 in the Morning.

Sine of $38^{\circ} 28'$ Compl. Lat.	9.7938317
Sine of $66^{\circ} 30'$ Compl. of Declin.	9.9623978
Logarithm of the Number 2.	0.3010300
The fixed Logarithm	20.0572595
Log. Sine of 15° doubled, add	18.8259924
The nearest nat. Sine against it is 764290	8.8832519

The Summer Meridian Altitude is $61^{\circ} 58'$
And its natural Sine is 8826743
From which subtract 764290 the Difference found.

Remains 8062453 the nat. Sine of $53^{\circ} 44'$
The Summer Altitude for the Hours 10 and 2.
The Winter Meridian Altitude is $14^{\circ} 58'$, and its natural Sine } 2588190
Subtract the former Difference 764290

Remains the natural Sine of $10^{\circ} 30'$ the } 1823900
Winter Altitude
for the Hours of 2 and 10.

The

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The same Day for the Altitude of 5 and 7.

	Fixed Number	200572595
Sine of half the Hour from Noon	}	195688942
37° 30' doubled		

The natural Sine against it is 4228819 396261537
 The Winter Meridian Altitude 2588190

Refts 1640629 the nat. Sine
 of 9° 26', Summer Altitude for 5 in the Morning, or
 7 at Night.

The Summer Meridian Altitude as before	8826743
The former Difference	4228819

Refts the natural Sine of 27° 22' 4597924
 the Summer Altitude for 7 in the Morning, or 5
 in the Afternoon.

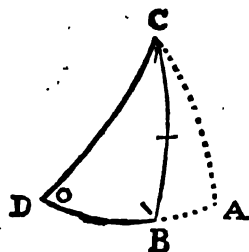
The Side sought may be greater than a Quadrant,
 and so be doubtful; but we may determine,

That when the Legs are of the same kind, and the
 Angle comprehended acute, the Side sought is less than
 a Quadrant.

And when the Legs or containing Sides are of a dif-
 ferent kind, and the Angle obtuse, the Side sought is
 greater than a Quadrant.

C A S E X.

Two Angles B and C, and the interjacent Side BC, being given; to find the third Angle D.



Let the Angles B be 104° ,
C $36^\circ 08'$, and BC $30^\circ 00'$.

In the Triangle ABC, there
is given the Angle ABC, and
the Hypothenufe BC; to find
the vertical Angle ACB.

As ct. ABC $76^\circ 00'$	9.39677
To Radius	10.
So is cs. BC $30^\circ 00'$	9.93753

To ct. ACB $16^\circ 04'$ 10.54076

Add ACB $16^\circ 04'$ to BCD $36^\circ 08'$, the Sum is $52^\circ 12'$, the whole Angle ACD.

Then in the Triangle ABC, the oblique Angles,
and the Perpendicular AC being compared, the Comp.
of the Angle ABC is the middle Part, and the Comp.
of the Angle ACB, and AC are Extremes Disjunct;
and in the Triangle ACD, the Angle D is the middle
Part: therefore it will be,

As s. ACB $16^\circ 04'$
To cs. ABC $76^\circ 00'$
So is s. ACD $52^\circ 12'$

To cs. D $46^\circ 19'$

Ar. Com. 0.557903
9.383675
9.897712
9.839290

Mr.

Mr. Collins delivers these following Proportions to solve this Case.

I. If both the Angles are equal, it will be,

As Radius to the Sine of the Angle given, so is the Co-sine of half the given Side, to the Co-sine of half the Angle sought.

In all other Cases not belonging to right angled Triangles,

If one or both of the given Angles be acute, it holds,

As the Radius to the Co-sine of the interjacent Side, So is the Tangent of the lesser Angle, to the Tangent of a 4th Arch.

If the interjacent Side be more than 90° , subtract the 4th Arch from the other Angle; but if less than 90° , subtract the 4th Arch from the other Angle's Complement to 180° , noting the residual Arch.

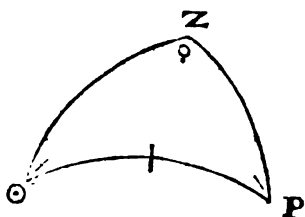
Then,

As the Co-sine of the 4th Arch, to the Co-sine of the Arch remaining;

So the Co-sine of the lesser Angle, to the Co-sine of the Angle sought.

When the interjacent Side is less than a Quadrant, and the residual Arch more; or when the interjacent Side is greater than a Quadrant, and the residual Arch less, the Angle sought is obtuse; in all other Cases acute.

Example. In the Triangle $\odot ZP$, let there be given, the Angle of Position at $\odot$ $21^\circ 28'$; the Angle at P the Hour from Noon $33^\circ 47'$; and the Side $\odot P$, the Sun's distance from the elevated Pole $103^\circ 00'$, to find the Sun's Azimuth $\odot ZP$.



As Radius to cs. of $77^\circ 00'$ (the	} 9.352088
Compl. of 103°)	
So is t. $Z\odot P$ $21^\circ 28'$	9.594656
To t. of the 4th Arch $5^\circ 03'$	8.946744

From $33^\circ 47'$ the greater Angle,
Subtract $5^\circ 03'$ the 4th Arch.

Remains $28^\circ 44'$ the residual Arch.

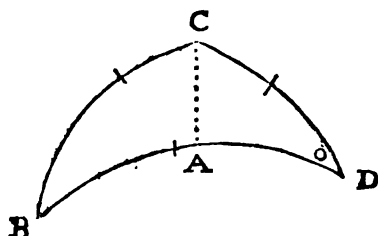
Then,

As cs. of the 4th Arch $5^\circ 03'$	Ar. Com.	9.001689
To cs. of the residual Arch $28^\circ 44'$		9.942933
So is cs. of the lesser Angle $21^\circ 28'$		9.968777
To cs. of $35^\circ 00'$		9.913399

Whose Complement to 180° is 145° , the Angle sought; this Angle is obtuse, because the interjacent Side is more than a Quadrant, and the residual Arch less.

CASE XI.

The three Sides BC , CD , and BD , being given; to find the Angle D .



Let BC be $30^{\circ} 00'$, CD $24^{\circ} 04'$, and the Base BD $42^{\circ} 09'$.

BC $30^{\circ} 00'$

CD $24 \ 04$

Sum	<u>54 04</u>	half Sum	27 02	} half the Base $21^{\circ} 4\frac{1}{2}'$.
Diff.	5 56	half Diff.	2 58	

Then to find the Segments of the Base, say,

As t. of half the Base $21^{\circ} 4\frac{1}{2}'$	Ar. Com.	0.4141259
To t. of half the Sum of the Sides $27^{\circ} 02'$		9.7077902
So is t. of half the Difference of the Sides $2^{\circ} 58'$		8.7145345

To t. of half the Diff. of the Segments of the Base $3^{\circ} 56'$		8.8364506
Add and subtract $21 \ 04\frac{1}{2}'$ the half Base.		

Sum 25 00 $\frac{1}{2}$ the Segment AB .

Diff. 17 08 $\frac{1}{2}$ the Segment AD ,
 $\quad \quad \quad L \ 4$

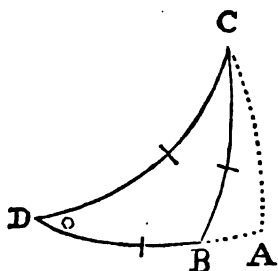
Then,

Then,

As Radius to t. AD $17^{\circ} 08'\frac{1}{2}$	9.4891655
So is ct. CD $24^{\circ} 04'$	10.3500583

To cs. the Angle D $46^{\circ} 19'$	9.8392238
-------------------------------------	-----------

Again,



In the Triangle BCD, let the Side BD be $24^{\circ} 04'$, BC $30^{\circ} 00'$, and DC $24^{\circ} 9'$; and let the Angle D be required.

Half the Base BD is $12^{\circ} 02'$, and

Half the Sum of the Sides $36^{\circ} 4'\frac{1}{2}$, and

Half the Diff. of the Sides $6^{\circ} 4'\frac{1}{2}$.

Then,

As t. of half the Base $12^{\circ} 02'$	Ar. Com.	0.6712847
To t. of half the Sum of the Sides $36^{\circ} 04'\frac{1}{2}$		9.8624560
So is t. of half the Diff. of the Sides $6^{\circ} 4'\frac{1}{2}$		9.0270550

To t. of half the Diff. of the Seg- ments $19^{\circ} 59'$	} 9.5607957
Add the half Base $12^{\circ} 02'$	

Sum $32^{\circ} 01'$, the Base DA.

Again,

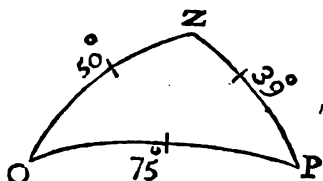
As Rad. to t. Base AD $32^{\circ} 01'$	9.7960703
So is ct. DC $42^{\circ} 09'$	10.0432767

To cs. of the Angle D required $46^{\circ} 18'$	9.8393470
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To

To solve this Case without a Perpendicular, by the 4th Axiom.

In the Triangle $\odot ZP$, let $\odot Z$ the Complement of the Sun's Altitude be $50^{\circ} 00'$, $\odot P$ $75^{\circ} 00'$ the Sun's distance from the elevated Pole, and ZP $39^{\circ} 00'$ the Complement of the Latitude; to find the Angle Z the Sun's Azimuth from the North.



1. By the first Proportion, Page 76.

$\odot Z$ $50^{\circ} 00'$	} $\odot P$ opposite to $Z = 75^{\circ} 00'$, } the half is	} $37^{\circ} 30'$		
ZP $39^{\circ} 00'$				
			Half Difference add and subtract	$5^{\circ} 30'$
Diff. $11^{\circ} 00'$			Sum	$43^{\circ} 00'$
Half $5^{\circ} 30'$		Diff. $32^{\circ} 00'$		

Then it will be, As the Rectangle of the Sines of $50^{\circ} 00'$ and $39^{\circ} 00'$, the Sides comprehending the Angle required, is to the Square of Radius; so is the Rectangle of the Sines of $43^{\circ} 00'$ the Sum, and $32^{\circ} 00'$ the Difference, to the Square of the Sine of half the Angle Z required.

Therefore, first set down the Arithmetical Complements of the Sines of the containing Sides, and under that the Sines of the Sum and Difference; half the Sum of all them will be the Sine of half the Angle required. Thus:

$\odot Z$

☉Z 50° 00' }	Comp. Arithm. {	Sine	0.115746
ZP 39 00 }		Sine	0.201128
The Sum 43° 00' }		Sine	9.833783
The Diff. 32 00 }		Sine	9.724209
The Sum			<u>19.874866</u>

The half of the Sum of the Log. Sines is } 9.937433
the Sine of 59° 59'

Doubled is 119 58 the Angle Z, the Azimuth from the North part of the Meridian.

Again, Let the Angle P, the Hour from Noon, be required,

☉P 75° 00' }	☉Z opposite to P 50° 00', the half is }	25° 00'
ZP 39 00 }	Half Diff. add and subtract	<u>18 00</u>
Diff. <u>36 00</u> }	Sum	<u>43 00</u>
Half 18 00 }	Diff.	<u>7 00</u>

☉P 75° 00' }	Comp. Arithm. {	of the Sine	0.015056
ZP 39 00 }		of the Sine	0.201128
The Sine of the Sum 43°			9.833783
The Sine of the Diff. 7°			<u>9.085894</u>
Sum			<u>19.135861</u>
21° 42'			<u>9.567930</u>

Doubled is 43 24 the Angle P, which is the hour from Noon, that is 53 min. 36 seconds past 2 in the Afternoon; or 6 min. 24 sec. past 9 if it was in the Morning.

2. By

2. By the second Proportion. Page 79.

Add all the three Sides together, and take half the Sum, and from that subtract the Base (or Side opposite to the Angle required) then,

As the Rectangle of the Sines of the comprehending Sides, is to the Square of Radius; so is the Rectangle of the Sines of the half Sum and Difference, to the Square of the Co-sine of half the Angle required.

☉Z	50° 00'	☉Z	50° 00'	Ar. Com.	0.115746
ZP	39 00	ZP	39 00	Ar. Com.	0.201128
Base ☉P	75 00	Half Sum	82° 00'	Sine	9.995753
		Diff. of the Base and			
		half Sum,	Sine		9.085894
The Sum	164 00				
Half	82 00			Sum	19.398521
Diff.	7 00			30° 01'	9.699260

Which doubled is 60 02 whose Complement to 180 is 119° 58', the Angle Z, the same as before.

This Case is usually solved upon the Sliding Rule, or upon Gunter's Scale, by the Method following, (which is the same in effect as the Proportion above) viz.

As Rad. to one of the containing Sides 50°, so is the other containing Side 39° to a fourth 28° 49'. Then again,

As that fourth to the half Sum of the Sides 82°, so is the Difference 7° to a seventh Sine 14° 30'; then divide the space between 14° 30' and 90° into two equal Parts, the middle Point between them will be at 30° 01', half the Complement of the Angle, as before.

But

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But if you have versed Sines upon your Rule, you will find against the Sine of $14^{\circ} 30'$, the versed Sine of $119^{\circ} 58'$, the Angle required, the same as before.

3. *By the third Proportion, Page 80.*

Where it is; As the Rectangle of the Sines of the half Sum of the three Sides, and of the Difference of the Base, and the said half Sum,

To the Square of Radius ;

So is the Rectangle of the Sines of the half Sum of the Base, and Difference of the Sides, and of the Difference of the Base and Difference of the Sides,

To the Square of the Tangent of half the Angle sought.

⊙Z $50^{\circ} 00'$	half the Base $37^{\circ} 30'$	}	⊙Z $50^{\circ} 00'$
ZP 39 00	half diff. Sides 5 30		ZP 39 00
Diff. 11 00	Sum 43 00	}	Base ⊙P 75 00
Half 5 30	Diff. 32 00	}	Sum 164 00
			Half 82 00
			Diff. 7 00

Half the Sum of the three Sides $82^{\circ} 00'$	}	0.004247
Sine		
Diff. of the half Sum and the Base $7^{\circ} 00'$	}	0.914106
Sine		
Half Sum of the Base and Diff. of the Sides $43^{\circ} 00'$	}	9.833783
Sine		
Half the Diff. of the Base, and Diff. of the Sides $32^{\circ} 00'$	}	9.724209
Sine		
		20.476345

Half the Sum is the Tangent of $59^{\circ} 59'$
half the Angle } 10.238172

doubled 119 58 is the Angle
CASE

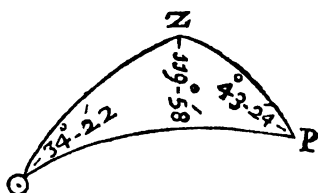
Z required.

C A S E XII.

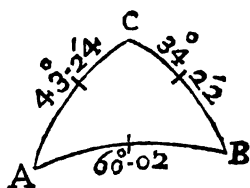
The three Angles given, to find the Sides.

Let the Angle Z the Sun's Azimuth be $119^{\circ} 58'$, the Angle P the Hour from Noon be $43^{\circ} 24'$, and the Angle of Position at $\odot$ be $34^{\circ} 22'$; to find the Complement of the Latitude ZP.

First, We must reduce the Angles into Sides, thus. Take the Complement of the greatest Angle $119^{\circ} 58'$, and that will be the greatest Side of the new Triangle, the other two Sides will be the same as the other two remaining Angles. See Definit. 6. Chap. 6.



So the Sides of the Triangle ABC, are equal to the Angles of the other Triangle $\odot ZP$, each Side of the last Triangle to its opposite Angle in the first; only the Side AB in the last, is the Complement of the Angle Z in the first to 180 : so likewise will the Angles in the latter be equal to the Sides in the former, but the Angle C in the latter will be the Complement of the Side $\odot P$ in the former.



Having thus reduced the Angles into Sides, we may find the Angle A, which will be equal to the Side ZP in the former Triangle, by any of the Proportions in the fourth Axiom, or by letting fall a Perpendicular, as in the last Case.

By

By the first Proportion, Axiom IV.

AB 60° 02'	}	Half the Base CB	17° 11'
AC 43 24		half Diff. add and sub.	8 19
Diff. 16 38			
Half 8 19			
		Sum	25 30
		Diff.	8 52

The Side AB 60° 02'	Sine	Ar. Com.	0.062324
The Side AC 43 24	Sine	Ar. Com.	0.162988
The half Sum of the Base and Diff. of the Sides 25. 30'	Sine		9.633984
The half Diff. of the Base and Diff. of the Sides 8 52	Sine		9.187903
			<u>19.047199</u>

The half Sum is the Sine of half the Angle A 19° 30' } 9.523594
 doubled is the Angle A 39 00
 Again, let the Angle C (equal to the Comp. of the the Side CP in the first Figure) be required.

By the second Proportion, Axiom IV.

Base 60° 02'	}	AC 43° 24'	Sine	Ar. Com.	0.162988
43 24		BC 34 22	Sine	Ar. Com.	0.248346
34 22		The half Sum 68° 54'	Sine		9.969860
		The Difference 8 52	Sine		9.187903
Sum 137 48					<u>19.969097</u>
Half 68 54					
Diff. 8 52					

Half Sum is the Sine of 37° 36' } 9.784548

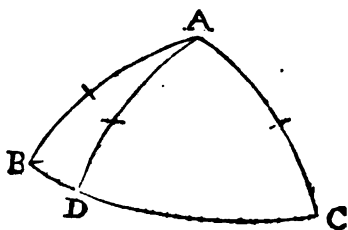
Which doubled is 75 00 the Side CP.
 Whose Complement to 180, is the Angle C.

The

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The First, Second, Fifth, Sixth, Seventh, and Eighth Cases, may be term'd the doubtful Cases, because that three given Terms are not sufficient Data to find one single Answer without the quality of a fourth : which is thus demonstrated.

Let AD and AC be two equal Sides including the Angle DAC, and both of them less or greater than a Quadrant.



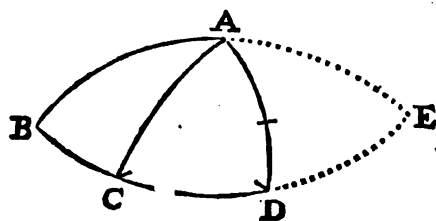
Draw thro the Points C and D, the Arch of a great Circle CD; continue it, and draw thereunto another Arch or Side from A, namely AB, neither thro the Poles of the Arch CD, nor thro the Poles of AD, so that the Angles B, and BAD may not be right Angles, nor the Angle ADB : If then each of these Sides AD and AC, be less than a Quadrant, the two Angles C and ADC will be acute; and if these two Arches be greater respectively than a Quadrant, the two Angles C, and ADC, will be obtuse; whence it comes to pass, that the Angle ADB is obtuse, when the Angle ADC is acute, and the contrary: Now forasmuch as AD and AC are equal to each other, the other Data, viz. the Side AB, and the Angle at B, are common to both; for in each Triangle ABD, and ABC, there are given two Sides, with the Angle at B, opposite to one of them: now these are not sufficient Data to find the Angle opposite to the other Side, which may be either the acute Angle C, or the obtuse Angle ADB, the Complement thereof to a Semicircle: Nor to find the third Side, which may be either BD, or the whole Side BC; nor the Angle included, which may be either BAD or BAC: therefore in those three Cases we have required the quality of

*

of

of the Angle opposite to the other given Side AB; and tho it be not so much observed in letting fall Perpendiculars, without the knowledge of the quality of the said Angle, it could not be determined whether the Perpendicular would fall within or without the Triangle, nor whether the Angle found in the first Case, be the thing sought, or its Complement to 180° ; nor whether the Angles or Segments found by the first and second Operations in the other Cases, are to be added together, or subtracted from each other, to obtain the Side or Angle sought.

So also two Angles, with a Side opposite to one of them, are not sufficient Data to obtain a fourth thing in the said Triangle, without the affection of the Side opposite to the other given Angle.



Let AB and AC be two unequal Sides containing the Angle BAC, both together equal to a Semicircle, one being greater, the other less than a Quadrant; draw thro the Points B and C, the Arch of a great Circle BC; continue it, and draw thereto from A another Side AD, but not thro the Poles of AC, nor thro the Poles of BC; so that the Angles D, and CAD may not be right Angles, nor the Angle ACD a right Angle; for if it were a right Angle, the Angle ABC, whereto it is equal, should be also a right Angle; and so the two Sides AB and AC, by reason of the Right Angles at B and C, should be equal, and the Quadrants contrary to the Supposition. Now that
the

the Angles ACD and ABC are equal, may be thus proved: Suppose the two Sides AB and BD to be continued to a Semicircle at E, then will the said Angle be equal to its opposite at B; the Side AC by Supposition is equal to the Side AE, the Complement of the Side AB to a Semicircle: but equal Sides subtend equal Angles; therefore the Angle at C is equal to the Angle at B, or at E: which being admitted, retaining the Side AD and Angle at D, we have another Angle opposite thereto, either C or B, which are equal and common to both Triangles; and so, if the Side opposite to the given Angle at D, were sought, a double Answer should be given, either the Side AC, or the other Side AB, its Complement to 180° : and the interjacent Side might be CD or BD, and the third Angle the lesser Angle CAD, or the greater BAD. So it is plain, that by only the three Terms given, these Cases are doubtful without the affection of a fourth.

In all such Cases, where the affection of the Side or Angle required is not known, it will be the best way to delineate the Triangle truly; and by that you may commonly discern whether it be more or less than a Quadrant. How to delineate any Triangle by the three Terms given, shall be shewed in the next Chapter.

I shall next shew how to vary the second Proportions in each of the Ten first Cases: Which may be done by the Theorems laid down in Page 108, &c.

The first Proportions being right angled Triangles, the Variations are already shewn.

C A S E I.

The Proportion is,

As $s. BD : s. C :: s. CB : s. D$. But it may be,
As $se.c. C : se.c. BD :: s. CB : s. D$ By the Inverse
of Theor. 6th.

And As $s. C : s. BD :: se.c. CB : se.c. D$ By the 5th.
As $se.c. BD : se.c. C :: se.c. CB : se.c. D$ By Theor.
the 4th.

C A S E II.

The Proportion is,

As $s. C : s. BD :: s. D : s. CB$. But it may be,
As $se.c. BD : se.c. C :: s. D : s. CB$ By the In-
verse of Theor. 6th.

And As $s. BD : s. C :: se.c. D : se.c. CB$ By the 5th.
As $se.c. C : se.c. BD :: se.c. D : se.c. CB$ By the 4th.

C A S E III.

The second Proportion is,

As $s. AB : ct. B :: s. AD : ct. D$
As $t. B : se.c. AB :: s. AD : ct. D$ By the 7th Theor.
As $se.c. AB : t. B :: se.c. AD : t. D$ By the 7th Theor.

C A S E IV.

The second Proportion is,

As $cs. ACB : ct. BC :: cs. ACD : ct. DC$
As $t. BC : se. ACB :: cs. ACD : ct. DC$ By the 7th.
As $se. ACB : t. BC :: se. ACD : t. DC$ By the 7th.

C A S E

C A S E V.

The second Proportion is,

As ct. BC : cs. BCA :: ct. CD : cs. ACD
 As fe. BCA : t. BC :: ct. CD : cs. ACD } By the Inverse
 As t. BC : fe. BCA :: t. CD : fe. ACD } of the 7th The.

C A S E VI.

The second Proportion is,

As cs. BC : cs. AB :: cs. CD : cs. AD
 As fe. AB : fe. BC :: cs. CD : cs. AD By the 6th Theo.
 As cs. AB : cs. BC :: fe. CD : fe. AD By the 5th.
 As fe. BC : fe. AB :: fe. CD : fe. AD By the 4th.

C A S E VII.

The second Proportion is,

As ct. B : s. AB :: ct. D : s. AD
 As fe.c. AB : t. B :: ct. D : s. AD By the 7th Theor:
 As t. B : fe.c. AB :: t. D : fe.c. AD By the 7th.

C A S E VIII.

The second Proportion is,

As cs. B : s. ACB :: cs. D : s. ACD
 As fe.c. ACB : fe. B :: cs. D : s. ACD By the 6th.
 As s. ACB : cs. B :: fe. D : fe.c. ACD By the 5th.
 As fe. B : fe.c. ACB :: fe. D : fe.c. ACD By the 4th.

C A S E IX.

The second Proportion is,

As $\text{cs. AB} : \text{cs. BC} :: \text{cs. AD} : \text{cs. CD}$

As $\text{fe. BC} : \text{fe. AB} :: \text{cs. AD} : \text{cs. CD}$ By the 6th Theo.

As $\text{cs. BC} : \text{cs. AB} :: \text{fe. AD} : \text{fe. CD}$ By the 5th.

As $\text{fe. AB} : \text{fe. BC} :: \text{fe. AD} : \text{fe. CD}$ By the 4th.

C A S E X.

The second Proportion is,

As $\text{s. ACB} : \text{cs. ABC} :: \text{s. ACD} : \text{cs. D}$

As $\text{fe. ABC} : \text{fe.c. ACB} :: \text{s. ACD} : \text{cs. D}$ By the 6th.

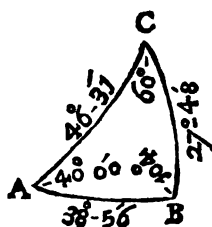
As $\text{cs. ABC} : \text{s. ACB} :: \text{fe.c. ACD} : \text{fe. D}$ By the 5th.

As $\text{fe.c. ACB} : \text{fe. ABC} :: \text{fe.c. ACD} : \text{fe. D}$ By the 4th.

Thus have I shewed all the Variations, and sufficiently explained all the Cases both in right and oblique angled spherical Triangles: I shall next set down a Synopsis of all the Cases in both.

A SYNOPSIS of the Sixteen Cases of right angled Spherical Triangles, and their Proportions.

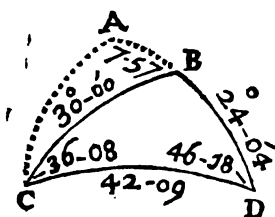
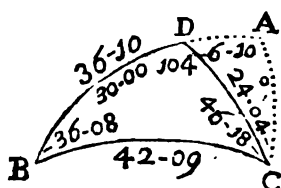
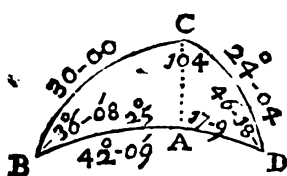
Note, That the middle Part in this Synopsis, whether given or sought, is noted with the Old English Letter.



Cases	Given	Req.	Proportions.
I.	AC. A	$\angle C$	$R : s. AC :: s. A : s. BC$ $90^\circ : 46^\circ 31' :: 40^\circ 00' : 27^\circ 48'$
II.	AC. $\angle C$	BC	$ct. AC : R :: cs. \angle C : t. BC$ $46^\circ 31' : 90 :: 60^\circ 00' : 27^\circ 48'$
III.	$\angle C$. A	C	$ct. A : R :: cs. \angle C : ct. C$ $40^\circ 00' : 90 :: 46^\circ 31' : 60^\circ 00'$
IV.	AC. BC	A	$s. AC : R :: s. BC : s. A$ $46^\circ 31' : 90 :: 27^\circ 48' : 40^\circ 00'$
V.	AC. BC	$\angle C$	$R : ct. AC :: t. BC : cs. \angle C$ $90 : 46^\circ 31' :: 27^\circ 48' : 60^\circ 00'$
VI.	$\angle C$. AB	BC	$cs. AB : R :: cs. \angle C : cs. BC$ $38^\circ 56' : 90 :: 46^\circ 31' : 27^\circ 48'$
VII.	BC. A	$\angle B$	$R : ct. A :: t. BC : s. \angle B$ $90 : 40^\circ 00' :: 27^\circ 48' : 38^\circ 56'$
VIII.	BC. $\angle B$	C	$cs. BC : R :: cs. \angle B : s. C$ $27^\circ 48' : 90 :: 40^\circ 00' : 60^\circ 00'$
IX.	$\angle C$. A	AC	$s. A : R :: s. \angle C : s. AC$ $40^\circ 00' : 90 :: 27^\circ 48' : 46^\circ 31'$
X.	$\angle B$. A	BC	$ct. A : R :: s. \angle B : t. BC$ $40^\circ 00' : 90 :: 38^\circ 56' : 27^\circ 48'$
XI.	BC. C	$\angle B$	$R : cs. BC :: s. C : cs. \angle B$ $90 : 27^\circ 48' :: 60^\circ 00' : 40^\circ 00'$
XII.	AB. $\angle B$	AC	$t. AB : R :: cs. \angle B : ct. AC$ $38^\circ 56' : 90 :: 40^\circ 00' : 46^\circ 31'$
XIII.	$\angle B$. BC	A	$t. BC : s. \angle B :: R : ct. A$ $27^\circ 48' : 38^\circ 56' :: 90 : 40^\circ 00'$
XIV.	AB. BC	$\angle C$	$R : cs. BC :: cs. AB : cs. \angle C$ $90 : 27^\circ 48' :: 38^\circ 56' : 46^\circ 31'$
XV.	$\angle B$. C	BC	$s. C : R :: cs. \angle B : cs. BC$ $60^\circ 00' : 90 :: 40^\circ 00' : 27^\circ 48'$
XVI.	A. C	$\angle C$	$R : ct. A :: ct. C : cs. \angle C$ $90 : 40^\circ 00' :: 60^\circ 00' : 46^\circ 31'$

A Synopsis of the First Ten CASES of oblique angled spherical Triangles. See the three following Figures.

Cases	Given	Req.	Proportions.
I.	$\begin{array}{ c } \hline BD \\ \hline BC \end{array}$ C	D	$s. BD : s. C :: s. BC : s. D$ $42^{\circ} 09' : 104^{\circ} 00' :: 30^{\circ} 00' : 46^{\circ} 18'$
II.	$\begin{array}{ c } \hline C \\ \hline D \end{array}$ BD	BC	$s. C : s. BD :: s. D : s. BC$ $104^{\circ} 00' : 42^{\circ} 09' :: 46^{\circ} 18' : 30^{\circ} 00'$
III.	$\begin{array}{ c } \hline BC \\ \hline BD \end{array}$ B	D	$ct. BC : R :: cs. B : t. AB$ $s. AB : ct. B :: s. AD : ct. D$
IV.	$\begin{array}{ c } \hline B \\ \hline C \end{array}$ BC	CD	$ct. B : R :: cs. BC : ct. BCA$ $cs. ACB : ct. BC :: cs. ACD : ct. CD$
V.	$\begin{array}{ c } \hline BC \\ \hline CD \end{array}$ B	C	$ct. B : R :: cs. BC : ct. ACB$ $ct. BC : cs. ACB :: ct. CD : cs. ACD$
VI.	$\begin{array}{ c } \hline BC \\ \hline CD \end{array}$ B	BD	$ct. BC : R :: cs. B : t. AB$ $cs. BC : cs. AB :: cs. CD : cs. AD$
VII.	$\begin{array}{ c } \hline B \\ \hline D \end{array}$ BC	BD	$ct. BC : R :: cs. B : t. AB$ $ct. B : s. AB :: ct. D : s. AD$
VIII.	$\begin{array}{ c } \hline B \\ \hline D \end{array}$ BC	C	$ct. B : R :: cs. BC : ct. ACB$ $cs. B : s. ACB :: cs. D : s. ACD$
IX.	$\begin{array}{ c } \hline BC \\ \hline BD \end{array}$ B	CD	$ct. BC : R :: cs. B : t. AB$ $cs. AB : cs. BC :: cs. AD : cs. CD$
X.	$\begin{array}{ c } \hline B \\ \hline C \end{array}$ BC	D	$ct. ABC : R :: cs. BC : ct. ACB$ $s. ACB : cs. ABC :: s. ACD : cs. ADC$



C H A P. X.

Shewing how to resolve all the Cases both of right and oblique angled spherical Triangles by Projection; that is, to project any Spherical Triangle to answer any of the Cases in spherical Trigonometry, by what is given in each Case.

BEfore I begin with the several Cases, it will be necessary to shew how to project any Angle within a primitive Circle, according to any number of Degrees given; and also to project any greater or lesser Circle of the Sphere, and to find its Pole, and to measure any part of a great Circle; and to measure any spherical Angle, &c.

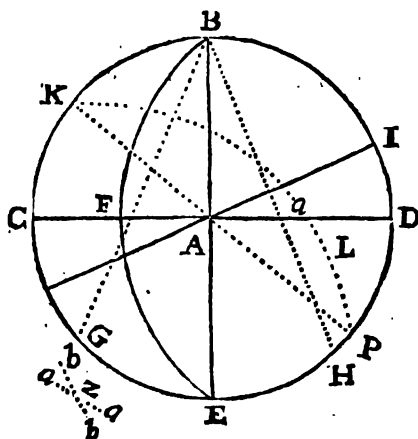
PROBLEM I.

To find the Pole of any great Circle.

In this Problem are three Cases.

CASE 1. The Pole of the primitive Circle is required.

By Definition the second, the Pole of a great Circle is 90 Degrees from it, therefore the Center of the primitive Circle is the Pole thereof.



CASE 2. The Pole of a right Circle is required.

Rule. From the Chords lay 90 Deg. on the primitive Circle from C or D, both ways, to B or E; so is B or E the Pole required.

CASE 3. The Pole of an oblique Circle is required.

Rule. The Pole of the oblique Circle BFE is required: CD being drawn at right Angles to the oblique

oblique Circle, and BE at right Angles to that; draw BG thro F, and from G lay 90 deg. of Chords to H, and draw BH, which will cut AD in a, the Pole of the oblique Circle required. Or the half Tangent of the Complement of AF, set from A to a, gives the Pole.

PROBLEM II.

To describe a spherical Angle.

In this Problem are two Cases.

CASE 1. To make an Angle whose angular Point may be at the Center of the primitive Circle.

Such an Angle is made in all respects like a plain Angle.

Example. It is required to make the Angle DAI equal to $23^{\circ} 30'$, whose angular Point A is the Center.

1. With a Chord of 60° (on the Center A) describe the primitive Circle BDEC; draw the Diameter CD thro the Center, take $23^{\circ} 30'$ from the same Line of Chords, and set from D to I, and draw AI; and it is done.

CASE 2. To make an Angle whose angular Point may be at the primitive Circle.

Rule. With the Secant of the Angle draw a Circle within the primitive Circle.

Example. It is required to project an Angle of 46° deg. whose angular Point may be at B, in the primitive Circle.

From the Center A, set the half Tangent of 44° (the Comp. of 46°) to F, and with the Secant of 46° set from F upon the Diameter CD (extended if need be) describe the oblique Circle BFE, which shall cut the primitive Circle in the Points B and E, making the Angles

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 Angles CBF and CBF, each equal to 46 deg. as was required.

PROBLEM III.

To draw a great Circle thro any Point, so that it shall make any given Angle with the primitive Circle.

Rule. With the Tangent of the given Angle, and one Foot in the Center of the primitive Circle, strike an Arch; and with the Secant of the Angle, and one Foot in the given Point, cross the former Arch; which Crossing will be the Center of the Circle to be drawn.

Example. Let it be required to draw an oblique Circle thro the point L, so that it may make with the primitive Circle an Angle of $50^{\circ}00'$.

Note, That the point L must be so far from the Center A, that the Tangent from A, and the Secant from L, may intersect each other; otherwise it is impossible.

With the Tangent of 50° , and one Foot in A, strike an Arch *aa*; and with the Secant of 50° , cross it with the Arch *bb* in the Point Z; so shall Z be the Center of the Circle PLK required to be drawn; and if K and P be diametrically opposite, it is truly drawn, otherwise not.

PROBLEM IV.

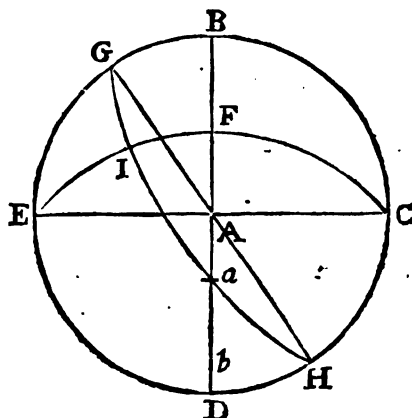
To draw a great Circle thro any two Points, given within the primitive Circle.

Rule. Draw a Line thro the Center of the primitive Circle, and also thro one (always the remotest) of the two Points; produce that Line till it cut the primitive Circle: cross it at right Angles with another Line thro the Center; and where this last Line cuts the primitive Circle, draw a Line from that Point, and thro the given Point, which the other Line passed thro;
 ON

In this Problem are Four CASES.

CASE 1. To draw the right Circle BAD at right Angles with the primitive Circle BCDE.

Draw the right Line thro the Center A, and it is done.



CASE 2. To draw a right Circle perpendicular to a right Circle.

This is done by drawing the Diameter EC at right Angles to the given right Circle BAD, the Points E and C being the Poles thereof.

CASE 3. To draw an oblique Circle at right Angles to a right Circle given.

Let BAD be the given right Circle; to draw the oblique Circle EFC.

Draw the oblique Circle thro the Poles E and C, and it is done. If it be required that the said oblique Circle shall make a given Angle with the primitive Circle, draw it, as by *Case 2. Prob. 1.* thus.

Suppose it to make an Angle of 40° , set off the half Tangent of 50° (its Complement) from A to F, and the

the half Tangent of 40° , set from A downwards to a , that Point will be the Pole thereof; and the Secant set from F to b , and upon b as a Center, strike the Circle EFC, which shall cut the right Circle BAD at right Angles, and the primitive Circle with an Angle of 40° ; as was required.

CASE 4. To draw an oblique Circle at right Angles to a given oblique Circle.

Rule. Find the Pole of the given oblique Circle, and thro that Pole draw a Circle that may cut the primitive Circle diametrically opposite; and it is done.

Example. Let it be required to draw another oblique Circle at right Angles to the oblique Circle EFC, in the last Figure.

Draw the Diameter GH, and draw a Circle thro the three Points G, a , H, and it will cut the Circle EFC at right Angles in the Point I, a being the Pole of the Circle EFC, as aforesaid.

Note, That if the Point I, in the oblique Circle be limited, then draw a Circle thro I, and the Pole a , by *Problem 3*. Or if it be required that the oblique Circle shall be drawn, so as to make a given Angle with the primitive Circle, then draw a Circle by *Prob. 2*.

PROBLEM VI.

To lay any number of Degrees on any great Circle.

In this Problem are three Cases.

CASE 1. To lay any Degrees on the primitive Circle.

Rule. This is done from a Scale of Chords, thus; to lay 40° from B, in the following Figure, take 40 from the Scale of Chords, and set from B to G; and it is done.

CASE 2. To lay any number of Degrees on a right Circle.

Rule.

the Diameter EG in a ; so shall a be the Pole of the Circle BHD ; or if the half Tangent of the Complement of AH be set from A towards E , it will fall upon the same Point a : then from C , set 51° (taken from the line of Chords) upwards to d ; then lay a Ruler upon a and d , and it will cut the oblique Circle in e ; so shall $H e$. contain 51 degrees, as was required.

PROBLEM VII.

To measure any part of a great Circle.

This Problem is the Converse of the last, and has likewise three Cases.

CASE 1. To measure any part of the primitive Circle.

Rule. Take the part to be measured in your Compasses, and lay it upon the Scale of Chords, and you will see how much: Thus, if EG be measured upon the Scale of Chords, it will be found to be 50 degrees.

CASE 2. To measure any part of a right Circle.

Rule. If the part to be measured lieth next the Center of the primitive Circle, measure it upon the first part of the Line of half Tangents; and if it lie next the primitive Circle, measure it upon the latter part of the half Tangents from 90 downwards. Thus if AF be measured upon the Line of half Tangents from the beginning of the Line, you will find it to be 50 deg. but if BF was to be measured, it will reach from 90° to 50° , so that it will contain 40° .

You may measure a part of the right Circle by Chords in the primitive Circle, thus: Lay a Ruler from E (one of the Poles of the right Circle BAD) and the Point F , and it will cut the primitive Circle in d ; then measure Cd upon the Chords, and you will find it 50° , as before.

CASE

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CASE 3. To measure any part of an oblique Circle.

Rule. Find the Pole of the given oblique Circle, then lay a Ruler to the Pole and to the extremity of the Part to be measured, and reduce it to the primitive Circle, and so measure it upon the Scale of Chords, as before.

Example. Let $H\epsilon$, in the oblique Circle BHD, be measured. Upon (a) the Pole, and (ϵ) , the Extremity of the part to be measured, lay a Ruler; and it will cut the primitive Circle in (d) ; measure Cd on the Line of Chords and you will find it 50° .

Again, Suppose the Arch ϵg (in the oblique Circle BHD) was to be measured; lay a Ruler from the Pole (a) to (ϵ) , and it will cut the primitive Circle in (d) ; and lay the Ruler from (a) to (g) , and it will cut the primitive Circle in (b) ; then measure db upon the Line of Chords, and you will find it $80^\circ 30'$, which is the quantity of ϵg required.

PROBLEM VIII.

To measure any Spherical Angle.

In this Problem are four Cases; and this is

A General Rule. A spherical Angle is measured by the Arch of a great Circle, intercepted between the two containing Sides, the angular Point being the Pole of that Circle.

Or, The distance of the Poles of the containing Sides, is equal to the measure of the contained Angle.

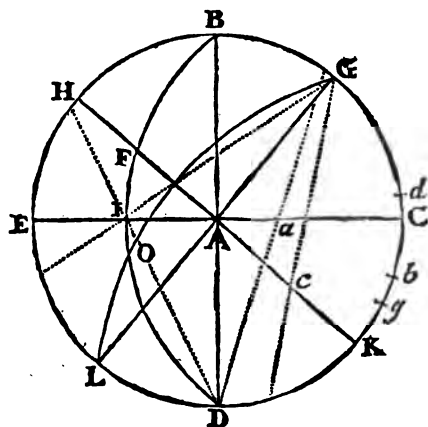
CASE 1. To measure an Angle, when the angular Point is the Center of the primitive Circle.

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Rule. Such an Angle is measured on the primitive Circle by a Line of Chords.

Example. Let the Angle GAC be to be measured.

Take CG, and measure it upon the Line of Chords, and you will find it to contain $50^{\circ} 30'$.



CASE 2. To measure an Angle, when the angular Point is at the primitive Circle.

Rule. Find the Poles of the two containing Sides; the distance of those two Poles, is the measure of the required Angle.

Example. Let the Angle FBH be to be measured; if the distance of A (the Pole of the primitive Circle) and (a), the Pole of the oblique Circle, be measured upon the Scale of half Tangents, you will find it to contain 38° . Or if EI be measured upon the half Tangents from 90, it will contain the same.

CASE 3. To measure an Angle, when the angular Point is not in the Center of, nor at the primitive Circle.

Rule. Find the two Poles of the two containing Sides, and reduce them to the primitive Circle; then measure the distance of them on the Line of Chords.

Example.

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Example. Let it be required to measure the Angle BFH. Lay a Ruler from the angular Point F, and (*a*) the Pole of the oblique Circle BFID; and it will cut the primitive Circle in (*b*): the distance of G (the Pole of the right Circle) and (*b*) measured upon the Scale of Chords, you will find to be 64° , the measure of the Angle required.

CASE 4. To measure an Angle, when both the containing Sides are Circles.

Let the Angle LOD be measured.

Lay a Ruler from the angular Point O, and the two Poles of the oblique Circles (*a*) and (*c*), and it will cut the primitive Circle in the Points *d*, and *g*: then measure the distance of *d g* upon the Line of Chords, and you will find it 35 degrees, which is the measure of the Angle LOD required.

PROBLEM IX.

To draw a Parallel Circle.

Definition. A Parallel, or lesser Circle, cutteth the Sphere into two unequal Parts, and lieth parallel to a great Circle.

In this Problem are three Cases.

CASE 1. To draw a Circle parallel to the primitive Circle.

Rule. With the half Tangent of its distance from the Pole, and one foot in the Center of the primitive Circle, draw a Circle.

Example. Let it be required to draw a Circle parallel to BCDE, and $66^{\circ} 30'$ distant from the Pole, or Center A.

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CASE 3. To draw a Circle parallel to an oblique Circle.

Rule. From the half Tangents lay the Parallel's distance from the Pole of the oblique Circle both ways, and note those two Marks, whose distance is the Diameter of the parallel Circle; then find the middle between those two Marks, and that shall be the Center.

Example. Let it be required to draw a Parallel to the oblique Circle BOD, at 25 deg. distance from it.

Find the Pole of the oblique Circle (*a*), and measure its distance by half Tangents from A the Center of the primitive Circle, which suppose to be 45° ; then add and subtract 45 to and from 65° (the Compl. of the distance) and the Sum is 110, and the Difference 20: then from the half Tangents set 110 from A, upon the Diameter EC extended, which will reach to (*b*) (a little beyond C;) and set 20° from A to *p*, the middle between *b* and *p* is at *c*, which is the Center: then with one Foot in *c*, and the other extended to *p*, draw the Parallel QP R, which will be the Parallel required.

I shall next project the several Triangles, from what is given, and resolve all the several Cases both in right and oblique angled spherical Triangles. And first, of right angled spherical Triangles.

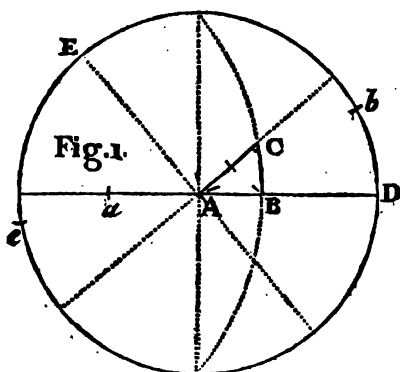
CASES 1, 2, 3.

There is given the Hypotenuse AC $46^\circ 31'$, and one of the oblique Angles, suppose A $40^\circ 00'$; with these to make a Rectangle spherical Triangle, and to find the other Parts.

1. To make the Triangle so, that the given Angle BAC may be at the primitive Circle's Center.

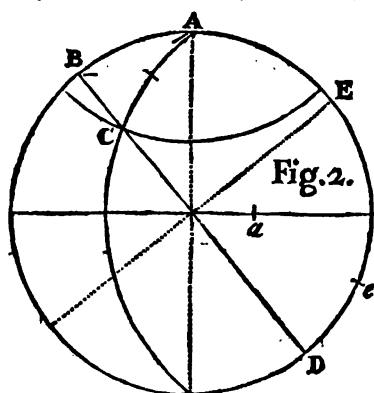
N 3

Having



Having drawn the primitive Circle, and cross'd it with the Diameter, make an Angle of $40^{\circ} 00'$ at the Center A; and set the half Tangent of $46^{\circ} 31'$, from A to C, and thro the point C; and at right Angles to AB draw the oblique Circle.

2. To make the Triangle so, that the given Angle may be at the primitive Circle.



After the primitive Circle is drawn and quarter'd, draw the oblique Circle (by *Prob. 2. Case 2.*) to make the Angle BAC 40° : and (by *Prob. 9. Case 2.*) at $46^{\circ} 31'$ distance from A, draw a Parallel to cut the oblique Circle in C; and thro C, and the Center of the

primitive Circle draw BD, to cut AB in B, at right Angles.

To measure the things required.

The Sides AB, and BC, are measured, by *Prob. 6.*
thus.

In *Fig.* the first, AB is measured by half Tangents, and BC is measur'd by laying a Ruler upon the Pole of the oblique Circle (*a*) and the point C, and it will cut the primitive Circle in *b*: then measure Db upon the Scale of Chords, and you will find it $27^{\circ} 48'$.

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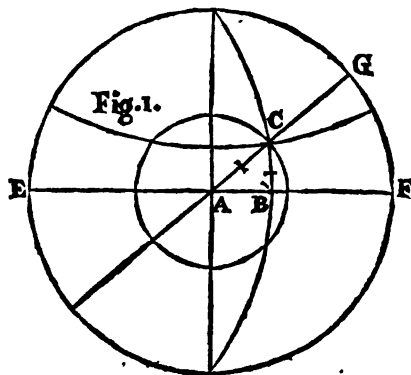
In Fig. 2. if BC be measured upon the half Tangents from 90 downwards, you will find it $27^{\circ} 48'$; and AB measured upon the Line of Chords, you will find $38^{\circ} 56'$.

The Angle C is measured by *Prob. 8.* thus.

Lay a Ruler upon the Pole of the oblique Circle (α), and upon C the angular Point, and it will cut the primitive Circle in (ϵ); the extent from (ϵ) to E measured upon the Scale of Chords will be $60^{\circ} 00'$, the quantity of the Angle C.

CASES 4, 5, 6.

There is given the Hypothenuſe AC, and one of the Sides, ſuppoſe BC : with theſe to make a right-angled Triangle, and to find the other Parts.



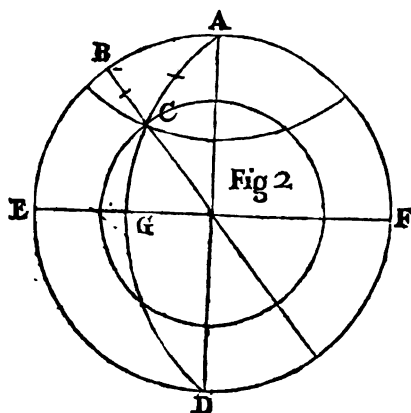
1. Having drawn the primitive Circle, and quartered it, by *Prob. 9.* draw a Circle parallel to the primitive, at the distance of $AC\ 46^{\circ}\ 31'$ from the Pole A; that is, with the half Tangent of $46^{\circ}\ 31'$, with one Foot on A, describe the Circle. Again, by

184 Spherical Trigonometry. Part I.

Prob. 9. draw a Circle parallel to EF , at the distance of $BC\ 27^{\circ} 48'$, which cuts the other Parallel in C . Then by *Case 3.* of *Prob. 5.* draw the great Circle at right Angles to AB , and thro the Point C , and it is done.

2. To make the Triangle, that one of the oblique Angles may be at the primitive Circle.

Draw a Circle parallel to the right Circle EF (by *Prob. 9.*) and at the distance of $46^{\circ} 31'$ from A ; then (by the same Problem) draw a Circle parallel to the primitive, at the distance of BC from it, which will cut the other in C . Then thro the three Points A , C and D , draw the oblique Circle; and lastly, draw the right Circle BC , thro the point C , and thro the Center of the primitive Circle.



To measure the things required.

The Side AB in *Fig. 1.* is measured by half Tangents, as in the last; and in the second Figure, is measured by Chords, *Prob. 7.*

The measure of the Angle BAC , *Fig. 1.* is FG measured upon the Chords; and the measure of BAC , *Fig. 2.* is EG , and is measured upon half Tangents from

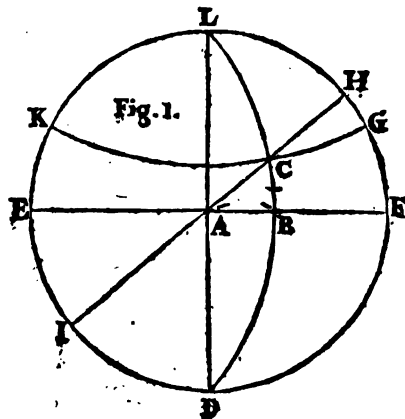
Chap. 10. Spherical Trigonometry. 185
 from 90 downwards; and the Angle C is measured
 by *Prob. 8*. See the last.

So the measure of the Side AB will be found $38^{\circ} 56'$,
 the Angle A $40^{\circ} 00'$, and the Angle C $60^{\circ} 00'$.

CASES 7, 8, 9.

*There is given the Side BC, and the opposite Angle
 A; to make a right angled spherical Triangle,
 and to find the other Parts.*

1. Make the Angle A 40° (by *Prob. 2.*) by setting
 the Chord of 40 from F to H, and draw IH thro
 the Center A; then draw the Circle GK parallel
 to EF, at the distance of BC $27^{\circ} 48'$, which will
 cut HI in C; then thro the three Points LCD, draw
 the oblique Circle.



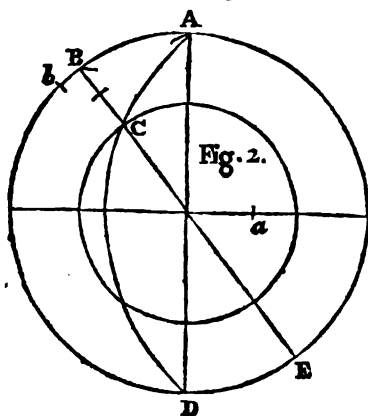
2. To make the Triangle, that one of the oblique
 Angles may be at the primitive Circle.

Draw

Spherical Trigonometry.

Part I.

Draw the oblique Circle ACD, making an Angle



at the primitive Circle A of 40° (by *Prob. 3 Case 2.*) Then draw a Circle parallel to the primitive Circle at $27^{\circ} 48'$ distance from it; that is, with the half Tangent of $62^{\circ} 12'$ upon the Center of the primitive Circle, draw the parallel, to cut the oblique Circle in C; then thro C, and the Center, draw the right Circle EB.

To measure what's required.

The Hypotenuse AC, and Side BA, are measured by *Prob. 7*.

In *Fig. 1.* they are both measured by half Tangents; and in *Fig. 2.* AB is measured upon the Scale of Chords; and AC is measur'd by reducing it to the primitive Circle, thus. Lay a Ruler upon *a*, the Pole of the oblique Circle, and upon the Point C, and it will cut the primitive Circle in *b*; then *Ab*, measured upon the Chords, will be found $46^{\circ} 31'$: The Angle C is measured as before directed. See *Prob. 8.*

CASES 10, 11, 12.

There is given the Side AB, and the adjacent Angle A; to make the right angled spherical Triangle, and to find the other Parts.

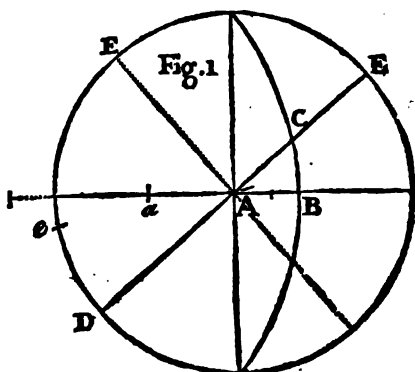
1. To make the Triangle, that the Angle A may be at the Center of the primitive Circle.

Having

Chap. 10. Spherical Trigonometry. 187

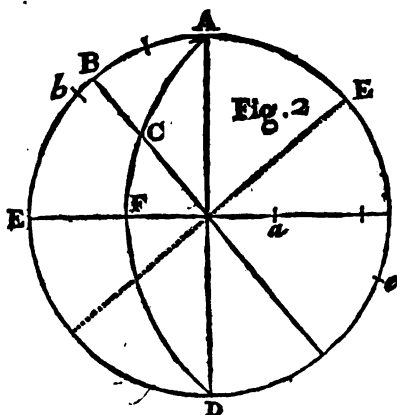
Having described the primitive Circle, and quarter'd it as before; make the Angle A $40^{\circ} 00'$ (by *Prob. 2. Case 1.*)

From the Scale of half Tangents take $38^{\circ} 56'$, and set from the Center A to B, and thro B (by *Prob. 5. Case 3.*) draw the oblique Circle at right Angles to AB, which will cut the right Circle AC in C.



2. To draw the Triangle ABC, so that the Angle A may be at the primitive Circle.

Draw the oblique Circle ACD, to make an Angle with the primitive Circle at A of 40° ; then from the Line of Chords, set $38^{\circ} 56'$, from A to B; and by the Point B, and Center of the primitive Circle, draw the right Circle, cutting AB at right Angles in B; and it is done,



To measure what is required.

The Side BC is measured by half Tangents, as before directed; the Hypothenuse AC is measured by reducing it to the primitive Circle, thus. Lay a Ruler

188 Spherical Trigonometry. Part I.

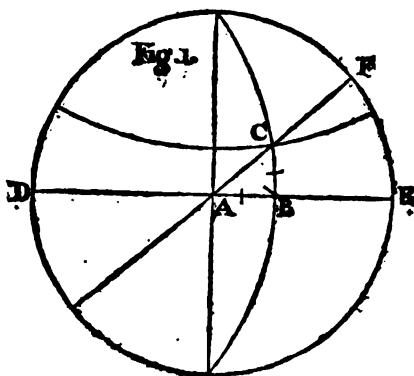
Ruler to the Pole of the oblique Circle a , and to the point C , and it will cut the primitive Circle in b ; measure Ab upon the Line of Chords, and you will find it $46^{\circ} 31'$, and so much is the Hypothenufe AC .

The Angle C is measured as before, that is, lay a Ruler upon the angular point C , and upon a the Pole of the oblique Circle, and it will cut the primitive Circle in e : then measure Ec upon the Line of Chords, and you will find it 60° .

CASES 13, 14.

The two Sides AB and BC being given, to make the right angled spherical Triangle, and to find the other Parts,

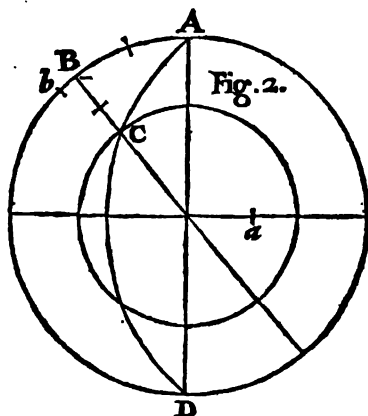
1. Having drawn the primitive Circle and quarter'd it, take $38^{\circ} 56'$ from the Scale of half Tangents, and set from A to B , and thro B draw the oblique Circle at right Angles to AB ; then draw a Circle parallel to DE , at the distance of BC $27^{\circ} 48'$: then thro the Point C , and the Center A , draw the right Circle AC .



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2. To draw the Triangle ABC, so that the Angle A may be at the primitive Circle.

With the half Tangent of $62^{\circ} 12'$ (the Comp. of BC) draw the Parallel; then take the Chord of $38^{\circ} 56'$, and set it from A to B, and draw the right Circle from the Point B, and thro the Center, to the Parallel in C; then thro C draw the oblique Circle ACD.



To measure the things required.

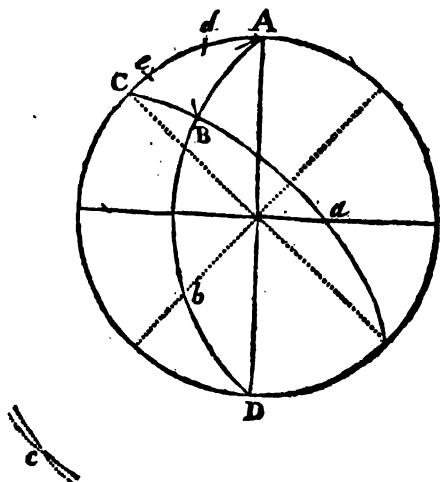
The Hypotenuse AC is measured by half Tangents in Fig. 1. and in Fig. 2. AC is measured by reducing of it to the primitive Circle, by laying a Ruler upon *a*, the Pole of the oblique Circle, and to the Point C, and it will cut the primitive Circle in *b*; measure *Ab* upon the Line of Chords, and you will find it $46^{\circ} 31'$: The Angle ACB is measured by reducing it to the primitive Circle, as before directed.

CASES

CASES 15, 16.

The two Angles A and C being given, to make the right angled spherical Triangle, and to find the other Parts.

Having drawn the primitive Circle, and quarter'd it, draw the oblique Circle ABD, to make an Angle with the primitive Circle at A of $40^{\circ} 00'$.



Then thro the Pole of the said oblique Circle a , draw another oblique Circle, to make an Angle with the primitive Circle at C , of $60^{\circ} 00'$. Thus, with the Tangent of 60° , one Foot in the Center of the primitive Circle, with the other Foot strike an Arch, as at c ; and with a Secant of 60° , and one Foot in a , cross the said Arch, and where that crossing is, shall be the Center of the oblique Circle $C B a$; which oblique Circle will cut the other oblique Circle at right Angles in B , (by *Definis. 3. Chap. 6.*) and will cut the primitive

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primitive Circle with an Angle of 60 Degrees, by Problem 3.

To measure the things required.

The Hypothenufe AC is measured upon the Chords; the other Sides are measured by reducing them to the primitive Circle, thus. Lay a Ruler upon b , the Pole of the Circle CB a , and upon B, and it will cut the primitive Circle in d : measure Cd upon the Chords, and it will be found $27^{\circ} 48'$, the measure of the Side BC; and a Ruler laid upon a and B, will cut the primitive Circle in e : measure Ae upon the Line of Chords, and you will find it $38^{\circ} 56'$, the measure of the Side AB required.

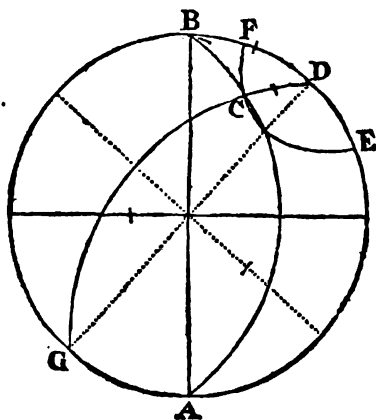
Thus have we resolved all the Cases of right angled spherical Triangles; and next,

Of Oblique Angled Spherical Triangles.

CASES 1, 5, 6.

There are given two Sides, and an Angle opposite to one of them; to find the rest.

Let BD $42^{\circ} 09'$, CD $24^{\circ} 04'$, and the Angle B $36^{\circ} 08'$, be given, to project the Triangle BCD.



Note,

Note. The Angle opposite to the other given Side, ought to be foreknown, whether acute or obtuse; otherwise two several Triangles may be made from what is given.

On the primitive Circle make BD (the given Side joining to the given Angle) equal to $42^{\circ} 09'$; then (by *Prob. 2. Case 2.*) draw the oblique Circle ACB, making an Angle with the primitive Circle at B, of $36^{\circ} 8'$ (equal to the Angle given). Draw the Parallel ECF, to cut off CD, equal to $24^{\circ} 04'$. Lastly, draw the oblique GCD, thro the two Points C and D, and it is done.

If this last oblique Circle had been drawn thro the other Point, where the Parallel cuts the oblique Circle ACB, then the Angle C would have been acute, whereas now it is obtuse.

To measure the things required.

The Side is found by *Prob. 6.* and the Angles by *Prob. 7.* as is fully explained in the Cases foregoing; the Side $30^{\circ} 00'$, the Angle CDB $46^{\circ} 18'$, and the Angle BCD $104^{\circ} 00'$, or $76^{\circ} 00'$, according to what it is, whether obtuse or acute.

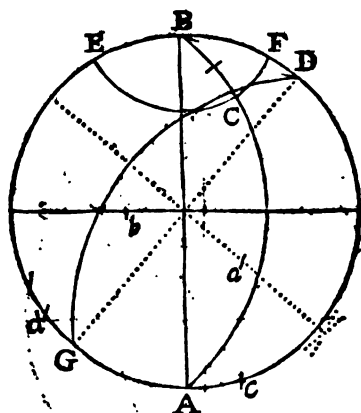
CASES 2, 8.

There are given two Angles, and a Side opposite to one of them; to find the rest.

Let the Angle B $36^{\circ} 08'$, and the Angle D $46^{\circ} 18'$, and the Side BC $30^{\circ} 00'$, be given, to project the Triangle BDC.

Draw

Draw the oblique Circle ACB, to make an Angle with the primitive Circle at B of $36^{\circ} 08'$; then at the distance of BC $30^{\circ} 00'$ draw the Parallel ECF, to cut the oblique Circle in C; then thro the Point C draw the oblique Circle GCD, to make an Angle of $46^{\circ} 18'$ at D, by *Prob 3*.



To measure the things required.

Find the Angle BCD (by *Prob. 8*.) thus: Lay a Ruler upon the Angle C, and upon *a* the Pole of DCG (one of the Legs,) and it will cut the primitive Circle in *c*; also lay it upon C, and upon *b*, (the Pole of the other Leg) and it will cut the primitive Circle in *d*; then measure *cd* upon the Scale of Chords, and you will find it 76° degr. whose Comp. to 180 is 104° , the Angle required.

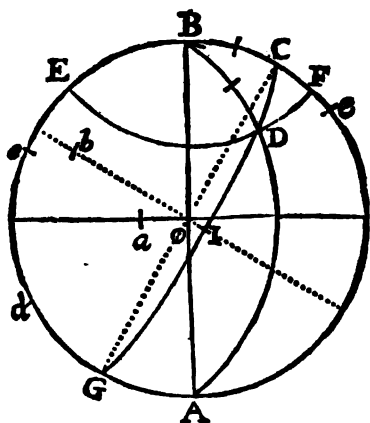
The Side CD is measured (by *Prob. 7*.) by reducing of it to the primitive Circle, and measuring it upon the Chords, and will be found $24^{\circ} 04'$.

The Side BD is measured upon the Chords, and will be found to be $42^{\circ} 09'$.

CASES 3, 9.

*There are given two Sides, and an Angle included;
to find the rest.*

Let the Side $BC\ 30^\circ\ 00'$, and the Side $BD\ 42^\circ\ 09'$, and the included Angle $B\ 36^\circ\ 08'$, be given, to make the Triangle BCD .



Make the Angle DBC equal to $36^\circ\ 08'$, by drawing the oblique Circle ADB ; then make $BC\ 30^\circ\ 00'$, and BD make $42^\circ\ 09'$, by drawing the Parallel EDF ; then thro C , D , and G , draw the oblique Circle.

To measure the things required.

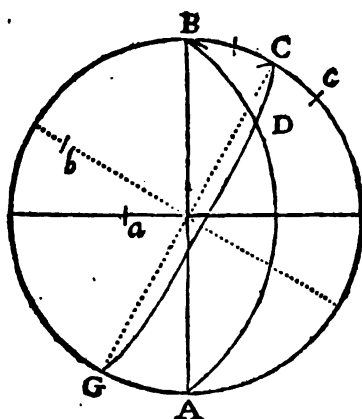
To measure the Angle D ; lay a Ruler upon D and a , and it will cut the primitive Circle in d ; and lay it upon D and b , and it will cut the primitive Circle in c ; measure dc upon the Chords, and you will find it $46^\circ\ 18'$: If OI be measured upon the half

half Tangents, you will find it $14^{\circ} 00'$, and so much is the Angle BCD above 90° , viz. $104^{\circ} 00'$. If a Ruler be laid upon b and D, it will cut the primitive Circle in e ; then measure Ce upon the Chords, and you will find it $24^{\circ} 04'$, which is the measure of DC.

CASES 4, 7, 10.

*There are given two Angles, and the interjacent Side ;
to find the rest.*

Let the Angle B $36^{\circ} 08'$, the Angle C $104^{\circ} 00'$, and the Side BC $30^{\circ} 00'$, be given, to make the Triangle BCD.



Make the Angle B equal to $36^{\circ} 08'$, by drawing the oblique Circle ADB; and make the Angle C 104° , by drawing the oblique Circle GDC, the Side BC being first set off from the Chords.

To measure the things required.

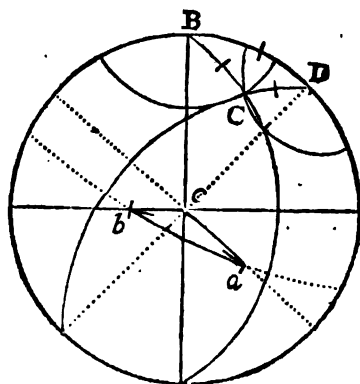
The Angle D and the Side DC are measured as in the last, and the Side BD is measured by laying a Ruler to the Pole of the oblique Circle *a*, and upon the Point D, which will cut the primitive Circle in *c*; then measure Bc upon the Line of Chords, and you will find it $42^{\circ} 09'$; and so much is the Side BD.

C A S E II.

The three Sides given, to find the Angles.

Let the Side BD $42^{\circ} 09'$, BC $30^{\circ} 00'$, and CD $24^{\circ} 04'$ be given, to make the Triangle.

On the primitive Circle make BD $42^{\circ} 09'$ the greater Side, then at the distance of BC $30^{\circ} 00'$ draw a Parallel, and at the distance of CD $24^{\circ} 04'$ draw another Parallel, and thro the Point of Interfection draw the two oblique Circles; and it is done.



If

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If the Great Circle ab be drawn thro the two Poles of the oblique Circles, and Lines be drawn from the said Poles to the Center or Pole of the primitive Circle, these three will constitute the Triangle abc whose Sides are equal to the Angles of the other Triangle BCD, and contrariwise the Angles of abc are equal to the Sides of BCD, only the greater Angle in the one, is equal to the Complement of the greater Side in the other to 180 degrees; that is, the Side bc is equal to the Angle CBD, and the Side ac to the Angle CDB, and the Side ab is equal to the Complement of the Angle BCD.

C A S E 12.

The three Angles being given, to find the Sides.

This Case is the same as the last in effect; for it is but changing the Angles into Sides, as is above shewed, (and as it is demonstrated Chap. 6. Def. 6.) and making a Triangle of those Sides, as in the last Case; and it is done.

Thus have I shewn how to resolve all Cases of spherical Triangles, both right and oblique-angled, by Projection.

A N

APPENDIX.

CHAP. I.

The Resolution of all the Cases of Plain Triangles, both right and oblique-angled, by Natural Arithmetick.

And first, of right-angled Triangles,

THE RULE.

1. **A** LWAYS divide 172 (with a competent Number of Cyphers annexed) by the Degrees and Decimal parts of a Degree contained in the lesser acute Angle.
2. From the Square of this Quotient always subtract 3, and out of the Remainder extract the square Root.
3. Subtract this square Root from the double of the Quotient, $\frac{1}{2}$ of the Remainder shall be the Hypotenuse,

4. Sub.

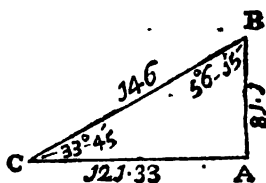
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4. Subtract the double of the Hypothenufe from the said Quotient, the Remainder shall be the greater of the Sides; the lesser Side being always assumed 1.000, &c.

From these assumed Sides we may easily find the true Sides, by the Rule of Three.

CASE I.

In the Triangle ABC, right-angled at A, there are given the Hypothenufe BC, and the acute Angles, B and C; to find the Base AC, and Perpendicular AB.



From the double Quotient (in § the following Page) 10.192
Subtract the square Root. 4.792

Remains 3)5.400

$\frac{1}{2}$ of the Remainder is the Hypoth. 1.800
The Hypoth. doubled is 3.600
Which subtr. from the Quot. 5.096

The Remainder is AC 1.496
And the Side AB is supposed 1.

By these three assumed Sides, the true Sides may be found (by Lib. 6. Pr. 4. of Euclid) by the Rule of Three; thus :

0 4

1.8

$$1.8 : 146 :: 1.00 : 81.1 \text{ AB}$$

$$1.8 : 146 :: 1.496 : 121.33 \text{ AC.}$$

That is, as the assumed Hypotenuse 1.8, is to the true one 146; so is the assumed Perpendicular 1.00, to the true one 81.1, &c.

$$33.75) 172.00000 (5.096$$

$$\begin{array}{r} 17200000 \\ 1087500 \\ \hline 632500 \\ 303750 \\ \hline 328750 \\ 212500 \\ \hline 116250 \\ 23250 \\ \hline 93000 \\ 18600 \\ \hline 74400 \\ 14880 \\ \hline 59520 \\ 11904 \\ \hline 47616 \\ 9523 \\ \hline 38093 \\ 7618 \\ \hline 30475 \\ 6095 \\ \hline 24380 \\ 4876 \\ \hline 19504 \\ 3901 \\ \hline 15603 \\ 3121 \\ \hline 12482 \\ 2496 \\ \hline 9986 \\ 1997 \\ \hline 7989 \\ 1598 \\ \hline 6391 \\ 1278 \\ \hline 5113 \\ 1023 \\ \hline 4090 \\ 818 \\ \hline 3272 \\ 654 \\ \hline 2618 \\ 524 \\ \hline 2094 \\ 419 \\ \hline 1675 \\ 335 \\ \hline 1340 \\ 268 \\ \hline 1072 \\ 214 \\ \hline 858 \\ 172 \\ \hline 686 \\ 137 \\ \hline 549 \\ 110 \\ \hline 439 \\ 88 \\ \hline 351 \\ 70 \\ \hline 281 \\ 56 \\ \hline 225 \\ 45 \\ \hline 180 \\ 36 \\ \hline 144 \\ 29 \\ \hline 108 \\ 22 \\ \hline 86 \\ 17 \\ \hline 69 \\ 14 \\ \hline 55 \\ 11 \\ \hline 44 \\ 9 \\ \hline 35 \\ 7 \\ \hline 28 \\ 6 \\ \hline 22 \\ 4 \\ \hline 18 \\ 4 \\ \hline 14 \\ 3 \\ \hline 11 \\ 2 \\ \hline 9 \\ 1 \\ \hline 7 \\ 1 \\ \hline 6 \\ 1 \\ \hline 5 \\ 1 \\ \hline 4 \\ 1 \\ \hline 3 \\ 1 \\ \hline 2 \\ 1 \\ \hline 1 \\ 1 \\ \hline 0 \end{array}$$

$$5.096$$

$$5.096$$

$$30576$$

$$45864$$

$$25480$$

$$25.969216$$

$$3 \text{ Subtract}$$

$$22.969216 (4.792$$

$$16$$

$$87) 696$$

$$609$$

$$949) 8792$$

$$8541$$

$$9582) 25116$$

$$19164$$

$$5952.$$

CASE

C A S E II.

*The Angle at the Base, and Perpendicular given ;
to find the Base and Hypothenuſe.*

33.75) 172.00000 (5.096
The Work of this Caſe is in all reſpects the ſame
as in Caſe 1.

C A S E III.

*The Hypothenuſe and Perpendicular given, to find
the Baſe and Angles.*

From the Square of the Hypo-	}	21316	}	Lib. 1. Pr. 47. of Euclid.
thenuſe				
Subtract the Square of the Per-	}	6577.21	}	
pendicular				
There remains		14738.79		

The Square Root of 14738.79 is 121.4, the Baſe.
To find the Angles.

To the Hypothenuſe add half the Baſe, or greater
Side; then it will be, As that Sum is to the leſſer
Side, ſo is 86 to the leſſer Angle.

To 146 Hypothenuſe
Add 60.7 half Baſe

Sum 206.7

Then 206.7 : 81.1 :: 86 : 33.75°
The Compl. of 33.75 is 56.25°, the greater Angle.

C A S E

C A S E IV.

The Base and Perpendicular being given, to find the Hypothennuse and Angles.

The Square of the Base	14738.79	}	By <i>Eucl. Lib. 1.</i>
The Square of the Perpen.	6577.21		Prop. 47.
The Sum			21316.00

The square Root of the Sum is 146, the Hypoth.
Find the Angles as in the last Case ; thus :

To 146 the Hypothennuse
Add 60.7 half the Base
206.7

Then, $206.7 : 81.1 :: 86 : 33.75 \text{ deg. or } 33^{\circ} 45'$,
Whose Complement is $56^{\circ} 15'$.

Thus all the Cases in right-angled Plain Triangles are easily and readily solved ; and by the same Rules, and with the like ease and facility, may oblique-angled Plain Triangles be solved, as I shall make appear in the following Cases.

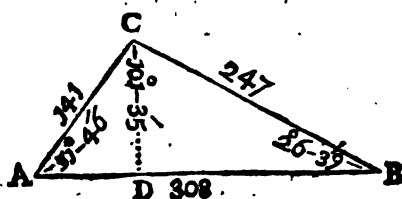
2. The

2. The Resolution of all the Cases of oblique-angled Plain Triangles by natural Arithmetick.

CASE I.

Two Angles, and a Side opposite to one of them, being given; to find the other two Sides.

Let the Angle A $51^{\circ} 46'$, and the Angle B $26^{\circ} 39'$, and the Side AC 141 be given; to find AB and CB.



First, we must reduce the given oblique-angled Triangle ABC into two right-angled ones, by letting fall the Perpendicular CD.

And first, in the right-angled Triangle ACD, we have given the Side AC, and the Angle A $51^{\circ} 46'$, whose Compl. $38^{\circ} 14'$ is the Angle ACD; then, according to the Rule, divide 172 by $38^{\circ} 14'$; thus:

$$\begin{array}{r} 38.23 \text{) } 172 \\ 3.82 \quad 17.2 \\ \hline 34.41 \text{) } 154.80 \text{ (4.4987} \end{array}$$

$$\begin{array}{r}
 2 \\
 \hline
 17160 \quad 8.9974 \\
 33960 \quad \hline
 4.4987 \quad 29910 \\
 78944 \quad 2382 \\
 \hline
 17995 \quad 20.238 \\
 1799 \quad 3. \quad \text{Sq. Root} \\
 405 \quad \hline
 36 \quad 17.238 \quad (4.1519 \\
 3 \quad \hline
 \end{array}$$

20.238 Q. fq.

From the double Quot.	8.9974
Subtr. the Square Root	4.1519

Remains 3)4.8455

$\frac{1}{4}$ is the Hypoth. AC	1.6151
---------------------------------	--------

The double of AC	3.2302
Which sub. from the Quot.	4.4987

Remains the Perpend. CD 1.2685

The Base AD being 1.

Then,

$$\begin{aligned} 1.6151 : 141 &:: 1 : 87.3 \\ 1.6151 : 141 &:: 1.2685 : 110.74 \end{aligned}$$

Then

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Then in the other Triangle CBD,
Divide 172 by the Angle B 26.65

$$26.65 \) \ 172.00 \ (6.454$$

$$\begin{array}{r} 12100 \\ 14400 \ 12.908 \\ 10750 \end{array}$$

$$\begin{array}{r} 6.454 \\ 6.454 \end{array}$$

$$\begin{array}{r} 25816 \\ 32270 \\ 25816 \\ 38724 \end{array}$$

$$\begin{array}{r} 41.654116 \\ 3 \text{ Sub.} \end{array}$$

$$38.654116 \ (6.217$$

$$\begin{array}{r} 122 \ 265 \\ 1241 \ 2141 \\ 12427 \ 90016 \\ 3027 \end{array}$$

From double Quot.	12.908
Sub. the square Root	6.217
Remains	3)6.691
$\frac{1}{2}$ of the Remainder is the Hypoth. CB	2.230
The Hypoth. doubled	4.460
Which Sub. from the Quot.	6.454
Remains the Side DB	1.994
	Then

3220) 17 20 00 4.4
15 29 2
1 37 10 0

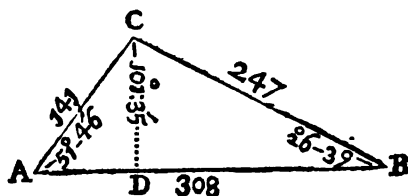
Then, $1 : 110.74 :: 2.23 : 247$; and

$1 : 110.74 :: 1.994 : 220.8$

Ans. $\left\{ \begin{array}{l} \text{The Segment of the Base AD is } 87.32 \\ \text{The Segment DB is } 220.85 \\ \text{The Perpendicular CD is } 110.74 \\ \text{The Side CB is } 247. \end{array} \right. 308$

CASE II.

Two Sides, with an Angle opposite to one of them, being given; to find the rest.



Let there be given the Sides AC 141, and CB 247, and the Angle B $26^{\circ} 39'$.

First, In the Triangle CBD,

$26.65) 172.00 (6.454$

Quot. squared 41.654116

Subtract 3

Remains $38.654116 (6.217)$

From the double Quot.

Subtract the square Root

12.908

6.217

Remains

6.691

2

$\frac{1}{2}$ of

Chap. 1. *by Natural Arithmetick.* 207

$\frac{1}{2}$ of the Rem. is the Hypoth. CB. 2.230

The Hypoth. doubled 4.460

Which subtr. from the Quot. 6.454

Remains the Side DB 1.994

Then to find the true Sides.

2.23 : 247 :: 1 : 110.74 = the Perpend. CD.

2.23 : 247 :: 1.994 : 220.8 = the Base DB.

141 = AC } Add and subtract.
110.74 = CD }

Sum 251.74

Diff. 30.26

Vid. Euclid, Lib. 3. Pr. 36.

151044

50348

75522

7617.6524 (87.3 Sq. Root = AD.

To find the Angles, and first in the Triangle CBD.

To 247 the Hypothennuse CB,

Add 110.4 half the Base DB

357.4 : 110.74 (CD) :: 86 : 26.65 degrees,
the Angle B, which is $26^{\circ} 39'$, whose Complement is
 $63^{\circ} 21'$, the Angle DCB.

To 141 the Hypothennuse AC,

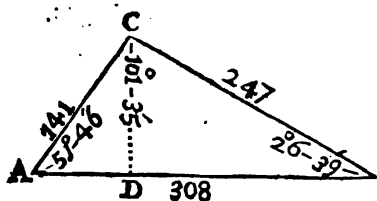
Add 55.37 half the Perpendicular CD.

196.37 : 87.3 (AD) :: 86 : 38.233 degrees,
that is $38^{\circ} 14'$, the Angle ACD, whose Complement
is $51^{\circ} 46' = CAD$.

CASE

C A S E III.

Two Sides, with the Angle comprehended by them, being given ; to find the rest.



Let the Sides AB 308, and AC 141, and the Angle A $51^{\circ} 46'$ be given.

In the Triangle ABCD, the Complement of the given Angle A is $38^{\circ} 14'$, the Angle ACD ; by which divide 172.

38.23)	172.00	(4.4987
	From the Sq. of the Quot.	20.2383
	Subtract	3

	Remains	17.2383
Whose sq. Root is		4.1519

From the double Q.	8.9974
Sub. the sq. Root	4.1519

Remains	4.8455
---------	--------

$\frac{1}{2}$ of the Rem. is Hyp.	1.6151
Which doubled is	3.2302
Which sub. from the Q.	4.4987

Remains CD	1.2685
------------	--------

Then, $1.6151 : 141 :: 1 : 87.3$, the Segm. AD.

And, $1.6151 : 141 :: 1.2685 : 110.74$, the Perpend. CD.

Then,

Then, in the Triangle CBD,
 From AB 308 To the Squ. of DB 48708.49
 Subtr. AD 87.3 Add the Squ. of CD 12263.3476
 Rem. DB 220.7 The Sum 60971.8376
 Whose square Root 247 is CB.

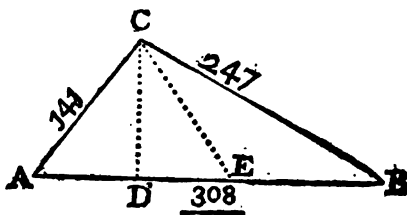
Then, to find the Angles,
 To 247 CB
 Add 110.4, half DB.

357.4 : 110.74 (CD) :: 88 : 26.65 degrees,
 the Angle B, which is 26° 39', whose Complement
 is 63° 21' the Angle DCB; then if DCB 63° 21' be
 added to ACD 38° 14', the Sum will be 101° 35', the
 obtuse Angle ACB.

C A S E IV.

The Three Sides being given, to find the Angles.

247
 141
 Sum 388
 Diff. 106



As the Base 308, to the Sum of the Sides 388;
 so is the Difference of the Sides 106, to the Difference
 of the Segments of the Base EB 133.53; which sub-
 tracted from the whole Base 308, the Remainder is
 174.47, the half whereof is 87.23 AD. And if the
 said half be added to EB 133.53 the Difference, the
 Sum will be 220.76 the other Segment DB.

P

Then

Then in the Triangle CBD,
 To 247 CB
 Add 110.38 half DB.

357.38 : 110.74 (CD) :: 86 : 26.65 degrees,
 the Angle B, that is $26^{\circ} 39'$, whose Complement
 $63^{\circ} 21'$ is DCB.

To 141 AC
 Add 55.37 half CD.

196.37 : 87.23 (AD) :: 86 : 38.233 degrees,
 that is $38^{\circ} 14'$, the Angle ACD, whose Complement
 is $51^{\circ} 46'$ the Angle CAD.

If DCB $63^{\circ} 21'$ and ACD $38^{\circ} 14'$ be added together, the Sum is $101^{\circ} 35'$, the whole Angle ACB.

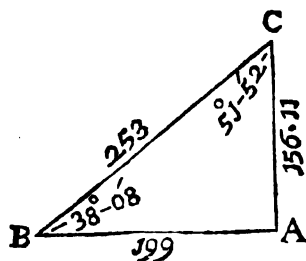
C H A P. II.

To find the Area of any Plain Triangle, by having certain Sides and Angles given.

1. Of Right-angled Triangles.

C A S E I.

The two acute Angles and the Base given; to find the Area.



In the right-angled Triangle ABC, let there be given the Angle B $38^{\circ} 08'$, and the Angle C $51^{\circ} 52'$, and the Base AB; to find the Area.

T H B

THE RULE.

To the double Logarithm of the Base, add the Log. Sine of the Angle at the Base; and from the Sum subtract the Log. Sine of the other acute Angle, the Remainder is the Logarithm of the double Area.

Log. of 199	2.298853
The same again	2.298853
The Sine of $38^{\circ} 8'$	9.790632
Sum	14.388338
The Sine of $51^{\circ} 52'$	9.895741
The Log. of 31088	4.492597

The half 15544 is the Area.

Demonstration. As the Sine of the Angle C, is to the Base; so is the Sine of the Angle B, to the Perpendicular. Here we must add the Logarithm of the Base, and Sine of the Angle at the Base B, and from the Sum subtract the Angle at the Perpendicular C, the Remainder is the Log. of the Perpendicular; to which if we add the Log. of the Base, we shall have the Log. of the double Area: so that it is plain, here is the Sine of the Angle at the Base B, and twice the Log. of the Base to be added, and the Sine of the Angle C to be subtracted. Which was, &c.

But we may find the Perpendicular otherwise, thus: As Radius, to the Base; so is the Tangent of the Angle at the Base, to the Perpendicular; and if to this Log. of the Perpendicular be added the Log. of the Base, the Sum is the Log. of the double Area; which affords us another Rule for finding the Area, by having the Base and acute Angles, viz. To the double Log. of the Base, add the Tangent of the

Angle at the Base, the Sum, abating Radius, is the Log. of the double Area.

The Log. of the Base 199	2.29885
The same again	2.29885
The Tangent of B $38^{\circ} 08'$	9.89489

The Log. 31088	4.49259
The half is 15544 the Area, the same as before.	

C A S E II.

The Hypothenufe and acute Angles are given; to find the Area.

THE RULE.

To the double Logarithm of the Hypothenufe, add the Sines of the acute Angles; the Sum, abating double Radius, is the Logarithm of the double Area.

In the former Triangle ABC, let the Hypothenufe 253, the Angle B $38^{\circ} 08'$, and the Angle C $51^{\circ} 52'$ be given; to find the Area.

The Log. of the Hypoth. 253	2.40312
The same again	2.40312
The Sine B $38^{\circ} 08'$	9.79063
The Sine C $51^{\circ} 52'$	9.89574

The Log. of 31088	4.49261
-------------------	---------

the same as before.

Demonstration. As Radius to the Hypothenufe, so is the Sine of the Angle at the Base to the Perpendicular.

And

Chap. 2. of Plain Triangles. 213

And as Radius, to the Hypothenuſe; ſo is the Sine of the Angle at the Perpendicular, to the Baſe.

If theſe two Proportions be compounded, there will be twice the Log. of the Hypothenuſe, and the Sines of the acute Angles to be added, and two Radius's to be ſubtracted: Which was, &c.

C A S E III.

The Baſe and Hypothenuſe, and Angle at the Baſe, given; to find the Area.

THE RULE.

Add the Logarithm of the Hypothenuſe, the Logarithm of the Baſe, and Sine of the Angle at the Baſe; and the Sum, abating Radius, is the Logarithm of the double Area.

In the former Triangle ABC, there is given the Hypothenuſe BC 253, the Baſe AB 199, and the Angle B $38^{\circ}08'$; to find the Area.

The Log. of the Hypoth.	253	2.40312
The Log. of the Baſe	199	2.29885
The Sine of the Angle B	$38^{\circ}08'$	9.79063

The Log. of 31088	4.49260
-------------------	---------

the ſame as before.

Demonſtration. As Radius, to the Hypothenuſe; ſo is the Sine of the Angle at the Baſe, to the Perpendicular. To which if the Log. of the Baſe be added, the Sum will be the Log. of the double Area: ſo here is the Log. of the Hypoth. the Sine of the Angle at the Baſe, and Log. of the Baſe, all added; and Radius ſubtracted. Which was, &c.

C A S E IV.

The Base and Hypothenuſe given, to find the Area.

T H E R U L E.

The Log. of the Sum, and Log of the Difference of the Baſe and Hypothenuſe being added together, take half their Sum; and to that add the Log. of the Baſe, this laſt Sum is the Log. of the double Area.

In the former Triangle AEC, there is given the Hypothenuſe 253, and Baſe 199; to find the Area.

Hypoth.	253	Log. of the Sum	452	2.655138
Base	199	Log. of the Diff.	54	1.732394
The Sum	452	The Sum		4.387532
The Diff.	54	The half		2.193766
		The Log. of the Baſe		2.298853
		The Log. of 31088		4.492619
		the ſame as before.		

Demonſtration. If the Sum and Difference of the Baſe and Hypothenuſe be multiplied together, and out of the Product the ſquare Root be extracted, that ſquare Root will be the Perpendicular (by *Eucl. Lib. 3. Pr. 36.*) Therefore half the Sum of the Log. of the Sum and Difference, is the Log. of the Perpendicular; to which add the Log. of the Baſe, the Sum is the Log. of the double Area. Which was, &c.

2. Of Oblique-angled Triangles.

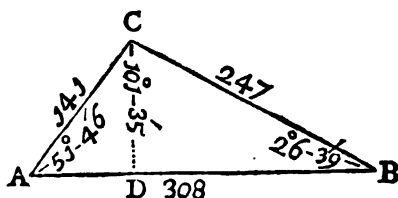
C A S E I.

All the Angles and one Side being given, to find the Area.

THE RULE.

Add the double of the Logarithm of the Side given, and the Sines of the two adjacent Angles, and from the Sum of these subtract the Sine of the opposite Angle, the Remainder (abating Radius) is the Logarithm of the double Area.

Let the Angle C $101^{\circ} 35'$ the Angle B $26^{\circ} 39'$ the Angle A $51^{\circ} 46'$ and the Side CB 247, be given; to find the Area.



The Log. of CB 247	2.392697
The same again	2.392697
The Sine of C $101^{\circ} 35'$ (or its Comp. $78^{\circ} 25'$)	9.991064
The Sine of B $26^{\circ} 39'$	9.651800

	The Sum	24.428258
The Sine of A $51^{\circ} 46'$ subtract		9.895144

Remains the Log. of 34128, the double }
Area } 4.533114

The half thereof 17064 is the Area required.

Demonstration. As Radius is to the Hypothennuse CB, so is the Sine of the Angle B to the Perpendicular CD.

And, as the Sine of the Angle A, is to the Side CB; so is the Sine of the Angle C, to the Base AB. Now if these two Proportions be compounded, we shall find the Log. of BC twice, the Sine of the Angle B, and Sine of the Angle C, all added; and the Radius, and Sine of the Angle A, to be subtracted. Which was to be proved.

C A S E II.

Two Sides, and the Angle comprehended by them, being given; to find the Area.

THE RULE.

Add the Logarithms of the given Sides, and the Sine of the Angle, and the Sum (abating Radius) is the Logarithm of the double Area.

Let there be given the Side AB 308, and the Side BC 247, and the comprehended Angle B $26^{\circ} 39'$; to find the Area.

The Log. of AB 308	2.488551
The Log. of BC 247	2.392697
The Sine of the Angle B $26^{\circ} 39'$	9.651800

The Log. of 34123, the double Area,	4.533048
-------------------------------------	----------

The half thereof, 17061.5, is the Area required.

Demonstration. As Radius to the Side CB, so is the Sine of the Angle B to the Perpendicular; to which

Chap. 2. of Plain Triangles, 217

which add the Log. of the Base AB, and you have the Logarithm of the double Area; so here is the Log. of the Side CB, the Sine of the Angle B, and the Log. of the Side AB, all added; and only Radius subtracted. Which was, &c.

C A S E HL

The three Sides being given, to find the Area.

THE RULE.

From the half Sum of the Sides subtract each particular Side; then add the Logarithms of the half Sum, and the three Differences together; half their Sum will be the Logarithm of the Area required.

Let the Side AB be 308, BC 247, and AC 141.

308	40	} Differences.
247	101	
141	207	

Sum 696

Half 348

The Log. of the half Sum 348	2.541579
The Log. of 207	2.315970
The Log. of 101	2.004321
The Log. of 40	1.602060

The Sum 8.463930

The Area 17060	Log.	4.231965
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This

This Rule is to be found in most Books of Measuring, and is demonstrated in my *COMPLETE MEASURER*, Part 2. Chap. 1. Sect. 5. and therefore I shall omit it here.

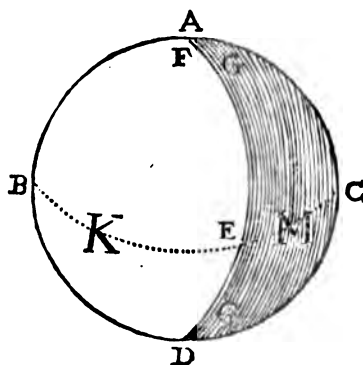
C H A P. III.

Of the Mensuration of the Area of a Spherical Triangle.

LEMMA I.

THE Lunary Superficies of the Hemisphericks, are as the Angles of the same Superficies.

The Proof of this Lemma (amongst many other ways) may be this :



Let the Meridian Semicircle AED be imagined to be equally mov'd over the Longitude of the Equator BEC, upon the Poles A, and D; then must the Angles on the other Side of A and D, (*viz.* F and G,) be as the Times; that is, F shall be to G, as the Super-

Superficies K to the Superficies M; and G is to F, as the Superficies M to the Superficies K &c. and so it will be howsoever they are taken. For

Those things that agree to a Third, agree amongst themselves.

COROLLARY.

Therefore, by Composition,

$$F+G : G :: K+M : M. \text{ Or,}$$

$$F+G : F :: K+M : K.$$

That is,

As two right Angles are to G,

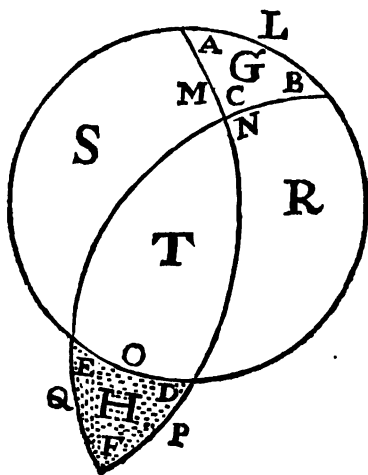
So is half the spherical Superficies to M;

And, as two right Angles are to F,

So is half the spherical Superficies to K.

LEMMA II.

The Triangle G is equal to the Triangle H, because the Angles and Sides of one are equal to the



Angles

Angles and Sides of the other, viz. $A = D$, $B = E$, $C = F$; and also $L = O$, $M = P$, $N = Q$; therefore they are congruous, and equal.

THEOREM.

The Excess of the three Angles over and above two right Angles, divided by 720, shews what the Area of the Triangle is in respect of the whole spherical Superficies.

For by
Lemma I. $\left\{ \begin{array}{l} 180 : A :: \frac{1}{3} \text{ Spherick} : G + R \\ 180 : B :: \frac{1}{3} \text{ Spherick} : G + S \\ 180 : C :: \frac{1}{3} \text{ Spherick} : G + T = H + T \end{array} \right.$

Therefore by Composition it will be, $180 : A + B + C :: \text{half the Spherick} : 3G + R + S + T$; and (by *Euc. 5. Lib. 24. Pr.*) $180 : A + B + C - 180 :: \text{half the Spherick} : 3G + R + S + T - \frac{1}{3} \text{ Spherick}$. But $G + R + S + T$ is equal to half the Spherick: Therefore, $3G + R + S + T - \text{half the Spherick} = 2G$. And the antecedent Terms being quadrupled, it will be, $720 : A + B + C - 180 :: 2 \text{ Sphericks} : 2G :: 1 \text{ Spher.} : G$. Therefore,

$\frac{A + B + C - 180}{720}$ shews what part the Triangle is of the whole Spherick.

These things are likewise true in all spherical Polygons of what ordinate Figure soever they be, or inordinate; so that all the Angles be given. And the Reason is, because all Polygons may be reduc'd into Triangles; therefore, this Rule following shall hold in all Multangles,

THE RULE.

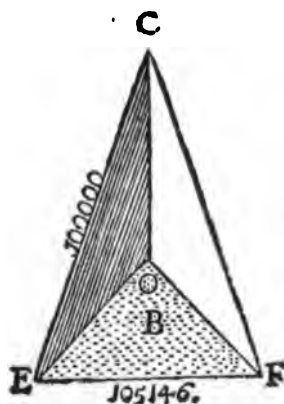
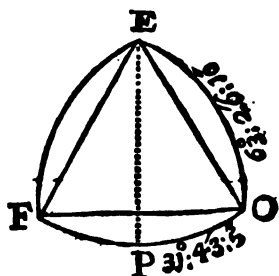
Multiply 180° by the number of Angles; subtract the Product out of the Aggregate of all the Angles

Chap. 3. *Of the Completion, &c.* 221
 increased by 360° ; the Residue divided by 720, gives
 the Area of the Polygon.

Of the Completion of a Solid Body.

From the foregoing Mensuration of the Area of a spherical Triangle, this *Corollary* may be deduc'd. If the Radius of the Sphere be 100000, the Side of an inscribed Icosiedron shall be 105146.22, equal to the Subtense of $63^\circ 26' 10''$. Therefore the plain equilateral Triangle FEO (in the Icosiedron) answers to the equilateral spherical Triangle in the Sphere; whose three spherical Triangles are connected in the plain Angles, in the same Point F, E, and O. And the Sides of this spherical Triangle are separately taken $63^\circ 26' 10''$, because their Subtenses FE, EO, OF, in the plain Triangle, are equal one to another.

Let fall the Perpendicular EP; then will the Spherical Triangle EPO be right-angled at P; where there is given (over and above the right Angle) EO $63^\circ 26' 10''$, and PO $31^\circ 43' 05''$ equal to half EO; wherefore the vertical Angle PEO will be found 36 deg. just, and the whole Angle E 72 deg. and the Sum of the three equal Angles E, F, and O, shall be 216 deg. from whence taking two right Angles, equal to 180 deg. there will remain 36 deg. therefore the



Triangle

222 *Of the Completion, &c.* Append.

Triangle EFO is $\frac{1}{12}$ of the whole Spherick, that is $\frac{1}{12}$ part; and this most truly, for 20 Pyramids FEOC, fill the solid place of the Icosiedron, and so 20 spherical Bases (cover'd over with 20 Triangular plain Bases) compleat the whole Spherick.

FEOC is one of the 20 Pyramids in the Icosiedron, and the plain Triangle B is one of the Bases; C is the Center of the Body, or Sphere, that circumscribes it.

TRIGO-

TRIGONOMETRY.

PART II.

CHAP. I.

In the first Part of this Treatise you have all the Cases of Plain and Spherical Trigonometry, very plainly laid down and demonstrated. In this second Part, I shall apply the Doctrine of Plain and Spherical Triangles to Practice, in all the most common Parts of the Mathematicks. And first,

Of Altimetria, or Measuring of Altitudes.

THE Object to be measured may be either accessible or inaccessible; or it may be upon level Ground, or standing upon a Hill.

SECT. I.

To measure an accessible Altitude.

Let AB represent a Tower, or Steeple, whose Height is required. First,

First, with a Quadrant, or other Instrument, find the Quantity of the Angle C, which suppose to be $52^{\circ} 30'$; then measure the Distance AC, which suppose to be 85 Feet: then by the first Case of Plain Triangles.

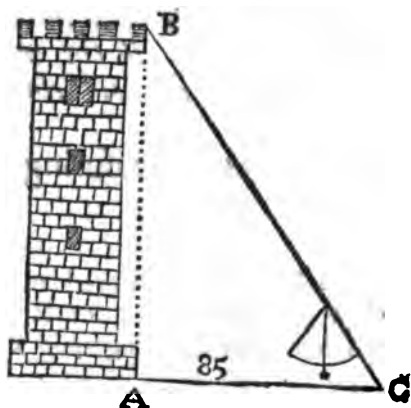
Ass. of $\angle B$ $37^{\circ} 30'$	A. C.	0.215553
To the Base AC 85	Log.	1.929419
So is s. of $\angle C$ $52^{\circ} 30'$		9.899466

To the Altitude AB 110.8	2.044438
--------------------------	----------

Or thus,

As Rad.	10.
To the Base AC 85	1.929419
So is t. of $\angle C$ $52^{\circ} 30'$	10.115019

To the Altitude 110.8	2.044438
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Note here, that in this and all such Cases, you must add the Height of your Eye, or Instrument, to the Altitude before found.

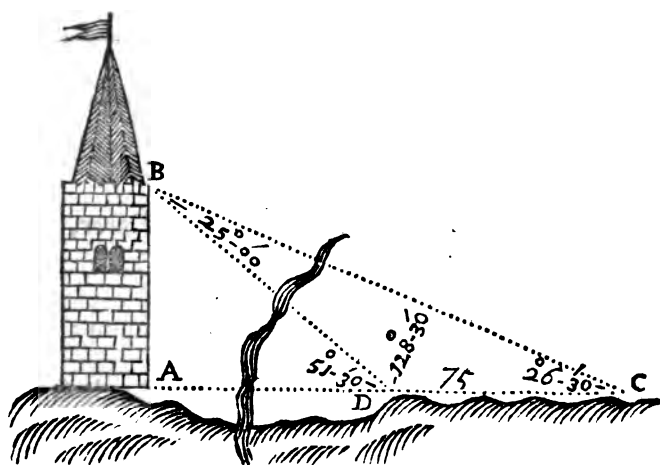
SECT.

S E C T. II.

To measure an inaccessible Altitude.

Let AB (in the following Fig.) be a Church-Steeple, whose Height is requir'd: but by reason of a River, or some other Obstacle, it is inaccessible, that is, you cannot come to the Foot of it at A.

First, with your Quadrant at C, take the Angle of Altitude, which suppose $26^{\circ} 30'$; then measure in a right Line towards the Steeple to D, which suppose to be 75 Feet; and at D, observe again the Angle of Altitude, which let be $51^{\circ} 30'$.



Now the two visual Lines CB, and DB, and the measured Distance CD, do form the oblique-angled Plain Triangle CBD, wherein are given all the Angles and the Side CD, the Angle BCD being $26^{\circ} 30'$; and the Complement of ADB $51^{\circ} 30'$ to 180 is the obtuse Angle BDC $128^{\circ} 30'$, and consequently the third Angle CBD is $25^{\circ} 00'$. But this Angle CBD

Q

may

may more readily be found by subtracting BCD from ADB (by *Eucl. Lib. 1. Pr. 32.*) Then, by Case the first of oblique-angled Plain Triangles, find the Side BD, thus :

As s. of $25^{\circ} 00'$ CBD	0.374052
To the distance of CD 75	1.875067
So is s. of $26^{\circ} 30'$ the $\angle C$	9.649527

To the visual Line BD 79.18	1.898640
-----------------------------	----------

Then, by Case 3. of right-angled Plain Triangles,

As Rad. to BD 79.18	1.898640
So is s. of ADB $51^{\circ} 30'$	9.893544

To the Altitude AB 61.97	1.792184
--------------------------	----------

S E C T. III.

To measure the Altitude of a Steeple, Tower, &c. standing upon a Hill.

Let EC represent a Steeple standing upon a Hill, whose Height is required.

First, with your Instrument find the Angle CAB, 44 degrees; and also the Angle EAB 26° .

Then measure in a strait Line towards the Steeple from A to D 134 Feet.

Then again at D, with your Instrument, find the Angle CDB $67^{\circ} 50'$; and also the Angle EDB 51 degrees.

Then, by the first Case of oblique-angled Plain Triangles, find the visual Line CD, in the Triangle ACD, wherein are given the Angles DAC, and ACD, and the Side AD; thus :

From the whole perpendicular Height BC 213.3
 Subtract the perpend. Height of the Hill BE 107.3

There remains the Height CE of the Steeple 106.

C H A P. II.

Of Longimetria, or measuring of Distances.

THE Theodolite, or Semicircle, is the best Instrument for taking of Distances of Trees, Steeples, Towers, &c. either of one, or many together; as in the Examples following.

S E C T. I.

To measure one single Distance.

Let A be a Tree; and you being a distance off at B, would find how far the Tree is from you, without approaching nearer to it, than you are at B.

1. Set up your Theodolite, or Semicircle, upon its Staff at B, as level as you can, laying the Index with the Sights upon it on the North and South Diameter of the Instrument.

2. Turn the Instrument about upon the Staff (the Index still lying upon the Diameter) till thro the Sights you see the Tree at A, and there fix the Instrument with the Screw, the beginning of the Degrees towards A.

3. From the Foot of your Instrument at B, measure with your Chain or Rod any convenient distance on either Side, as to C 50 Rods; and there cause a Mark or Staff to be set up.

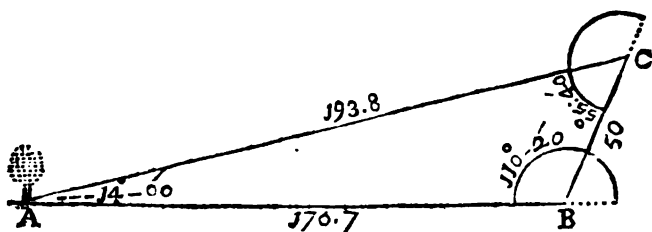
4. Turn

4. Turn the Index about till thro the Sights you see the said Mark at C, and note what number of Degrees and Minutes the Index cuts, which suppose $110^{\circ} 20'$, which set down.

5. Then set up some visible Mark at B, where the Instrument stood, and remove it to C; and there setting it up, with the Index upon the North and South Diameter as before, turn the Instrument about till thro the Sights you see the Mark set up at B, and there fix it, the beginning of the Degrees toward the Mark at B.

6. Turn the Index about till thro the Sights you see the Tree; and note what Degrees the Index cuts, which let be $55^{\circ} 40'$, which note down; and so have you done your Work in the Field.

And now to find the Distance from B, or C, to the Tree; you may perceive that the Tree, and the two Stations or Marks at B, and C, do make an oblique-angled Triangle ABC: in which is given, 1. The Angle ABC $110^{\circ} 20'$ observed at the first Station B. 2. The Distance measured BC 50 Rods. 3. The Angle CBA, observed at the second Station C $55^{\circ} 40'$. By which you may find the Distances AB, and AC, by the first Case of oblique-angled Plain Triangles. For, having the two Angles at B $110^{\circ} 20'$, and at C $55^{\circ} 40'$, their Sum is 166 degrees, which taken out of 180° , there remains 14 deg. for the Angle A. Then,



As s. BAC $14^{\circ} 00'$	Ar. C. 0.616325
To the measured Side BC 50 Rods	1.698970
So s. ACB $55^{\circ} 40'$	9.916859

To the Length of the Side AB 170.7	2.232154
------------------------------------	----------

Again,	
As s. BAC $14^{\circ} 00'$	Ar. C. 0.616325
To the Side BC 50	1.698970
So s. of the Angle ABC $110^{\circ} 20'$ (Com. $69^{\circ} 40'$)	9.972058

To the Length of the Side AC 193.8	2.287353
------------------------------------	----------

So the Distance from the first Station to the Tree is 170.9 Rods, and from the second Station to the Tree 193.8 Rods.

S E C T. II.

How to take the Distances of divers things remote from you ; as Churches or particular Places in a City, a Squadron of Ships at Sea, or the like ; and to make a Draught of the same.

Suppose that A, B, C, D, E, F, and G, were a Squadron of Ships lying at an Anchor, and you being on Shore, were desirous to take their Distances and to make a Draught of them, representing their Situation one from another.

1. Seek out two convenient Places upon the Shore, from either of which you may see all the Ships at one View, and make those two Places your two Stations for Observation, as Q and P.

Then

Then set up your Instrument at O upon its Staff, as level as you can. Turn it about upon the Socket, till the Needle hang directly over the Meridian Line of the Card, and the North-end of the Needle over the Flower-de-luce; and then screw the Instrument fast. Your Instrument being thus fix'd at O, call that your first Station; and turn the Index about till thro the Sights you see the first Ship A, and note what Degrees of the Instrument the Index cuts, which suppose 60 deg. which note down; then turn the Index about till thro the Sights you see the Ship at B, and mark what Degrees are cut by the Index, as $74^{\circ} 30'$, and note them down. Do thus with all of them, be there ever so many.

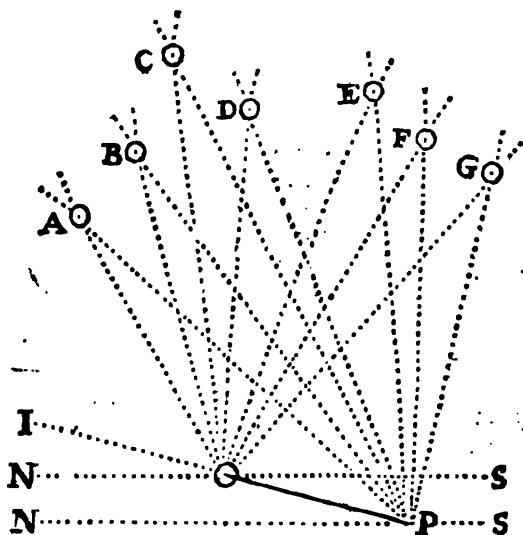
Then measure the Distance between O your first Station, and P your second Station, which is 240 Rods; turn your Index about till thro the Sights you see a Mark set up at P, and note what Degrees are cut, which are $15^{\circ} 50'$, which is the Angle your Station-line makes with the Meridian; then leave a Mark where your Instrument stood, and remove to your second Station at P, and set up your Instrument there, with the Needle over the Meridian-line, as before: and if you lay your Index upon $15^{\circ} 50'$, and look back thro the Sights, if you see the Mark left at your former Station, then your Instrument is plac'd right; if not, you must turn it till you can see the said Mark thro the Sights, and there screw it fast; then turn the Index about till thro the Sights you see the first Ship at A, and note what Degrees the Index cuts, which suppose $43^{\circ} 45'$, which note down; then turn the Index about to B, C, D, &c. noting the Degrees cut by the Index at every moving, and set them down in a Table, as is done in the following Page.

	At the first Station, the Index cut,		The Stationary distance	At the second Station, the Index cut,	
	De.	M.		De.	M.
At { A	60	00	240 Rods.	43	45
B	74	30	The Angle	54	00
C	82	30	the Stationary-	63	30
D	93	00	line makes with	69	00
E	110	20	the Meridian is	85	45
F	120	00	15° 50'.	92	00
G	130	00		103	00

How to protract, or lay down upon Paper, the Observations last taken.

Upon a Sheet of Paper draw an obscure Line as NS, for a meridian Line, upon which assume any Point as O for your first Station; unto which Point apply the Center of your Protractor, laying the strait Line of the Protractor upon the Line NS; then, keeping the Protractor in that Position, lay your Table of Observations before you, wherein you will find 60 deg. the first Observation made to the Ship A; therefore make a Mark at 60 deg. by the edge of the Protractor. The next Observation was 74° 30', therefore by the edge of the Protractor make a Mark at 74° 30'; do the like at 82° 30', at 93° 00', at 110° 20', at 120° 00', and at 130° 00': Then take off your Protractor, and from the Center O draw obscure Lines thro every one of those Marks, as the Lines OA, OB, OC, &c. Lay the Center of your Protractor upon O, and the strait Line upon NS, as before, and make a Mark at 15° 50' (being the Angle the stationary Line makes with the Meridian) and from the Center O thro that Mark draw the stationary Line, and out of some Scale of equal Parts (answerable to the largeness of your Paper)

per) take 240 (the stationary Distance) with your Compasses, and set from O to P, so shall P represent your second Station; then thro P draw another meridian Line parallel to the former, and lay the Center of your Protractor upon P, and the strait Line upon NS; keeping it fast in that Position, lay your Ta-



ble of Observations at your second Station before you, and with your Protracting-pin make Marks at $43^{\circ} 45'$ your first Observation, at $54^{\circ} 00'$ your second, at $63^{\circ} 30'$ the third, &c. till you have prick'd down all your Observations at your second Station P; then take away your Protractor and draw Lines from P, thro the several Marks, as PA, PB, PC, &c. cutting the former Lines in the Points A, B, C, D, E, F, and G, which Points do represent the several Ships, as they lie at Anchor, every one at its true Distance and Position from another, as they lie in the Sea or Harbour; as also from either of your Stations O or P.

This

This Section may otherwise be perform'd, without taking any notice of the Needle; thus:

As in the last Example, make choice of two Places, from either of which you may conveniently see all those Ships or other Objects whose Distances you would find; which two Places or Stations let be O and P, in the following Scheme.

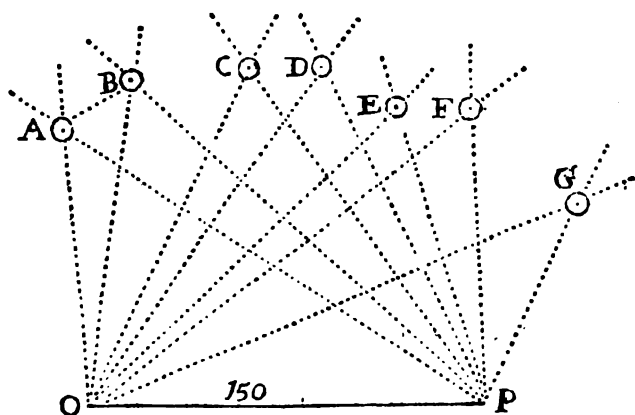
First place your Instrument at O, laying the Index on the Diameter, and turn the whole Instrument about, till thro the Sights you see your second Station P; then fixing the Instrument there, direct your Sights to the several Ships A, B, C, D, E, F, and G, noting the Degrees cut at each Observation, which are as below in the Table.

Then remove your Instrument to P, laying the Index on the Diameter, and turn it about till thro the Sights you espy your former Station at O; there fix your Instrument; then direct your Sights to every one of the Ships at A, B, C, D, &c. noting the Degrees cut at every Observation, as in the Table, and also note down the stationary Distance.

	At the first Station, the Index cut,			At the second Station, the Index cut,	
	De.	M.		De.	M.
At { A B C D E F G	82	30	The Stationary distance 150 Rods.	33	20
	95	40		42	30
	114	30		55	00
	124	10		63	40
	135	30		72	20
	140	00		86	10
	156	30		112	30

The Protraction of these Observations will be much like that in the former Example; for

First,



First, you must lay the Center of the Protractor upon the Center O, with the Diameter thereof upon the Line OP; keeping it fast, make Marks by the Limb at $82^{\circ} 30'$, at $95^{\circ} 40'$, at $114^{\circ} 30'$, at $124^{\circ} 10'$, at $135^{\circ} 30'$, at $140^{\circ} 00'$, and at $156^{\circ} 30'$; then take up your Protractor, and draw Lines from the Center O thro each of the Marks; then upon the Line OP set off 150 from O to P, taken from some Scale of equal Parts, and upon P place the Center of your Protractor, and the Diameter thereof upon the Line OP; keeping it fast there, make Marks by the Limb at $33^{\circ} 20'$, at $42^{\circ} 30'$, at $55^{\circ} 00'$, at $63^{\circ} 40'$, at $72^{\circ} 20'$, at $86^{\circ} 10'$, and at $112^{\circ} 30'$; then take up your Protractor, and draw Lines from the Center P thro every one of these Marks, and where these Lines cut the former each its correspondent Line, there are the Points representing the Ships.

To measure any of the Distances, thus laid down, by Trigonometrical Calculation.

In the last Figure are several right-lined Triangles made by the Intersection of the several visual Lines, together

gether with the Stationary-Line; in each of which there will be enough given to find what Distance you shall require.

Example. Let it be required to find the Distance from O your first Station, to the first Ship A.

By the visual Line OA, made at your first Station, and the visual Line PA, at your second Station, intersecting at A, and the Stationary-Line OP, is constituted the oblique-angled Triangle AOP : in which is given, 1. The Angle AOP $97^{\circ} 30'$ (the Complement of $82^{\circ} 30'$ to 180) and, 2. The Angle APO, observ'd at the second Station $33^{\circ} 30'$; consequently the Angle OAP must be $49^{\circ} 00'$ (being the Complement of the other two to 180) and, 3. The measured Distance between the Stations 150 Rods, to find the Side AO, which you may do by the first Case of Plain oblique-angled Triangles; thus :

As s. of OAP $49^{\circ} 00'$	0.1222201
To OP 150	2.1760912
So is s. APO $33^{\circ} 30'$	9.7418895
To AO 109.7	2.0402608

Again, to find the Distance AP,

As s. of the Angle OAP $49^{\circ} 00'$	0.1222201
To the Line OP 150	2.1760912
So is the Angle AOP $97^{\circ} 30'$	9.9962686
(Com. $82^{\circ} 30'$)	

To the Side AP, 197 Rods,	2.2945799

Again, to find the Distance from the second Station P, to the second Ship B.

In the Triangle OBP, there is given, 1. The Angle BOP $84^{\circ} 20'$ (being the Complement of the second Observation made at the first Station O) 2. The

Angle

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Angle OPB $42^{\circ} 30'$, the Angle taken at the second Observation at P; and 3. The Stationary distance 150.

Then by the first Case of oblique-angled Plain Triangles,

As s. of the Angle OBP $53^{\circ} 10'$	0.0967023
To the stationary Distance 150	2.1760912
So s. of the Angle BOP $84^{\circ} 20'$	9.9978725
To the Distance BP, 186.5	2.2706660

Then in the Triangle ABP, there is given the Angle APB $9^{\circ} 10'$ (being the Difference between the first and second Observations at the second Station) and the two comprehending Sides AP 197, and BP 186.5, to find AB.

Work by Case the 3d of oblique-angled Plain Triangles.

AP 197	From $180^{\circ} 00'$
BP 186.5	Subtr. 9 10
Sum 383.5	Rem. 170 50
Diff. 10.5	Half 85 25 $\frac{1}{2}$

Then,

As 383.5 the Sum of the Sides	7.416235
To 10.5 the Difference of the Sides	1.021189
So is t. $85^{\circ} 25' \frac{1}{2}$, $\frac{1}{2}$ the Sum of opp. Angle,	11.096807

To t. of 18 53 half their Difference	9.534231
Sum 104 18 $\frac{1}{2}$ = the Angle ABP.	

Diff. 66 32 $\frac{1}{2}$ = the Angle BAP.

Then,

Then, by Case I.

As s. $66^{\circ} 32' \frac{1}{2}$ the Angle BAP

0.037465

To 186.5 the Side BP

2.270679

So s. $9^{\circ} 10'$ the Angle APB

9.202234

To 32.39 the distance between A and B 1.510378

After the same manner you may proceed to find the rest of the Distances, there being enough given in each Triangle to find the rest; or you may take any of the Distances in your Compasses, and measure it upon the same Scale you laid down the stationary Distance by, and that will show you the Distance.

By either of those ways you will find the several Distances, as they are expressed in the Table below.

AO 109.7	CP 158.4	CD 27.7	OF 185.4
AP 197	OC 142.6	EP 117.8	EF 28.75
BP 186.5	BC 46.8	OE 160.2	PG 86.1
AB 32.4	DP 142.6	DE 31.6	OG 200.5
OB 126.6	OD 154.5	FP 119.4	FG 56.95

To resolve the first Example by Trigonometrical Calculation, you must note, That every one of the Observations made at the first and second Stations, are too much by $15^{\circ} 50'$ (the Angle which the Stationary-line makes with the Meridian) and therefore must be subtracted from each: Thus, from $60^{\circ} 00'$ (the Degrees cut at the first Observation, made at the first Station O) subtract $15^{\circ} 50'$, and there remains $44^{\circ} 10'$ for the Angle AOL, whose Complement to 180° is $135^{\circ} 50'$ the Angle AOP; and $15^{\circ} 50'$ subtracted from $43^{\circ} 45'$ (the Degrees cut at the first Observation, at the second Station) there will remain $27^{\circ} 55'$ for the Angle APO; and if you subtract $27^{\circ} 55'$ from $44^{\circ} 10'$, there will remain $16^{\circ} 15'$ the Angle OAP; and having all the Angles in the Triangle APO, and

one

one side OP, you may easily find the other Sides, as was shewed in the other Example: but the working I shall leave to the young Artift to perform, and proceed next to shew how the most remarkable Places in a Town or City may be taken and laid down in a Map.

S E C T. III.

How to take the Distances of the most remarkable Places in a Town or City.

Let A, B, C, D, E, F, G, H, K, L, and M be some eminent Places in a City, such as are nominated in the Table following; of which make choice of two, from each of which all the rest may be seen, as L St. *Giles's*, and M St. *Helen's*; then upon St. *Giles's* at L, set up your Theodolite, or Semicircle, laying the Index upon the North and South Diameter thereof; and then turn the Instrument about, till thro the Sights you see St. *Helen's* at M, and there fix it.

Then move the Index till thro the Sights you see the North-Gate, the Index cutting $37^{\circ} 30'$. Then direct it to the School-house, the Index cutting $77^{\circ} 00'$, and so to all the remarkable Places about the Town, noting down what Degrees the Index cut at every Place, which suppose to be such as are exhibited in the second Column of the following Table under Station I. St. *Giles's*.

Then removing from St. *Giles's* to St. *Helen's*, there set up your Instrument, laying the Index upon the North and South Diameter thereof, (as before) and turn the Instrument about till thro the Sights you see St. *Giles's*, and there fix it.

Then

Then turning the Index about, direct your Sights to the North-Gate, the Index cutting $16^{\circ} 30'$, which note down; then to the School-house, the Index cutting $36^{\circ} 45'$, and so to all the remarkable Places, as before; so shall you find the Degrees cut by the Index at the several Directions, to be such as are set down in the last Column of the Table, under Station II. *St. Helen's*.

The Names of the Places you observe.	Station I.		Station II.	
	St. Giles's		St. Helen's	
	deg.	min.	deg.	min.
A. North-Gate	37	30	16	30
B. School-house	77	00	36	45
C. Christ-Church	115	20	27	45
D. Prison-House	133	45	60	00
E. St. Benedi ~~ct~~ 's	168	00	140	15
F. South-Gate	187	30	197	30
G. The Hospital	210	45	269	00
H. The Town-Hall	216	30	248	45
K. The Old Tower	317	30	342	30

Having thus finished your Table of Observations, the next thing that you are to do, is to find the Distance between L *St. Giles's*, and M *St. Helen's*; which to do, (if you cannot conveniently measure it within the Town) you may go out thereof into some near-adjoining Field, or other open Place, where you may conveniently see both the Places; as suppose in a Field at O, and R: where,

1. Set up your Instrument at O, the Index lying on the North and South Diameter thereof, and turn the Instrument about till thro the Sights you see *St. Giles's* at L; and then fix the Instrument.

2. The Instrument thus fixed, direct the Sights to any Mark set up at a distance as at R, the Index there cut-

cutting 106 deg. and then to *St. Helens*, the Index cutting 40 deg.

3. Measure the Distance between O and R, which let be 470 geometrical Paces (of 5 Foot to a Pace) or 2350 Feet.

4. Set up your Instrument at R, the Index lying upon the Diameter, as before, turning it about till thro the Sights you see a Mark set up at your former Station at O, and there fix it: Then move the Index, till thro the Sights you see *St. Giles's* at L, the Index cutting 36 deg. then move it forwarder, till you see *St. Helens* at M, the Index cutting $65^{\circ} 30'$. And by these two Observations, the Distance between *St. Giles's* and *St. Helens* may be obtain'd by the Directions in the foregoing Section. For,

In the Triangle LOR, you have given, (1.) The distance OR 2350 Feet. (2.) The Angles LOR 106 deg. and LRO 36 deg. the Sum of them is 142 deg. whose Complement to 180° is 38 deg. for the Angle OLR, to find LO. Then,

By the first Case of oblique-angled Triangles;

As s. $38^{\circ} 00'$ OLR	0.2106580
To OR 2350	3.3710679
So s. $36^{\circ} 00'$ LRO	9.7692187
To LO 2244	3.3509446

Again,

Ass. OLR $38^{\circ} 00'$	0.2106580
To OR 2350	3.3710679
So s. LOR 106° (Com. 74°)	9.9828416
To LR 3669	3.5645675

R

Again,

Again, in the Triangle OMR, there is given,
 (1.) The Side OR 2350. (2.) The Angle ORM
 $65^{\circ} 30'$. (3.) The Angle MOR 66° deg. and also the
 Angle OMR $48^{\circ} 30'$, to find MR, by Case I. thus :

As s. OMR $48^{\circ} 30'$	0.1255439
To OR 2350	3.3710679
So s. MOR $66^{\circ} 00'$	9.9607302
To MR 2866	3.4573420

Then for the Distance LM, in the Triangle LRM;
 there is given, (1.) The Side LR 3669. (2.) The Side
 MR 2866. (3.) The Angle LRM $29^{\circ} 30'$, compr-
 hended between LR and MR ; to find the Side LM, by
 Case III. of oblique-angled Triangles.

As the Sum of the Sides 6535	6.1847544
To the Difference of the Sides 803	2.9047155
So is t. of half the Sum of the op- posite Angles, $75^{\circ} 25'$	10.5795854
To the t. of half their Diff. $25^{\circ} 01'$	9.6690553

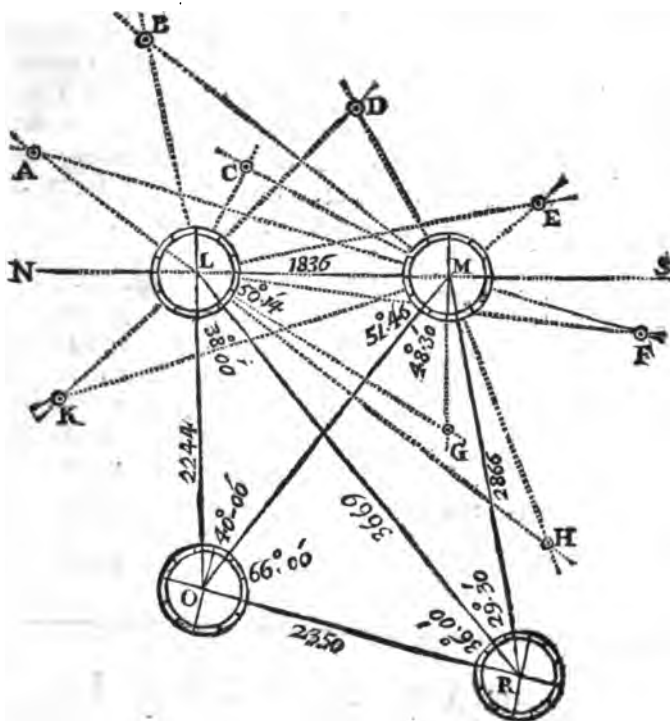
Sum	100 16 = LMR
Diff.	50 14 = MLR

Then by Case the first,

As s. MLR $50^{\circ} 14'$	0.1142681
To MR 2866	3.4573420
So is s. LRM $29^{\circ} 30'$	9.6923388
To LM 1836	3.2639489

Thus

Thus having found the Distance between *St. Giles's* at L, and *St. Helens* at M, to be 1836 Feet, you may lay down all the other Places, according to their true Distances and Situations one from another, by the Directions of the second Section before-going, and as you see done in the Figure below.



For the Trigonometrical Calculation of the Distances, you must observe the Directions in the second Section of this Chapter, and second Example; only observe that all Places which fall below the Line LM, as the Places represented by F, G, H, and K, you must take 180° out of the Degrees and Minutes taken at *St. Giles's*. Thus the Degrees cut in making the Ob-

ervation to F from St. *Giles's*, are $187^{\circ} 30'$, out of which take 180° , and the remainder is $7^{\circ} 30'$, the Angle MLF: and to find the Angle at M, take the Complement of the Degrees to 360. Thus the Degrees observed at St. *Helens* to F are $197^{\circ} 30'$, which subtracted from 360, the remainder is $162^{\circ} 30'$, which is the Angle LMF; and the Sum of $162^{\circ} 30'$ and of $7^{\circ} 30'$ is 170° deg. which taken from 180° , there remains $10^{\circ} 00'$ for the Angle LFM; so have you all the Angles in the Triangle FLM, and also the Side LM, to find the Sides FL and FM. I think I shall not need to say any thing more about it, but shall leave the working to the Ingenious Reader, and only set down the Distances of each Place from St. *Giles's*, and St. *Helens*, as below in the Table.

	Feet
From St. <i>Giles's</i> to	A 1455
	B 1700
	C 856
	D 1656
	E 2522
	F 3180
	G 2159
	H 3207
	K 1306

	Feet
From St. <i>Helens</i> to	A 3119
	B 2769
	C 1661
	D 1382
	E 820
	F 1380
	G 1104
	H 2047
	K 2936

CHAP. III. SECT. I.

Of a Fort, or Fortification.; and of several Axioms to be observed therein.

1. **A** Fort is made to the intent that a few Men might be able to defend themselves, and the Place, against a greater Number. 2. Therefore the Place is environed with a Rampire or Wall, and a Ditch,

Ditch, of sufficient height, breadth, and depth, to impede the Assaults of an Enemy. 3. And because the Sides thus inclosing a Fort are not of themselves sufficient, they have therefore Flanks to defend them; which Flanks are also themselves flanked by the Curtains or Sides. 4. And for the better defence of each Curtain, it is requisite that every Side of a Fort should have two Flanks; and if the Curtain be very long, it may have four, six or more. 5. And so there will be two Flanks placed near together, the one scouring the Side towards the right Hand, and the other towards the left, either Flank standing perpendicular to the Curtain that they flank. 6. And seeing the Curtains and Fronts of a Fort are especially defended; therefore the greater the Flanks, and the Gorge between them are, the better they are. 7. And forasmuch as the Front of a Bulwark needs the most defence, it ought so to be drawn, that it may be defended by Shot from as great a part of the Curtain as may be, which part of the Curtain is called the second Flank. 8. The outward flanking Angle ought never to exceed 150 Degrees; for by how much the lesser, so much the better it is: neither should the outward, or Diamond-point of a Bulwark be greater than 90 Degrees, or lesser than 60 Degrees. 9. And as the inward flanking Angle, and the Angle of the Shoulder of the Bulwark increase and decrease together, one exceeding the other 90 Degrees; therefore, as the inward flanking Angle should never be less than 15 Degrees, so the Angle of the Shoulder should never be less than 105 Degrees. 10. The longest Line of Defence, drawn from the Angle of the Flank to the outward Angle of the Bulwark, should never exceed 720 Feet, for that it ought not to be without Musket-shot. 11. A regular Fort is such a Fort as consists of equal Sides and Angles. And 12. such a Fort doth enclose a greater quantity of Ground, and the Force is in all Parts alike; therefore a regular Fort, if the

Ground or Place will admit, is better than an irregular one of the like number of Sides and Angles, 13. And by what hath been already said, in the eighth foregoing, it is evident, that a Fort of three Sides and Angles is of no moment, nor one of four of any great value; but the more Sides and Angles a Fort hath, the better and stronger it is. Wherefore, 14. If the fixed Line of Defence be 720 or 72 Rods, then may the Curtain be about 42 Rods, the Front of the Bulwark about 28 Rods, and the Flank to the Gorge as 6 is to 7, and the Angle forming the Flank about 40 Degrees.

S E C T. II.

Explaining the Terms of Art used in Fortification, especially of the Parts or Members of a Fort.

- I. A Fort is a piece of Ground environed with a Rampire or Wall, and a Ditch to impede the Assaults of an Enemy.
- II. A Fortress is a small Fort, Castle, or Sconce,
- III. A Rampire is a Wall of Earth enclosing the Place fortified, the Foot or Foundation whereof is (in the Figure below) noted with *a h*.
- IV. A Curtain ON.
- V. A Bulwark NFGHE,
- VI. The Front of a Bulwark FG, or KL.
- VII. A Flank NF, or OL.
- VIII. The Gorge of a Bulwark, or the space between two Flanks, as NE.
- IX. The Gorge Line NC,
- X. The Head-Line CG.
- XI. The Shoulder F, or L.
- XII. The Diamond-Point of the Bulwark, or sometimes the flanked Angle of the Bulwark, G.

- XIII. The second Flank O ;
- XIV. The fixing-fixed, or longest Line of Defence,
OG.
- XV. The shortest Line of Defence, ; G.
- XVI. The inward flanking Angle F ; N.
- XVII. The outward flank Angle, KPG.
- XVIII. The False-bray, the Breadth whereof is noted
with BC.
- XIX. The Ditch, the Breadth whereof is noted with
d e.
- XX. The Covert-way, the Breadth whereof is noted
with *e f.*
- XXI. A Casemate.
- XXII. The Parapet, *viz.* of the Rampire, False-bray,
and Covertway.
- XXIII. The Walk on the Rampire.
- XXIV. The Scarp inward and outward, *viz.* of the
Rampire, Parapets, and Ditch,
- XXV. Palisadoes.
- XXVI. A Bank or Foot-path:
- XXVII. The Brim of the Ditch.
- XXVIII. The Counterscarp.
- XXIX. A Ravelin.
- XXX. An Half-Moon.
- XXXI. A Horn-work.
- XXXII. A Trench.
- XXXIII. Gabions.
- XXXIV. A Breach.
- XXXV. A Mine.
- XXXVI. A Counter-mine.

There are several other Terms used in this Art, which I shall here omit, my design being only to apply the Doctrine of Plain Triangles to Practice in this Art,

S E C T. III.

To find the Quantity of the Angles in all Parts of a Fort, of any number of Sides proposed.

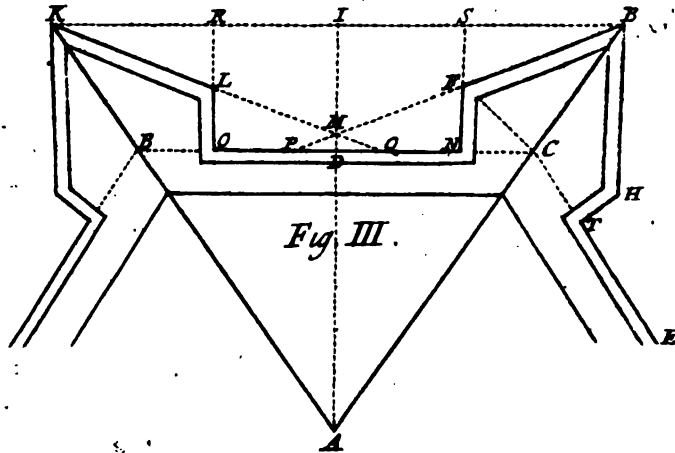
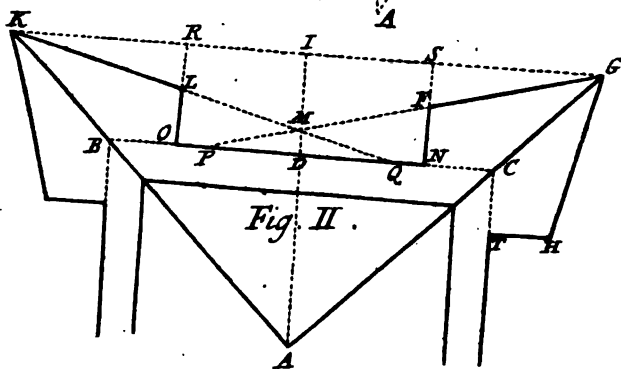
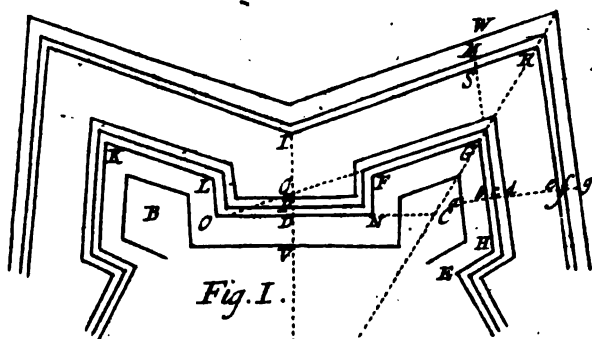
By the 13th Axiom foregoing, a Fort is to consist of four Sides at least; and by the 8th Axiom, the flanked Angle of a Bulwark ought to be at least 60 degrees; therefore in a regular Fort of four Sides, the flanked Angle of each Bulwark ought to be 60 deg.; and, by the 9th Axiom, the outward flanking Angle (exceeding the inward flanking Angle by 90 deg.) must needs be 150 deg.

In this Figure, let BC be one side of a Square fortified with four Bulwarks, one of which let be N, F, G, H, T; and seeing the flanked Angle of this Bulwark FGH is 60 deg. therefore the half thereof FGC is 30 deg. and IGC (being equal to DCA, namely, half the Angle of the Tetragon or Square) 45 deg. therefore SGF is 15 deg. and the Complement thereof SFG 75 deg. whereto is equal the Angle IMG, which is half KMG. Therefore the outward flanking Angle KMG is 150 deg. Which was to be proved.

And thus seeing in a quadrangular Fort, the flanked Angle is 60 deg. and the outward flanking Angle 150 deg. what these Angles will be in other Forts, consisting of more Sides than four, we may find by help of these in manner following: for which this is the

R U L E.

Subtract the Angle of the Square 90 deg. from the Angle of the Polygon; half the Remainder add to the flanked Angle of the Square (*viz.* 60 deg.) so have you the flanked Angle of the Polygon proposed. Also subtract the Half-remainder from the flanking Angle of the Square (*viz.* 150°) and the Remainder is the flanking Angle of the Polygon proposed,



EXAMPLE I.

Of a Pentagon.

	Deg.
The Angle at the Perimeter of the Pentagon is	108
The Angle of the Square subtract	90
The Remainder is	18
The half thereof is	9
Which added to the flanked Angles of the Square	60
Gives the flanked Angle of the Pentagon	69
From the flanking Angle of the Square	150
Subtract the aforesaid half Remainder	9
There remains the flanking Ang. of the Pentagon	141

EXAMPLE II.

Of a Hexagon.

From the Angle of the Hexagon, being	120
Subtract the Angle of the Square	90
And there remains	30
The half whereof is	15
Added to the flanked Angle of the Square	60
Gives the flanked Angle of the Hexagon	75
And from the flanking Angle of the Square	150
Subtract the foresaid half Remainder	15
There remains the flanking Angle of the Hexagon	135

And proceeding in this manner, you shall find, that the flanked Angle will not be 90 degrees, till you come to a Fort of twelve Sides.

Now the flanked Angle of a Bulwark being known, you may thereby come to the knowledge of all the other Angles requisite to be known.

In

In Fig. 3. Let BC be the Side of a Pen-	d. m.
tagon, whose Angle at the Center is BAC	72.00
The half whereof is CAD	36.00
The Complement thereof to 90 deg. is DCA	54.00
Now admit the Angle at the Bulwark FGH	69.00
The half thereof FCG	34.30
Subtracted from SGC, is equal to DCA	54.00
Remains the inward flanking Angle SGF	19.30
Equal to FPN, the Compl. of either SFG	70.30
Subtracted from two Right Angles	180.00
Leaves the Angle of the Shoulder NFG	109.30
Again, The same Angle SFG, or IMG	70.30
Doubled, gives the outward flanked Angle KMG	141.00
Lastly, From two right Angles	180.00
Subt. half the Ang. of the Polygon BCA	54.00
There remains the Angle DCG	126.00

And thus, from any flanked Angle proposed, you may find the Quantities of the other Angles.

But for any Polygon proposed, you may more compendiously set down the Angles of the Bulwark, and all the other Angles after the form of this Example following; remembring, that if the Polygon have more than 12 Sides, you make the Angles at the Bulwark a right Angle.

To half the Angle of the Polygon	BCA	d. m.
Add always		54.00
The Sum is the flanked Angle FGH		15.00
The half whereof FGC		69.00
Subtract from half the Angle of the Polygon BCA		34.30
There remains the inward flanking Angle SGF		54.00
Whose Complement is SFG		19.00
		70.30
		Which

Which subtracted from two right Angles	d. m.
Leaves the Angle of the Shoulder GFN	180.00
And the same Complement SFG, or IMG	109.00
Doubled, is the outward flanking Ang. KMG	70.30
	141.00

For the Angle forming the Flank, namely, the Angle FCN, it may be always about 40 Degrees. And according to this Rule is the following Table made.

A TABLE

TABLE of the Dimensions of the Angles observed in fortifying the regular Polygons following.

Number of the Sides of the Polygons.	V		VI		VII		VIII		IX		X	
	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.
Angle at the Center	BAC	72. 00	60. 00	51. 26	45. 00	40. 00	36. 00					
Angle of the Polygon	BCE	108. 00	120. 00	128. 34	135. 0	140. 0	144. 0					
Half the Angle of the Polygon	BCA	54. 00	60. 0	64. 17	67. 30	70. 0	72. 0					
To which (always) add		15. 00	15. 0	15. 0	15. 0	15. 0	15. 0					
The flanked Angle	EGH	69. 0	75. 0	79. 17	82. 30	85. 0	87. 0					
Half the flanked Angle	FGC	34. 30	37. 30	39. 38	41. 15	42. 30	43. 30					
Inward flank Ang. FPN, or	SGN	19. 30	22. 30	24. 39	26. 15	27. 30	28. 30					
Which add to a Right Angle		90. 00	90. 0	90. 0	90. 0	90. 0	90. 0					
Angle of the Shoulder	NFG	109. 30	112. 30	114. 39	116. 15	117. 30	118. 30					
Angle opposite to the Head Line	GFC	59. 30	62. 30	64. 49	66. 15	67. 30	68. 30					
Angle opposite to the Front	FCG	86. 00	80. 0	75. 43	72. 30	70. 00	68. 0					
Complement of SGF, viz.	SFG	70. 30	67. 30	65. 41	63. 45	62. 30	61. 30					
Outward flanking Angle	KMG	141. 0	135. 0	131. 22	127. 30	125. 0	123. 0					
Angle fronting the Flank	PCN	40. 0	40. 0	40. 0	40. 0	40. 0	40. 0					

S E C T. IV.

Of the Quantity of the Curtains, Flanks, Fronts, Gorges, and Lines and Sides, in regular Forts, of any determined number of Sides.

There is no necessity that the Angles in Forts should be exactly such as are found by the foregoing Rules, but they may be something more or less as the Place or other occasions shall require: But first, supposing them to be such, I will shew how to determine the quantity of the Sides and Lines of a Fort accordingly, by Trigonometrical Calculation.

The Length of the Curtain, and of the Front of the Bulwark given, to find what the other Sides and Lines should be.

As in the regular Pentagonal Fort, *Fig. III.* and so in others, to the intent the Line of Defence may be about 72 Rods, the Curtain 42, and the Front about 28, as is before noted in the 14th Axiom, and that the Proportion of the Flank to the Gorge be as 6 to 7; and let the Angles be as are expressed in the Table above: Then,

The Curtain is .	ON 420 Feet
The Front of the Bulwark	FG 280 Feet

Then will the rest of the Sides be found by Trigonometrical Calculation, as followeth.

In the right-angled Triangle SGF; by Case III.

As Radius 90°	10.
To the Front of the Bulwark FG 280	2.44713
So is s. of the inward flanking Angle	9.52350
SGF 19° 30'	

To the Line SF, 93.47	1.97063
	Again,

Again, by the same Case,

As Radius 90°

10.

To the Front of the Bulwark FG 280 }
Feet }

2.44715

So the Cosine of the inward flanking }
Angle $70^\circ 30'$ }

9.97435

To the Line SG 263.94 Feet

2.42150

Half the Curtain SI 210.00 add

The Sum is 473.94 for the Line IG ;
which doubled, is the Side of the outward Polygon,
or the Distance of the Diamond-Points of the Bul-
wark KG 947.88 Feet.

In the Triangle IAG, by Case II. of right-angled
Triangles ;

As the s. of half the Angle at the Cen- }
ter IAG $36^\circ 00'$ }

9.76922

Is to half the Side of the outward Pen- }
tagon 473.94 }

2.67572

So is Radius

10.

To the Semidiameter of the outward }

2.90650

Pentagon AG 806.3

In the same Triangle, by Case I.

As s. of the Angle IAG $36^\circ 00'$ Co. Ar.

0.23078

To half the Side of the Pentagon 473.94

2.67572

So is c. s. of half the Angle at the }

9.90796

Center AGI $54^\circ 00'$

To the Perpend. of the outward Pen- }

2.81446

tagon AI 652.3

In the Triangle FCG, by Case I. of oblique-angled Triangles;

As the s. of the Angle FCG $86^{\circ} 00'$ C. Ar. 0.00106

Is to the Front FG 280 2.44715

So s. of the Angle FGC $34^{\circ} 30'$ 9.75313

To the Line FC 158.98 2.20134

In the same Triangle, by the same Case;

As s. of the Angle FCG $86^{\circ} 00'$ Ar. C. 1.00106

To the Front FG 280 2.44715

So s. of the Angle GFC $59^{\circ} 30'$ 9.93532

To the Head-Line CG 241.44 3.38353

Which subtr. from AG 806.31 the Semidiam. of the inner Pentagon

There remains 564.87

In the Triangle FCN, by Case III. of right-angled Triangles;

As Rad. to the Line FC 158.98 before found 2.20134

So s. of the Angle forming the Flank FCN $40^{\circ} 00'$ 9.80807

To the Flank 102.19 2.00941

The Flank FN being 102.19

Add to it the Line first found, SF 93.47

The Sum is the Diff. of the Pentagons ID 195.66

Which subtract from the Perpend. AI 652.32

There remains 456.66
for AD the Perpend. of the inner Pentagon.

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In the Triangle FNC, by Case III. of right-angled Triangles.

As Radius to the Line FC 158.98 2.20134

So s. of the Ang. forming the Flank FCN 9.88425

To the Gorge Line NC 121.78 2.08559

To the Gorge Line NC 121.78

Add half the Curtain DN 210

The Sum is the Line DC 331.78

Which doubled is BC 663.56

for the Side of the Inner Pentagon.

In the Triangle FPN, by Case I. of right-angled Triangles.

As s. of the inner flanking Angle FPN } 0.47650
16° 30'

To the Flank FN 162.19 2.00941

So is the Cosine of the inner flanking } 9.97435
Angle 16° 30'

To the Line PN 288.58 2.46026

Which subt. from the Curt. ON 420

The remaind. is the 2d Flank OP 131.42

In the Triangle ROG, by Case IV. of right-angled Triangles.

To the Line before found SG 263.94

Add the Curtain ON 420

The Sum is the Line RG 683.94

S

Then

Then First,

As the Line RO, or ID, 195.66

7.70850

To the Line RG 683.94

2.83502

So is Radius t. 45°

10.

To t. of the Angle ROG $74^{\circ} 02'$

10.54352

Secondly,

As the s. of the Angle ROG $74^{\circ} 02'$

9.98291

Is to the Line RG 683.94

2.83502

So is Radius

10.

To the length of the Line of Defence

OG 711.4

}

2.85211

Thus have you the manner how to find the Sides, Lines, and Angles of any regular Fortification, by Trigonometrical Calculation, whereby the Excellency of Plain Trigonometry in some measure doth appear.

CHAP. IV. OF NAVIGATION.

I SHALL shew the Usefulness of Trigonometry in this most excellent Art, in as brief a Method as I can, yet so full and clear, as that it may be easily understood by a Learner; and first I shall begin with

SECT. I PLAIN SAILING.

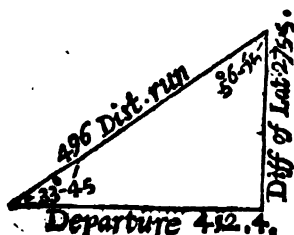
CASE I

The Course and Distance run being given; to find the difference of Latitude and Departure.

Admit a Ship sails SW b W. 496 Miles, I demand her difference of Latitude and Departure.

1. As Rad. to the Dist. run 496	2.69548
So s. of the Course $56^{\circ} 15'$	9.91985

To the Departure 412.4	2.61533
------------------------	---------



S 2

2. As

2. As Rad. to the Dist. run 496
So s. of the Rumb $33^{\circ} 45'$

2.69548
9.74474

To the Diff. of Lat. 275.5

2.44022

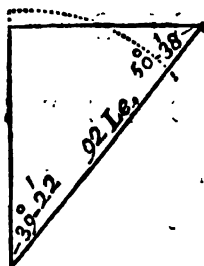
Admit a Ship in the Latitude 32° North, and sails away NE b N $\frac{1}{2}$ E 92 Leagues: I demand the Latitude and Departure.

1. As Rad. to the Dist. run 92 L.
So is c.s. of the Course $50^{\circ} 38'$

2.96378
9.88823

To the Diff. Lat. 71.1

2.85201



2. As Rad. to the Dist. 92 L.
So s. of the Course $39^{\circ} 22'$

2.96378
9.80228

To the Departure 58.4

2.76606

Divide the Diff. of Lat. and Departure by 20, (because 20 Le. is 1 Degree.)

2,0)7,1

2,0)5,8

3—11

2—18

3

3

33 min.

54

So

So the Latitude the Ship is in is $35^{\circ} 33'$ North,
And she is departed from the Merid. 2 54 Easterly.

Upon *Gunter's Scale* there is a Line of Sines, and Rumbs, marked with SR, which is a Line divided to every quarter Point of the Compass in the Nature of a Line of Sines. If you extend your Compasses in that Line from Radius, which is at 8 Points, to the Complement of the Course, which in this Example is 4 Points and a half; that extent will reach from the Distance run 92, to the Difference of Latitude 71.1. And extending from Radius to the Course $3\frac{1}{2}$ Points, that will reach from 92 the Distance run, to 58.4 the Departure.

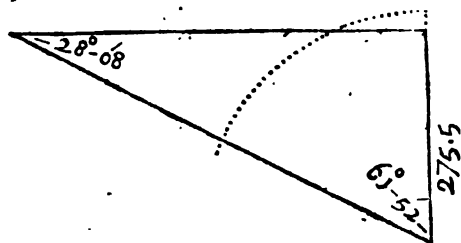
C A S E II.

*The Course and Difference of Latitude being given,
to find the Distance and Departure.*

Admit a Ship sails NW b W $\frac{1}{2}$ W, until her Difference of Latitude be 275.5 Miles: I demand her Distance run, and Departure.

1. Asc s. Course $28^{\circ} 8'$	9.67350
To the Diff. of Lat. 275.5	2.44011
So is Radius	10.

To the Distance run 584.3	2.76661
---------------------------	---------



2. As Rad. to the Distance run 584.3
So is the s. of the Course $61^{\circ} 52'$

1.76661
9.94539

To the Departure 515.2

2.71200

Admit a Ship in the Latitude of 40° North, sails away NW b $N \frac{1}{2} W$, until she comes into the Latitude of $43^{\circ} 56'$: I demand her Distance run, and Departure.

$43^{\circ} 56'$
 $40^{\circ} 00'$

1. As cs. of the Course $50^{\circ} 37'$
To the Diff. of Lat. 236
So is Radius

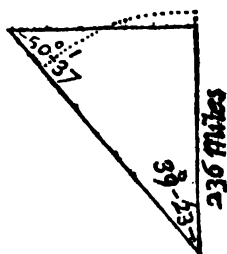
9.88813
2.37291
10.

$3^{\circ} 56'$
 60

To the Distance run 305.3

2.48478

236 Miles



2. As Radius to the Dist. run 305.3
So is the Sine of the Course $39^{\circ} 23'$

2.48478
9.80243

To the Departure 193.75

2.28721

If you extend the Compasses in the Line SR from $4 \frac{1}{2}$ Points to Radius, that extent will reach from 236 to 305.3 in the Line of Numbers; and in the second Proportion extend from Radius to $3 \frac{1}{2}$ parts, that will reach in the Line of Numbers from 305.3 to 193.75.

CASE

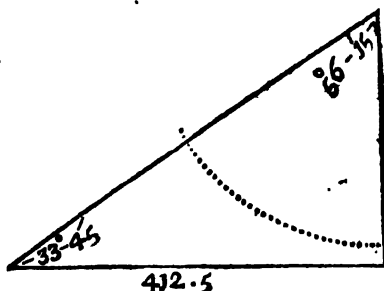
C A S E III.

The Course, and Departure being given; to find the Distance run, and Difference of Latitude.

Admit a Ship sails SW b W until her Westing be 412.5 Miles: I demand her Distance sail'd, and Difference of Latitude.

1. As s. of the Course $56^{\circ} 15'$	9.91985
To the Departure $412 \frac{1}{2}$	2.61542
So is Radius	10.

To the Distance run 496.1	2.69557
---------------------------	---------



2. As Rad. to the Distance 496.1	2.69557
So cs. of the Course $33^{\circ} 45'$	9.74474
To the Diff. of Latitude 275.6	2.44031

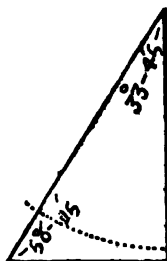
Admit a Ship in the Latitude of 50° North, and sails SW b S until she be departed from the Meridian 62 Leagues: I demand the Distance run, and what Latitude the Ship is now in.

1. As s. of the Course $33^{\circ} 45'$
 Is to the Departure 63
 So is Radius

9.74474
 1.79239
 10.

To the Distance run 111.6

2.04765



2. As Rad. to the Distance run. 111.6
 So cs. of the Course $56^{\circ} 15'$

2.04765
 9.91985

To the Diff. of Latitude 92.8

1.96750

Lat. come from $50^{\circ} 00'$
 Diff. Lat. subtr, 4 38

Lat. come to 45 22

C A S E IV.

*The Distance run, and difference of Latitude given;
 to find the Course and Departure.*

Admit a Ship in the Latitude of 6° North, and sails away between the N. and E. 98 Leagues, until she comes into the Latitude of $9^{\circ} 45'$. I demand the Course and Departure.

As

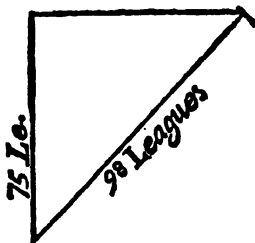
Chap. 4. Of Plain Sailing.

265

As the Distance run 98
Is to Radius
So is the Diff. Lat. 75

$$\begin{array}{r} 1.99123 \\ 10. \\ 1.87506 \\ \hline 9.88383 \end{array} \left\{ \begin{array}{l} 9^{\circ} 45' \\ 6 \text{ } 00 \\ \hline 3 \text{ } 45 \\ 20 \\ \hline 75 \end{array} \right.$$

To cs, of the Course $40^{\circ} 04'$



As Radius to the Distance run 98
So s. of the Course $40^{\circ} 04'$

$$\begin{array}{r} 1.99123 \\ 9.80867 \\ \hline \end{array}$$

To the Departure 63.1

$$\begin{array}{r} 1.79990 \end{array}$$

The Course is NE b N $6^{\circ} 19'$ Easterly; that is, a little above $\frac{1}{2}$ a point Easterly.

The increase of Longitude is $3^{\circ} 9'$.

Admit a Ship in 40° N. Latitude, sails away between the S. and W. 105 Leagues, until she comes into the Latitude of $36^{\circ} 06'$. I demand her Course, and Departure.

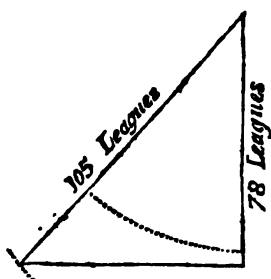
$$\begin{array}{r} 40^{\circ} 00' \\ 36 \text{ } 06 \\ \hline 3 \text{ } 54 \\ 20 \\ \hline \end{array}$$

78 Diff. Lat,

As the Distance run 105 Le. 2.021189
Is to Radius 10.
So is the Diff. of Lat. 78 1.892094

To cs, of the Course $42^{\circ} 1'$ 9.870905

As
2



As Radius to the Distance run 105	2.021189
So s. of the Course $42^{\circ} 01'$	9.825651

To the Departure 70.3	1.846840
-----------------------	----------

The Ship's Course is SW b S near $\frac{1}{2}$ Westerly, and $3^{\circ} 31'$ decrease of Longitude.

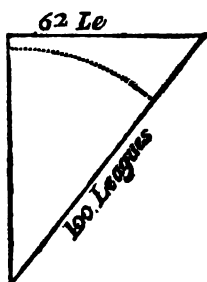
C A S E V.

The Distance run, and Departure being given; to find the Course, and Difference of Latitude.

Admit a Ship in the Latitude of 40° S. and sails between the N. and E. 100 Leagues, until she be departed from the Meridian 62 Leagues: I demand the Course, Difference of Latitude, and what Latitude the Ship is come to.

As the Distance run 100	2.000000
Is to Radius	10.
So is the Departure 62	1.79239
To s. of the Course $38^{\circ} 19'$	9.79239

As



As Radius to the Distance run 100
So cs. of the Course $51^{\circ} 41'$

2.00000
9.89464

To the Difference of Latitude 78.5

1.89464

40° 00' Lat. departed from
3 55 Diff. of Lat. subtr.

36 05 Lat. the Ship is come to.

Admit a Ship in the Latitude of 1° South, and sails away between the N. and W. 96 Leagues, until she be departed from the Meridian 56 Leagues. I demand her Course, Difference of Latitude, and what Latitude she is come into.

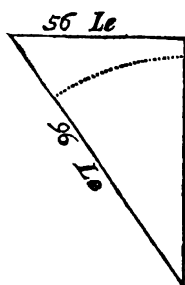
As the Distance run 96
Is to Radius
So is the Departure 56

1.982271
10.
1.748188

To s. of the Course $35^{\circ} 41'$

9.765917

As



As Radius to the Distance run 96
So cs. of the Course $54^{\circ} 19'$

1.982271
9.909691

To the Difference of Latitude 78

1.891962

$3^{\circ} 54'$ Difference of Latitude
1 00 Lat. come from subtr.

2 54 Lat. North come to.

C A S E VI.

The Difference of Latitude, and the Departure being given; to find the Course and Distance run.

Admit a Ship in the Latitude of 20° South; and sails away between the North and East until she comes into the Latitude $16^{\circ} 15'$, and be departed from the Meridian 59 Leagues. I demand the Course and Distance run.

$20^{\circ} 00'$
 $16 15$

3 45 Diff. Lat. = 75 Leagues.

As

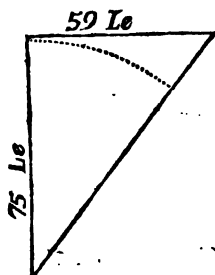
Chap. 4. Of Plain Sailing.

As the Diff. of Lat. 75 Leagues
Is to Radius or Tangent of 45°
So is the Departure 59 Leagues

269
1.87506
10.
1.77085

To the t. of the Course $38^\circ 11'$

9.89579



Ass. of the Course $38^\circ 11'$
Is to the Departure 59
So is Radius

9.79111
1.77085
10.

To the Distance run 95.4

1.97974

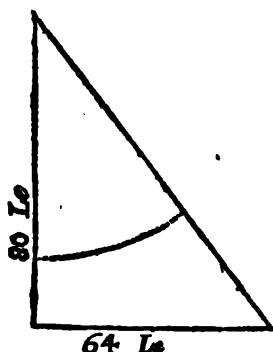
The Course is NE b N $4^\circ 26'$ Easterly.

Admit a Ship in the Latitude of 20° North, and
sails away between the South and East, until she
comes into the Latitude 16° , and be departed from
the Meridian 64 Leagues. I demand her Course and
Distance run.

20
16 As the Diff. of Lat. 80 Leagues
Is to Radius
4 So is the Departure 64 Leagues
20
80 To t. of the Course $38^\circ 40'$

1.90309
10.
1.80618
9.90309

As



As s. of the Course $38^{\circ} 40'$
Is to the Departure 64
So is Radius

9.79573
1.80618
10.

To the Distance run 102.4

2.01045

S E C T. II.

Shewing how to resolve a Traverse, or bring several Courses into one.

A Ship in the Latitude of 11° North, is bound for a Port bearing due South; but by reason of contrary Winds sails as followeth, SW b S 30 Miles, S. 33 Miles, NNW $\frac{1}{2}$ W 32 Miles, SW 39 Miles, SE 9 Miles, ESE 11 Miles. What is her Difference of Latitude, Departure and direct Course, and Distance?

Courses

Courses.	Distance.	Points from the Merid.	Points for lay down.	Differ.		Departure.	
				N.	S.	E.	W.
SW $\frac{1}{2}$ S	30	3	3		24.94		16.67
S	33	0	13		33.00		
NNW $\frac{1}{2}$ W	32	2 $\frac{1}{2}$	2 $\frac{1}{2}$	28.22			15.08
SW	39	4	6 $\frac{1}{2}$		27.56		27.56
SE	9	4	8		6.36	6.36	
ESE	11	6	14		4.21	10.16	
Summ'd up				28.22	96.07	16.62	59.31
Subtract					28.22		16.62
Difference of Latitude					67.85	Deapar.	42.69

By Case I. of Plain-Sailing, find the Difference of Latitude and Departure from the Meridian, for all the Courses and Distances severally; thus in the first of them there is given the Course SW $\frac{1}{2}$ S, that is 3 Points from the Meridian, and the Distance run 30 Miles: Then as Radius to 30, so is cs. of the Course to the Difference of Latitude; and as Radius to 30, so s. of the Course to the Departure. Then because the Course is S. Westerly, place the Difference of Latitude 24.94 in the South Column under S. and the Departure 16.67 in the West Column under W; then find the Difference for the next Course which is S, and therefore the Difference of Latitude will be 33 the same as the Distance, which place under S. and because there is no Easting nor Westing, therefore there is nothing to put in the East or West Columns. Then for the next Course, find the Difference of Latitude and Departure by the last Sect. Case I. as before; and because the Course is North Westerly, place the Difference of Latitude in the N. Column, and the Departure in the W. Column, and so proceed in all the rest.

Having
2

Having found the Difference of Latitude and Departure from the Meridian for each Course, and placed them in their proper Columns; add up the North, South, East and West Columns, and subtract the North and South Columns one from the other, that is the least from the greatest, in like manner the East and West Columns. As in the foregoing Table, the Sum of the North Column is 28.22, subtract from the Sum of the South Column 96.07, the Remainder 67.85 is the Difference of Latitude Southerly: and the Sum of the East Column 16.62, subtracted from the Sum of the West Column 59.31, the Remainder 42.69 is the Departure from the Meridian Westerly.

To delineate or draw a Traverse by Scale and Compasses, Mr. *Atkinson* in his Epitomy gives us four Rules for finding the Angles between the Courses; which are as followeth.

Rule 1. Two Meridians, and two Parallels; the lesser Course subtracted from the greater, gives the Angle between both Courses.

Rule 2. Two Meridians, and one Parallel; add both Courses together.

Rule 3. One Meridian, and two Parallels; subtract the Sum of both Courses from 16 Points.

Rule 4. One Meridian, and one Parallel; add 8 Points, the lesser Course, and the Complement of the greater together; the Sum is the said Angle.

Note 1. North and South are called Meridians, and East and West are called Parallels.

2. By Complement of the greater Course, understand how much it wants of 8 Points.

3. To know the Meridians and Parallels, the first and second Courses are compared together, and the second and third, and the third and fourth, &c.

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In the foregoing Example, having set down the Courses, the Distances, and Points from the Meridian in the first, second, and third Columns, in the fourth Column put the Points for laying down, which are found by the four foregoing Rules, thus: The first Course is ever the same with the Points from the Meridian; for the second Course it is 13 Points, found by comparing the first and second Courses together, in which are 1 Meridian and 1 Parallel, and by the 4th Rule it makes 13 Points. Again, comparing the second and third, and it is 2 Meridians and 1 Parallel, which by the second Rule makes $2\frac{1}{2}$ Points; and so for the rest, as in the foregoing Table.

By these Points for laying down it is easy to delineate the Traverse, as in the following Scheme.

First draw the Meridian NS, and on N draw an Arch with 60 deg. of Chords; on which Arch lay 3 Points, and by it draw a SW $\frac{1}{2}$ S Line, laying thereon the Distance 30 Miles from N to A.

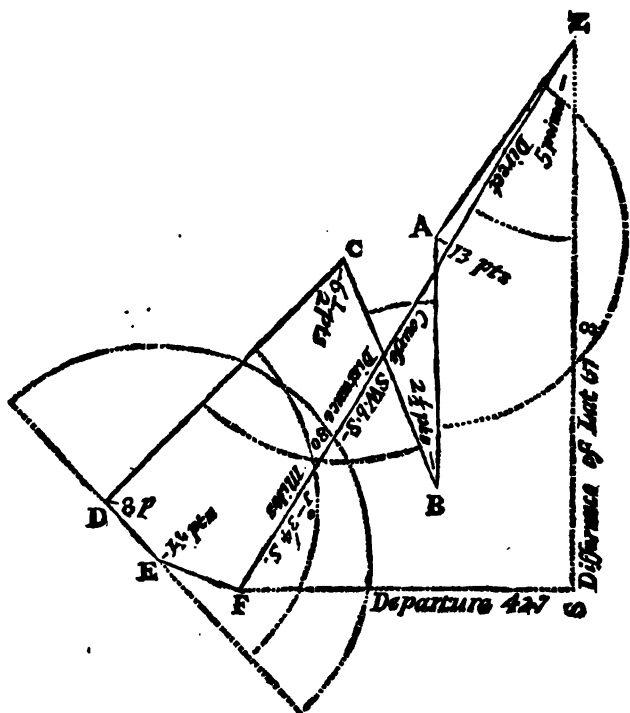
Secondly, With a Chord of 60, and one Foot in A, with the other draw an Arch, on which lay 13 Points; that is, lay 7 Points and 6 Points, and draw the Line AB, and upon it lay the Distance 33 Miles: proceed after the same manner with the rest.

11° 00'
1 08 Diff. Lat. subt.

9 52 Lat. come into.

Having the Difference of Latitude, and Departure, the direct Course and Distance may be thus found.

As Diff. Lat. 67.85	1.83155
To Radius	10.
So is the Depart. 42.7	1.63033
To t. Course 32° 11'	9.79878
T	As



As cs. Course $57^{\circ} 49'$
To Diff. Lat. 67.85
So is Radius

9.92755
1.83155
10.

To Distance 80.17

1.90400

Suppose a Ship in the Latitude of 45° N, and is bound for a Port bearing NW $\frac{1}{2}$ W and in the Latitude of $50^{\circ} 57'$, but by reason of contrary Winds fails as follows, NNW 35 Leagues, NE $\frac{1}{2}$ E 42 Le. WNW 38 Le. W 20 Le. NW $\frac{1}{2}$ N 23 Le. N 12 Le. ENE 39 Le. SW $\frac{1}{2}$ W 48 Le. N $\frac{1}{2}$ E 31 Le. NNW $\frac{1}{4}$ W 38 Le. and W $\frac{1}{2}$ N 45 Leagues: I demand the Difference

Difference of Latitude, and Departure ; and also the true Course and Distance that I have made my way good, and also the true Course and Distance to the Port I am bound to.

Courses.	Distance.	Points	Differ.	Lat.	Departure.	
			N.		E.	W.
NNW	35	2	32.34			13.39
NE $\frac{1}{2}$ E	42	5	23.34		34.92	
WNW	38	6	14.54			35.11
W	20	8				20.
NW $\frac{1}{2}$ N	23	3	19.12			12.78
N	12	0	12.			
ENE	39	6	14.92		36.03	
SW $\frac{1}{2}$ W	48	5		26.66		39.91
N $\frac{1}{2}$ E	31	1	30.40		6.05	
NNW $\frac{1}{2}$ W	38	2 $\frac{1}{2}$	33.51			17.91
W $\frac{1}{2}$ N	45	7	8.78			44.14
Summ'd up			188.95	26.66	77.00	183.24
Subtract			26.66			77.00
Difference of Latitude			162.29		Dejar.	106.24

Find the Difference of Latitude and Departure answerable to every one of the Courses, thus ;

As Radius	10.
To Dist. 35	1.54407
So s. Cour. 22° 30'	9.58284
To Dejar. 13.39	1.12691
As Radius	10.
To Dist. 35	1.54407
So cs. Course	9.96561
To Diff. L. 32.34	1.50968

T 2

Because

First, draw the Meridian Line SN, then take 32.34 from a Diagonal Scale, and lay it from S to (a), and take the Departure 13.39 from the same Scale, and set one foot of the Compasses in (a), and with the other strike a little stroke at b; then take the Distance 35 from the Scale, and set one foot in S, and with the other cross the former stroke at b, and draw Sb; then draw bc parallel to SN, and take 23.34 the Difference of Latitude, and lay from b to c, and take the Departure 34.92; and with one foot of the Compasses in c, with the other strike a stroke at d; and with 42 the Distance, with one foot in b, cross that stroke in d, and draw bd which will be NE b E, and so proceed with all the rest. But observe, that if the Difference of Latitude be in the N. Column you must set it upwards, and if in the S. Column it must be laid downwards; and if the Departure be in the E. Column, it must be laid to the Right-hand, but if it be in the W. Column it must be laid to the Left-hand: also observe, that the Difference of Latitude, the Departure, and the Distance, always make a right-angled Triangle, as you may see by the little prick'd Lines; upon those which represent the Meridians I have figured the Difference of Latitude, and upon those representing the Parallels, or East and West Lines, I have figured the Departure.

Then, by the Difference of Latitude and Departure upon a direct Course, say,

As the Diff. of Lat. 162.29	2.21029
Is to Radius	10.
So is the Departure 106.2	2.02612
To the t. of the Course 33° 12'	9.81583

As the s. of the Course $33^{\circ} 12'$
 Is to the Departure 106.2
 So is Radius

9.73843
 1.02612
 10.

To the Distance run 193.9

2.28769

Ascs. of the Course to the Port $33^{\circ} 45'$ 0.25526
 To the Diff. of Lat. 119 2.07553
 So s. Course to the Port $56^{\circ} 15'$ 9.91985

To the Depart. at the Port 178.1

2.25066

Next, to find the Course and Distance from the Ship where she is, to the Port.

$50^{\circ} 57'$
 $45^{\circ} 00'$

$5^{\circ} 57' = 119$ Le.

162.3 Diff. of Lat. at the Ship 178.1 Dep. at the Port.
 119 Diff. of Lat. at the Port 106.2 Dep. at the Ship.

43.3 Diff. of Lat. to the Port 71.9 Dep. to the Port.

Then, As the Diff. of Lat. 43.3

1.63649

To Radius

10.

So is the Departure 71.9

1.85673

To the t. of the Course $58^{\circ} 57'$

10.22024

As the s. of the Course $58^{\circ} 57'$

9.93284

To the Depart. 71.9

1.85673

So is Radius

10.

To the Distance run 83.9

8.92389

So the Course I have made my way good is NW $\frac{1}{2}$ N nearly.

The Distance run upon the same Course is 193.9 Leagues.

The direct Course to the Port is merely SW $\frac{1}{2}$ W $\frac{1}{4}$ W.

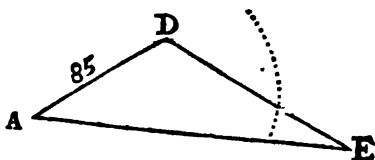
The Distance to the Port I am bound for is 83.9 Leagues.

S E C T. III.

Oblique Sailing; or the Doctrine of Oblique-angled Triangles applied in Problems of Plain Sailing.

Problem 1. Two Ships sail from the same Port, the one sails ENE 85 Miles, the other E $\frac{1}{2}$ S so far until she find the first Ship bears NW $\frac{1}{2}$ W; I demand the second Ship's distance from the Port, and the distance between the two Ships.

The Geometrical Protraction of the Triangle is thus; Consider first that the Angle A is 3 Points, AD being ENE is 2 Points from the East, and AE being E $\frac{1}{2}$ S is one Point from the East; therefore the whole Angle DAE is 3 Points: wherefore upon A with 60 deg. of Chords strike an Arch of a Circle, and lay upon it 3 Points



or $33^{\circ} 45'$, and draw the Lines AD and AE, and from A to D lay 85; then the Angle E being 2 Points, the Angle D must be 11 Points; therefore draw DE so as to make an Angle at D of 11 Points, or $123^{\circ} 45'$, so is the Triangle finished: then if you

measure DE upon the Scale of equal Parts, you will find it to contain 123.4, and AE will be 184.7.

The Trigonometrical Calculation is thus.

Ass. of the Angle E $22^{\circ} 30'$	0.41716
Is to the Side AD 85	1.92942
So is s. of the Ang. at the Port A $33^{\circ} 45'$	9.74474

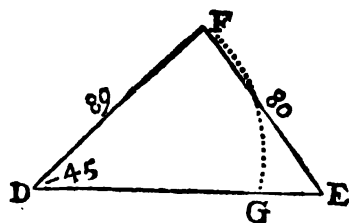
To the Side DE 123.4	2.09132
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As s. of the Angle E $22^{\circ} 30'$	0.41716
Is to the Side AD 85	1.92942
So s. of the Angle D $123^{\circ} 45'$ (or $56^{\circ} 15'$)	9.91985

To the Side AE 184.7	2.26643
----------------------	---------

So the second Ship's Distance from the Port is 184.7 Miles, and the Distance between the two Ships is 123.4 Miles.

2. There are two Ports that lie East and West one of another; one Ship sails from the Westernmost Port NE 89 Leagues; the other sails from the Eastermost Port 80 Leagues, and there meets the first: I demand the Course steered, and the Distance between the Ports.



Draw DE of any convenient length, and with 60 deg. of Chords one foot in D with the other strike an Arch, and lay G to F the Chord of 45° , and draw the Line DF;

and from a Scale of equal parts take 89, and lay from D to F; then from the same Scale take 80, and with one foot of the Compasses in F, with the other cross

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the Line DE in E, and draw the Line FE; measure the Angle E by a Line of Chords, and the Line DE by equal parts.

D represents the Port, and Angle of the first Ship's Course.

E the second Ship's Port, and Angle of her Course.

F the Place where they met.

The Trigonometrical Calculation is thus,

As the 2d Ship's Distance run 80	8.09690
Is to s. of the 1st Ship's Course 45°	9.84948
So is the 1st Ship's Distance run 89	1.94939

Toth e 2d Ship's Course 51° 52'	9.89577
45 00	

Sum 96 52

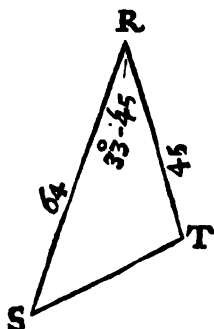
Com. 83 08 = < F

Ass. of the 1st Ship's Course 45°	0.15052
Is to the 2d Ship's Distance 80	1.90309
So is the Angle F 83° 08'	9.99687

To the Distance of the Ports 112.3	2.05048
------------------------------------	---------

3. Two Ships set sail from a certain Port, the one sails S b E 45 Leagues, the other SSW 64 Leagues. I demand their Bearing, and Distance from each other.

64 2 add	180° 00'
45 5	33 45
109 Sum of the Sides	146 15 Sum of the unknown <s
19 Difference	73 07 ½ half Sum.



As the Sum of the Sides 109	7.962574
Is to their Difference 19	1.278753
So t. of $\frac{1}{2}$ the unknown $\angle S 73^{\circ} 7' \frac{1}{2}$	10.518060

To t. of half their Differ. 29 53	9.759387
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Sum 103 00 $\frac{1}{2}$	= the Ang. T:
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Diff. 43 14	= the Ang. S.
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See Case 3. Chap. 4. Part 1.

Hence the Course or Bearing from S to T is SE $\frac{1}{4}$ E nearly; then for the Distance of the Ships from each other,

As s. of the Angle S $43^{\circ} 14'$	0.164328
Is to the Side RT 45	1.653212
So is s. of the Angle R $33^{\circ} 45'$	9.744739

To the Side ST, the Distance 36.5	1.562279
-----------------------------------	----------

4. If two Ports lie SW $\frac{1}{4}$ S, and NE $\frac{1}{4}$ N, being distant one from the other 456 Miles, the Wind S $\frac{1}{4}$ E,

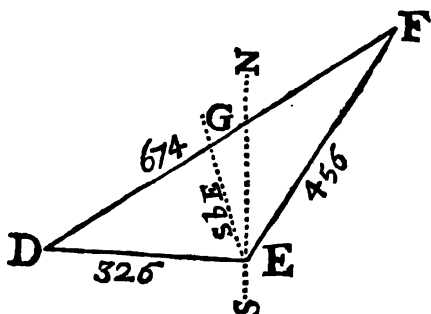
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S b E, and a Ship sails from the Northermost close upon a Wind 674 Miles with her Larboard-Tack aboard, 326 Miles with her Starboard-Tack aboard, and then arrives at the Southermost: I demand how near the Wind she makes her way good upon each Tack.

Note, That F represents the Northermost Port, and E the Southermost.

By Cafe 5. Chap. 4. Part 1. find the Angles.

674	54
456	272
326	402
1456	
728	



The half Sum 728 Log.	Ar. Com.	7.137869
Diff. of the half Sum and DF 54	Ar. C.	8.267606
Diff. of the half Sum and { EF 272		2.434569
{ DE 402		2.604226

The Tangent of $59^{\circ} 03' 10''$ is 20.444270

The double 118 06 20 = the Angle DEF.

The half Sum 728 Log.	Ar. C.	7.137869
Diff. of the half Sum and EF 272	Ar. C.	7.565431
Diff. of the half Sum and { DE 402		2.604226
{ DF 54		1.732394

The Tangent of $18^{\circ} 19' 11''$ is 19.039920

The double 36 38 22 = the Angle EDF.
From

From $118^{\circ} 06' =$ the Angle DEF

Subt. $45^{\circ} 00' =$ the Angle GEF, being 4 Points.

Rem. $73^{\circ} 06'$ } $=$ the $\angle$ between the Ship's Course
and the S b E Point, the 2d Tack.

Add $36^{\circ} 38' =$ the Angle EDF.

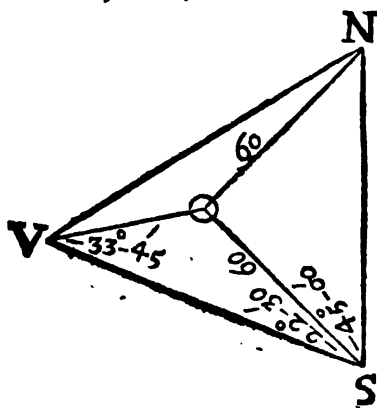
Sum $109^{\circ} 44'$

Com. $70^{\circ} 16'$ } $=$ the $\angle$ between the Ship's Course
and the S b E Point, the 1st Tack.

So the first Tack was SW b W $\frac{1}{4}$ W. and the second
Tack is E b S $\frac{1}{4}$ E.

5. A Ship sailing NW. two Islands appear in sight; one bears WNW, the other N. from the Ship; and when the Ship had sailed 60 Miles further, the first bears W b S, and the other NE: their Bearing and Distance is required.

V represents one Island, and N the other.
S the Ship at the first Observation, and O the Ship
at the second; then,



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As s. of OVS $33^{\circ} 45'$	0.255261
Is to OS 60	1.778151
So is s. OSV $22^{\circ} 30'$	9.582840

To the Side OV 41.33	1.616252
----------------------	----------

The Sum 101.33

The Diff. 18.67
 $180^{\circ} 00'$
 $146^{\circ} 15' =$ the Angle NOV

$33^{\circ} 45' =$ the Sum of the opp. Angles.

$16^{\circ} 52 \frac{1}{2}'$, half Sum.

As the Sum of the Sides 101.33	7.994263
Is to the Diff. of the Sides 18.67	1.271144
So t. of $\frac{1}{2}$ the Sum of the opposite An- gles, $16^{\circ} 52 \frac{1}{2}'$	9.481939

To t. of $\frac{1}{2}$ their Diff. 8.747346

Sum 20 $04 \frac{1}{2}' =$ the $\angle$ OVN

Diff. 13 $40' =$ the $\angle$ ONV

As s. of OVN $20^{\circ} 04'$	0.464533
Is to the Side ON 60	1.778151
So is s. of the Angle NOV $146^{\circ} 15' (33^{\circ} 45')$	9.744739

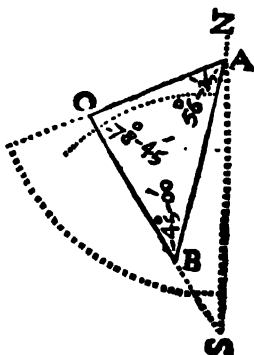
To the Distance of the Islands 97.15 1.987423

The Course, or Bearing of one Island to the other,
 is NE $\frac{1}{2}$ E very near,

6. Two Ships sail from one Port, the one sails S by W, the other WSW, and arrives at two several Ports; the Westermost Port bears from the Eastermost NW ½ N: then if the Distance run of both Ships, and Distance between the Ports, be together 130 Leagues, I demand them severally.

Assume $AB = 100$; then,

As s. C 78° 45'	9.991574
Is to AB 100	2.000000
So s. A 56° 15'	9.919846
To CB assumed 84.776	1.928272



Again,

As s. C $78^{\circ} 45'$	9.991574
Is to AB 100	2.000000
So s. B $45^{\circ} 00'$	9.849485
To AC assumed 72.097	1.857911

84.776
100

Sum 256.873

As

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As 256.873 the Sum of the assumed Sides 2.409718
 Is to 130 the Sum of the true Sides 2.113943
 So is 100 the assumed Side AB 2.000000

To 50.609 the true Side AB 1.704225

Again,
 As 256.873 Ar. C. 7.590282
 Is to 130 2.113943
 So is 84.776 the assumed Side CB 1.928272

To the true Side CB 42.904 1.632497

Again,
 As 256.873 Ar. C. 7.590282
 Is to 130 2.113943
 So is 72.097 the assumed Side AC 1.857911

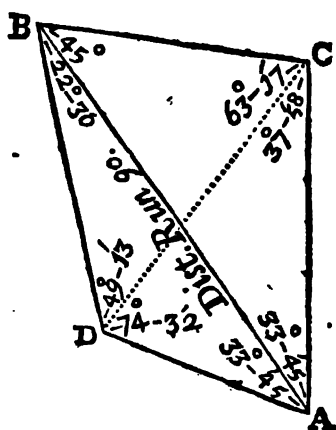
To the true Side AC 36.487 1.562136

7. Coasting along the Shore I saw two Capes of Land, the first by the Compass did bear North, the second WNW; then I stood away NW *b* N 90 Miles, and then I find the first to bear from me E *b* S, and the second S *b* E: the Bearing and Distance of both is required.

In the Triangle ABC,
 Ass. of ACB $101^{\circ} 15'$ (or $78^{\circ} 45'$) 0.00843
 Is to the Distance run 90 1.95424
 So s. of ABC $45^{\circ} 00'$ 9.84948

To the Side AC 64.9 1.81215

In



**In the Triangle ABD,
'As s. ADB 123° 45' (or 56° 15')
Is to the Distance run 90
So s. ABD 22° 30'**

0.080154
1.954242
9.582840

To the Side AD 41.42

1.617236

64.9
41.42

106.32 Sum of the Sides

23.48 Diff. of the Sides,

180 00
67 30 = the $\angle$ DAC

11230 = Sum of the opposite Angles.

56 15 = half Sum;

A

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As the Sum of the Sides 106.32 7.973388
 Is to the Diff. of the Sid. 23.48 1.376698
 So t. of $\frac{1}{2}$ the opp. Ang. 56° 15' 10.175108

To t. of half their Diff. 18 17 9.519192

Sum 74 32 = the greater Ang.

Diff. 37 58 = the lesser Ang.

See Case 3. Chap. 4. Part 1.

As s. the Angle CDA 74° 32' 0.016019
 Is to the Side AC 64.9 1.812150
 So s. of the Ang. DAC 67° 30' 9.965615

To the Distance of the Capes GD 62.2 1.793784

In the Triangle BCD,

As s. of the Angle DBC 67° 30' 0.034385
 Is to the Distance of the Capes 62.2 1.793784
 So s. of the Angle BDC 49° 13' 9.879202

To the Distance BC 50.98 1.707371

Again,

As s. of the Angle DBC 67° 30' 0.034385
 Is to the Distance of the Capes 62.2 1.793784
 So s. of the Angle BCD 63° 17' 9.950968

To the Distance BD 60.14 1.779137

So the Distance of the two Capes is 62.2 Miles;
 and the Bearing is SW $\frac{1}{4}$ W, and NE $\frac{1}{4}$ E.
 When I first made the Capes, the Northermoft was
 64.9 Miles distant, and the Distance of the Wester-
 moft was 41.4 Miles. And when I had sail'd 90 Miles;

U

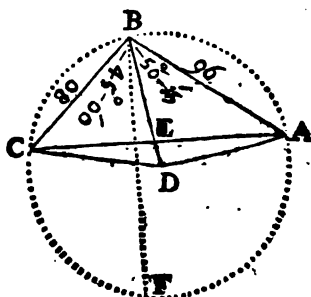
my

my Distance from the Northermost was 50.98 Miles,
and my Distance from the other was 60.14 Miles.

8. There are three Ships of equal Distance from one Port, and bound for the same place: The Eastermost is distant from the middlemost 96 Leagues, and bears SE $\frac{1}{2}$ E; the middlemost is distant from the Westermost 80 Leagues, and bears SW from her. I demand each Ship's Course to the Port, and how far they are distant from it.

Thro the three Points A, B, and C, strike a Circle, whose Center D represents the Port; then draw the Semidiameters DA, DB, and DC, which shall be the Distance of each Ship from the Port.

	96	56° 15'
	80	45 00
Sum	176	101 15
Diff.	16	78 45
		39 22½ half



Then,

Then,

As Sum of the Sides 176 7.75449

Is to their Diff. 16 1.20412

So t. of $\frac{1}{2}$ the Sum of the opposite Angle } 9.91443
 $39^{\circ} 22' \frac{1}{2}$ }

To t. of $\frac{1}{2}$ their Diff. 4 16 8.87304

Sum 43 38 = BCA

Diff 35 06 = BAC

Therefore the Course from C to A is *E 6 N* $9^{\circ} 53'$
E.

As s. of the Angle BCA $43^{\circ} 38'$ 0.161125

Is to the Side AB 96 1.982271

So s. of the Ang. ABC $101^{\circ} 15'$ (or $78^{\circ} 45'$) 9.991574

To the Side CA 136.45 2.134979

Which is the Distance of the two Ships A and C.

The Arch AFC is $202^{\circ} 30'$ (being double to the Angle ABC $101^{\circ} 15'$ by *Euc. 3. Lib. 20. Pr.*) therefore $202^{\circ} 30'$ subtracted from 360, the remainder is the Angle ADC $157^{\circ} 30'$; and because the Sides AD and CD are equal, therefore the Angles DAC, and DCA are equal; and so the Complement of $157^{\circ} 30'$ to 180 is $22^{\circ} 30'$, the half whereof $11^{\circ} 15'$ is the Angle DAC, or ACD. Then,

As s. of the Angle ADC $157^{\circ} 30'$ (or $22^{\circ} 30'$) } 0.417160

Is to the Distance CA 136.45 2.134979

So s. of the Angle $11^{\circ} 15'$ 9.290236

To the Side CD (= AD) 69.56 1.842366

Which is the Distance of each Ship from the Port.

U 2

To

To find the Course from C to D, the Angle BCE is $43^{\circ} 38'$, to which add $11^{\circ} 15'$, the Sum is $54^{\circ} 53'$, the Angle BCD; therefore the Course from C to D is $E 6 S 1^{\circ} 22' E$. and the Course from A to D is $W 6 S 1^{\circ} 21' Southerly$, and the Course from B to D is $S 6 E 1^{\circ} 21' Southerly$.

S E C T. IV.

Of CURRENTS.

Problem 1. Admit a Current runs East 6 Miles an hour, and a Ship sails West directly against it 6 Miles an Hour; I demand her compound Motion.

If the Current runs directly contrary to the Course of the Ship, and the Motion of the Current be equal to the Motion of the Ship, it is evident that the Ship makes no way, but stands still; for so much as she is forced forward by the Wind, she is driven backward by the Current.

2. Admit a Current runs East four Miles an Hour, and a Ship sails West 6 Miles an Hour: I demand her compound Motion.

From the Ship's simple Motion

Subtract the simple Motion of the Current $\left. \begin{array}{r} 6 \\ 4 \\ -2 \end{array} \right\} \text{Mil.}$

Remain the Ship's compound Motion

Thus the Ship advances forward 2 Miles every Hour.

3. A Current sets West 8 Miles an Hour, and a Ship sails East directly against it 5 Miles an Hour: I demand her compound Motion. Here

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Here it is evident, that because the Current moves faster than the Ship, the Current must force the Ship backwards, tho by the Log. she appears to gain ground. Therefore,

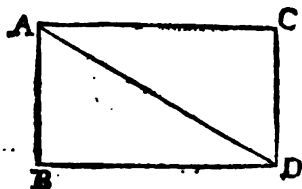
$$\begin{array}{r} \text{From the Current's simple Motion} \quad 8 \\ \text{Subtract the Ship's simple Motion} \quad 5 \\ \hline \text{There remains the Ship's comp. Motion} \quad 3 \end{array} \left. \vphantom{\begin{array}{r} 8 \\ 5 \\ 3 \end{array}} \right\} \text{Miles.}$$

And thus the Ship falls a-stern 3 Miles every hour.

4. A Ship sails East 4 Miles an hour, upon a Current which sets East 5 Miles an hour: I demand her compound Motion.

$$\begin{array}{r} \text{Add the Current's Motion} \quad 5 \\ \text{To the Ship's simple Motion} \quad 4 \\ \hline \text{That Sum is the Ship's comp. Motion} \quad 9 \end{array} \left. \vphantom{\begin{array}{r} 5 \\ 4 \\ 9 \end{array}} \right\} \text{Miles.}$$

5. A Ship sails South 3 Miles an hour, where there is a Current running East 5 Miles an hour: I demand the Ship's compound Motion, and which way.



U 3

As

A

As the Side AB 3, the Ship's motion	0.477121
To the Side BD = AC 5, the Car. mot.	0.698970
So is Radius	10.

Tot. of the Angle BAD $59^{\circ} 02'$	10.231849
--	-----------

Again, As s. of the Angle BAD $59^{\circ} 02'$	9.933217
To BD the Current's motion 5	0.698970
So is Radius	10.

To AD the Ship's compound Motion 5.83 0.765753

Hence it appears that the Ship's Course is SE $\frac{1}{2}$ E $\frac{1}{4}$ E, and her horary Motion is 5.83 Miles.

6. A Ship sails East 4 days together, by Log. 480 Miles, where there is a Current setting all this while to the South 2 $\frac{1}{2}$ Miles each hour: I demand her Angle of Deflection, and compound Motion.

2.5 As BD 480 (in the former Figure)	2.681243
24 Is to BA 240	2.380211
So is Radius	10.

160	
50 To the Tangent of the Angle BDA $26^{\circ} 33'$	9.698970

60.0	
4	Again,
As s. of the Angle BDA $26^{\circ} 33'$	9.650287
240 Is to BA 240, the Current's Motion	2.380211
So is Radius	10.

To the Ship's comp. Motion 537 2.729924

Hence the Ship's Course is ESE $4^{\circ} 3'$ Southward.

7. A Ship sails in 8 hours from a certain Cape towards B, bearing South, 18 Miles by Log. in a Current setting to the Eastward; and then observing the said Cape, she finds it to bear WNW. I demand how fast the Current sets, and the Ship's true Distance run.

In the former Figure,	
As s. of the Angle ADB $22^{\circ} 30'$	0.417160
Is to AB 18 Miles	1.255272
So is s. of the Angle BAD $67^{\circ} 30'$	9.965615
To BD 43.46, the Current's motion	1.638047

Which divide by 8, the Quotient is 5.43 Miles, the hourly Rate. Again,

As s. of ADB $22^{\circ} 30'$	0.417160
Is to AB 18 Miles, the Ship's Motion	1.255272
So is Radius	10.
To the Ship's comp. Motion AD 47.4	1.672432

S E C T. V.

Of MERCATOR'S Sailing.

To find the meridional Difference of Latitude, or the Difference of Latitude in meridional Parts.

1. If one Place be under the Equinoctial, and the other in North or South Latitude; the meridional Parts in the Table answering to the Degrees and Minutes of the Place with Latitude, is the meridional Difference of Latitude.

Or by the Meridian Line upon *Gunter's Scale*, look for the Latitude of the Place in the Meridian Line, and right against it in the Line of equal Parts you have the meridional Difference of Latitude.

Example. If one Place be under the Equinoctial, and the other in $35^{\circ}30'$ North Lat. I demand the meridional Parts of the Difference of Latitude.

Ans. The meridional Diff. of Lat. is 2281.

By *Gunter's Scale*.

Extend the Compasses from the beginning of the Meridian Line to $35^{\circ}30'$; then measure that extent upon the Line of equal Parts, and you will find it 38° , which brought to Minutes by multiplying by 60, it will make 2280, which is 1 less than by the Table.

2. If the two Places be both in North, or both in South Latitude; subtract the meridional Parts of the lesser Latitude from those of the greater, and the Remainder is the meridional Difference of Latitude.

Example.

Sup.	$\begin{cases} \text{Lizard} \\ \text{Barbadoes} \end{cases}$	$\begin{cases} \text{Lat. } \{ 50^{\circ}00' \\ 13^{\circ}00' \} \end{cases}$	Mer. Pts.	$\begin{cases} 3475 \\ 787 \end{cases}$
------	---	---	-----------	---

Mer. Diff. Lat.	<u>2688</u>
-----------------	-------------

By *Gunter's Scale*.

Extend your Compasses from 13° to 50° upon the Meridian Line, and measure that upon the Line of equal Parts, you will find it 44.8 , which multiplied by 60 makes 2688,

3. If the two Places have one North Latitude, and the other South; add the meridional Parts of both Latitudes together, the Sum is the Difference of Latitude in meridional Parts,

Example.

Chap. 4. Mercator's Sailing. 297

Example.

$$\begin{array}{l} \{ \text{Barbadoes} \\ \{ \text{St. Helena} \end{array} \} \text{Lat. } \begin{array}{l} \{ 13^{\circ} 00' \text{ N} \\ \{ 16^{\circ} 00' \text{ S} \end{array} \} \text{Mer. Pts. } \begin{array}{l} \{ 787 \\ \{ 973 \end{array}$$

The Sum is the Merid. Diff. Lat. 1760

By *Gunter's Scale*.

Extend your Compasses from the beginning of the Line to 13° , and lay that extent upward from 16° ; keep the uppermost Point where it falls, and extend the other to the beginning of the Line; that extent measured upon the Line of equal Parts, it will be 29.3, which multiplied by 60 makes 1758.

C A S E I.

Both Latitudes, and the Difference of Longitude between any two Places given; to find the Course and Distance.

What is the Course, Distance and Departure between the *Lizard* and *Barbadoes*?

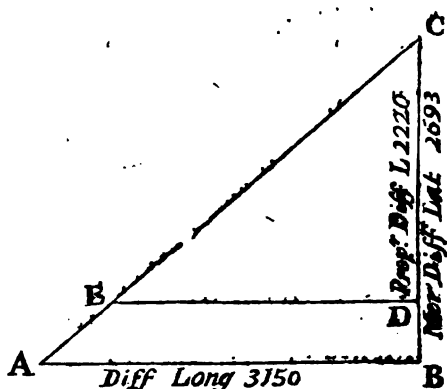
$$\begin{array}{l} \text{Lizard} \} \\ \text{Barbad.} \} \end{array} \text{Lon. } \begin{array}{l} \{ 5^{\circ} 24' \\ \{ 57^{\circ} 54' \end{array} \} \text{Lat. } \begin{array}{l} \{ 50^{\circ} 10' \\ \{ 13^{\circ} 10' \end{array} \} \text{M. p. } \begin{array}{l} \{ 3490 \\ \{ 797 \end{array}$$

$$\begin{array}{r} \text{Diff. Long. } 52 \ 30 \\ \hline 60 \\ \hline 31 \ 50 \end{array} \quad \begin{array}{r} \text{Diff. La. } 37 \ 00 \\ \hline 60 \\ \hline 22 \ 20 \end{array} \quad \begin{array}{r} \text{M. D. L. } 2693 \\ \hline \end{array}$$

$$\begin{array}{r} \text{As the Mer. Diff. Lat. } 2693 \\ \text{Is to Radius} \\ \text{So is the Diff. Long. } 31 \ 50 \end{array} \quad \begin{array}{r} 3.43024 \\ 10. \\ 3.49831 \end{array}$$

$$\begin{array}{r} \text{To t. of the Course } 49^{\circ} 28' \\ \text{So the Course is SW } \frac{1}{4} \text{ W nearly.} \end{array} \quad \begin{array}{r} 10.06807 \end{array}$$

In



In the Triangle CDE is given the proper Difference of Lat. 2220, and the Course ECD $49^{\circ} 28'$, to find EC the Distance.

As the cs. of the Course $49^{\circ} 28'$	9.81284
To the proper Diff. of Lat. 2220	3.34635
So is Radius	10.

To the Distance run EC 3416	3.53351
-----------------------------	---------

As Rad. to the Distance EC 3416	3.53351
So s. of the Course $49^{\circ} 28'$	9.88083

To the Departure ED 2595	3.41434
--------------------------	---------

CASE

C A S E II.

Both Latitudes, and Course given; the Distance, Difference of Longitude, and Departure required.

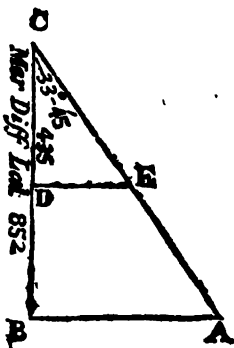
A Ship in the Latitude $62^{\circ}45'$ North sails S E $\frac{1}{2}$ S till she comes into the Latitude $55^{\circ}30'$ North; I demand the Distance run, Diff. of Longitude, and Departure.

$$\begin{array}{r} \text{Lat. } \left\{ \begin{array}{l} 62^{\circ}45' \\ 55^{\circ}30' \end{array} \right\} \text{ Merid. parts } \left\{ \begin{array}{l} 4872 \\ 4020 \end{array} \right\} \\ \hline 715 \text{ Mer. Diff. Lat. } 852 \\ 60 \end{array}$$

Diff. La. 435

As ca. of the Course $56^{\circ}15'$ CED	9.91984
Is to the proper Diff. of Lat. 435 DC	2.63849
So is Radius	10.

To the Distance run 513.2 EC	2.71865
------------------------------	---------



As
2

300 *Of Navigation.*

Part II.

As Rad. to Mer. Diff. Lat. 852 BC	2.93044
So t. of the Course 33° 45' BCA	9.82489
To the Diff. of Longitude 569.3 BA	2.75533
As Radius to the prop. Diff. Lat. 435	2.63849
So is t. of the Course 33° 45' DCE	9.82489
To the Departure 290.7 DE	2.46338

CASE III.

Both Latitudes and Distance given ; the Course, Departure, and Difference of Longitude required.

Admit a Ship sails from the *Lizard* in the Latitude 50° North, to *Port Royal* in *Jamaica* in the Latitude $17^{\circ} 40'$ North, the Distance supposed to be 4000 Miles; I demand the Course, Difference of Longitude, and Departure.

<i>Lizard</i> } Lat. { $50^{\circ} 00'$ } Mer. parts { 3474
<i>P. Royal</i> } { $17 \ 40$ } { } { 1077
$32 \ 20$ Mer. diff. 2397
60

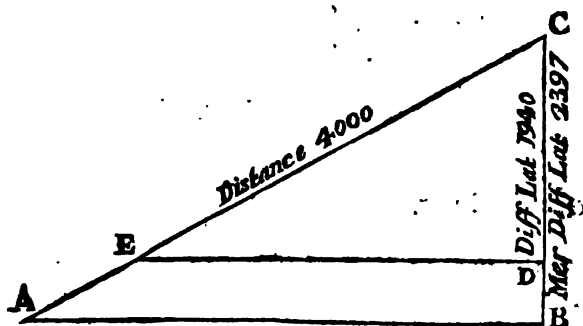
Prop. diff. Lat. 1940

As the Distance run 4000 EC	3.60206
Is to the proper Diff. of Lat. 1940 DC	3.28780
So is Radius	10.

Tocs. of the Course 29° 01' DEC 9.68574

So the Course is WSW $\frac{1}{2}$ W very near.

As



As Rad. to the Distance run 4000 EC	3.60206
So s. of the Course 60° 59' ECD	9.94175
To the Departure 3498 ED	3.54381
As the proper Diff. of Lat. 1940 DC	6.71220
Is to the Merid. Diff. of Lat. 2397 BC	3.37967
So is the Departure 3498 ED	3.54381
To the Diff. of Longitude 4322 AB	3.63568

C A S E I V.

Both Latitudes and Departure given; the Course, Distance, and Difference of Longitude required.

A Ship in the Latitude 50° 10' North, and Longitude 5° 24' West, sails South Westward till her Departure be 789 Miles, and she be in the Latitude 39° 20' North; I demand the Course, Distance, and Longitude she is in.

$\left. \begin{array}{r} 50^{\circ} 10' \\ 39 \ 20 \end{array} \right\} \text{Latitude}$
 $\left\{ \begin{array}{r} 3490 \\ 2570 \end{array} \right\} \text{Merid. parts}$

10 50
 60

920 Mer. Diff. Lat.

650 Diff. Lat.

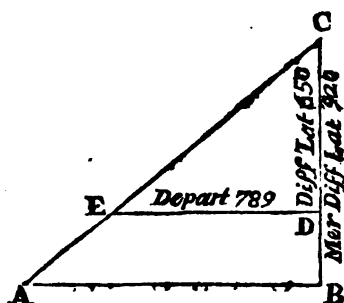
As the proper Diff. of Lat. 650
 Is to the Departure 789
 So is Radius

2.81291
 2.89708
10.

To t. of the Course $50^{\circ} 31'$

10.08417

So the Course is SW $\frac{1}{2}$ W.



As s. of the Course BCD $50^{\circ} 31'$
 Is to the Departure 789 ED
 So is Radius

9.88751
 2.89708
10.

To the Distance run EC 1022

3.90057

As

Chap. 4. Mercator's Sailing.		303
As the proper Diff. Lat. CD 650		7.18709
Is to the Departure ED 789		2.89708
So is the Merid. Diff. Lat. BC 920		2.96378
To the Diff. of Longitude AB 111.7		3.04795
	18 37	
The Longitude come from, add	5 24	
The Longitude the Ship is in	24 01	West.

C A S E V.

One Latitude, Course, and Distance given; to find the Difference of Latitude, Difference of Longitude, and Departure.

A Ship in the Latitude of $42^{\circ} 30'$ North, and Longitude $18^{\circ} 31'$, sails SE $\frac{1}{4}$ S 59° Miles; Demand the Latitude, and Longitude of the Ship, and how much she is departed from the Meridian.

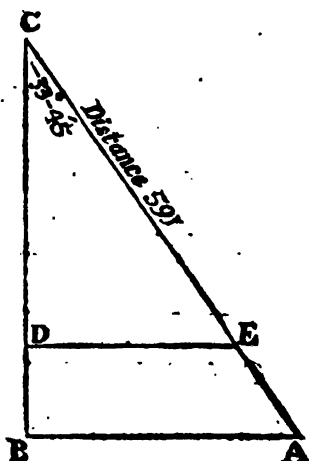
As Rad. to the Distance CE 59	2.771587
So is cs. of the Course $56^{\circ} 15'$	9.919846

To the Diff. Lat. CD 498.4	2.691433
----------------------------	----------

8° 11'

Lat. departed from $42^{\circ} 30'$	} Mer. parts {	2822
Diff. Lat. subtract 8 11		2195

Lat. the Ship is in 34 19	Mer. diff.	627
---------------------------	------------	-----



As Rad. to t. of the Course $33^{\circ} 45'$	9.82489
So is the Mer. diff. of Lat. 627	2.79727

To the diff. Longitude 419	2.62216
----------------------------	---------

As Rad. to the Distance run CE 591	2.77158
So is s. of the Course DCE $33^{\circ} 45'$	9.74474

To the Departure 328.3 DE	2.51632
---------------------------	---------

The Longitude departed from $18^{\circ} 31'$	
Difference of Longitude add 6 59	

The Longitude of the Ship	25 30
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C A S E

CASE VI.

One Latitude, Departure, and Course given; to find the Distance, Difference of Latitude, and Difference of Longitude.

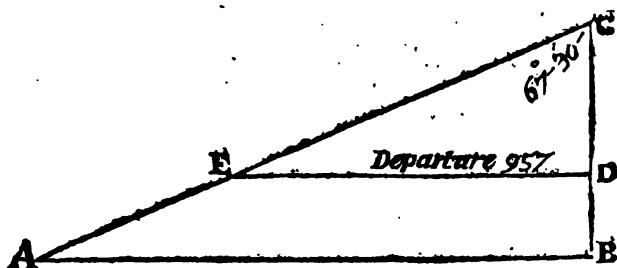
A Ship sails WSW from a certain Port in $50^{\circ} 10'$ North Latitude, and West Longitude $5^{\circ} 24'$, until her Departure from the Meridian be 957 Miles; I demand her Distance run, and the Latitude and Longitude she is in.

Lat. departed from $50^{\circ} 10'$
Diff. Lat. subtr. $6 36$

Lat. the Ship is in $43 34$

Mer. parts $\begin{cases} 54157 \\ 3490 \end{cases}$

Mer. diff. Lat. 667



Ass. of the Course ECD $67^{\circ} 30'$
Is to the Departure ED 957
So is Radius

9.965613
2.980912
10.

To the Distance run 1035.83 EC
X

3.015297
As

As Rad. to the Distance EC 103 5.85	3.015297
So is the cs. of the Course CED 22° 30'	9.582840

To the Difference of Latitude CD 396.4	2.598137
--	----------

6° 36'

As the proper Diff. of Lat. CD 396.4	7.401863
Is to the Departure ED 957	2.980912
So is the Merid. Diff. of Lat. CB 667	2.824126

To the Diff. of Longitude AB 1610	3.206901
-----------------------------------	----------

Longitude departed from	5° 24'
Diff. of Longitude add	26 30

Longitude the Ship is in 32 14

C A S E VII.

*One Latitude, Distance, and Departure given;
the Course, Difference of Latitude, and Difference of Longitude required.*

A Ship in the Latitude 49° 30' North, and West Longitude 14° 30', sails South Eastward 645 Miles, until her Departure be 500 Miles; I demand the Course sailed, and the Latitude, and Longitude the Ship is in.

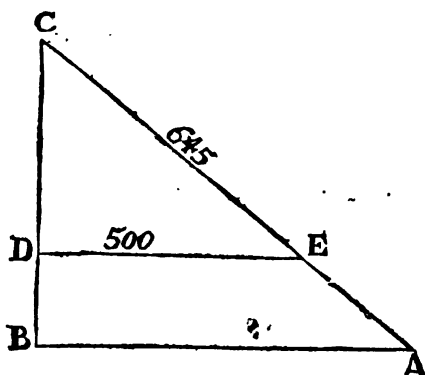
As the Distance CE 645	2.80956
To the Radius	10.
So is the Departure 500	2.69897

To s. of the Course 50° 50'	9.88941
-----------------------------	---------

Lat.

Lat. departed from 49 30
Diff. of Lat. subtr. 6 47

Lat. the Ship is in 42 43



As Rad. to the Diff. run CE 645	2.80956	3428	} Mer. pa.
So cs. of the Course 39° 10'	9.80043	2839	
		589	
To the Diff. of Lat. CD 407.3	2.60999		

As the proper Diff. of Lat. CD 407.3	7.39111
Is to the Departure DE 500	2.69897
So is the Merid. Diff. of Lat. BC 589	2.77011

To the Diff. of Longitude 724.7 AB	2.86019
------------------------------------	---------

Longitude departed from 14° 40'
Diff. of Longitude subtr. 12 04

Long. the Ship is in 2 36

C A S E VIII.

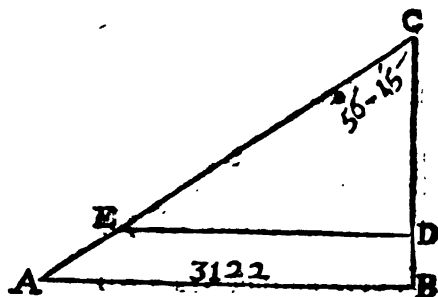
One Latitude, Course, and Difference of Longitude given; to find the Difference of Latitude, Distance, and Departure.

A Ship sails from the *Lizard* in the Latitude $50^{\circ} 10'$ North, and West Longitude $5^{\circ} 24'$ SW b W, until her Longitude be $57^{\circ} 26'$ West; I demand the Latitude the Ship is in, her Distance run, and Departure.

Longitude the Ship is in $57^{\circ} 26'$
Longitude departed from $5^{\circ} 24'$

52 02
60

Diff. of Longitude 3122



As t. of the Course $56^{\circ} 15'$
Is to Radius
So is the Diff. of Longitude 3122

10.17502

10.

3.49443

To the Mer. Diff. Lat. 2086

3.31941

3

As

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As cs. of the Course $33^{\circ} 45'$	9.74474
Is to the Diff. Lat. 1643	3.21584
So is Radius	10.

To the Distance 2958	3.47090
----------------------	---------

As Rad. to the Distance 2958	3.47090
So is the s. of the Course $56^{\circ} 15'$	9.91985

To the Departure 2459	3.39075
-----------------------	---------

Lat. <i>Lizard</i> $50^{\circ} 10'$	3490 Mer. parts.
Mer. diff. Lat. subtr.	2086

Remains	1404
---------	------

Look 1404 in the Table of Meridional Parts, and against it you will find $22^{\circ} 47'$, the Latitude required.

From $50^{\circ} 10'$
Subtr. 22 47

27 23
60

Diff. Lat. 1643

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5. Take the nearest Distance from e to the prick'd Line Bc , and lay that Distance on the Line AB from B to D , and draw the prick'd Line De .

6. Lay the Difference of Longitude on the prick'd Arch made with the Sine of 90° , from B to C , and draw the Line AC ; also with the Distance AD , and one foot in A , draw the prick'd Arch DE to cut AC in E ; likewise draw Lines from B to C , and from D to E ; measure AD upon the Line of Sines, and you will find it $42^\circ 30'$, which is the Complement of the Latitude; so the Latitude is $47^\circ 30'$.

Then,	
As the Difference of Longitude BC 370	2.56820
Is to Radius AB	10.
So is the Distance sailed DE 250	2.39794

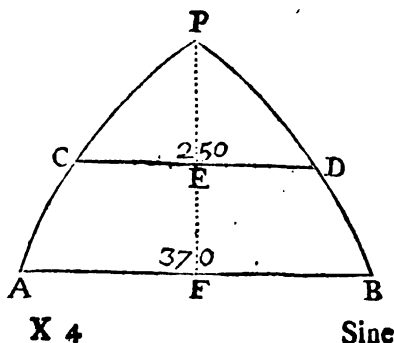
To s. of $42^\circ 30'$ AD , the Com. of the Lat. 9.82974

We may demonstrate this and the two following Cases thus.

Let AB (in the following Figure) represent an Arch of the Equator, P the Pole of the World, and AP and BP are Meridians each 90 degrees, CD is an Arch of the Parallel of Latitude the Ship sails in, CA being $47^\circ 30'$ the Latitude, or Distance of the Parallel from the Equator; and CP is the Complement $42^\circ 30'$.

Then AF is the Sine of the Arch AP , and CE is the Sine of the Arch CP . Therefore, by *Axiom 1. Ch. 7.* of the first Part,

As AF (the Sine of 90°) is to AB (the Difference of Longitude) so is CE (the



Sine Complement of the Latitude) to CD, (the Distance) and the contrary.

C A S E II.

The Difference of Longitude between two Places in the same Latitude given ; to find their Distance.

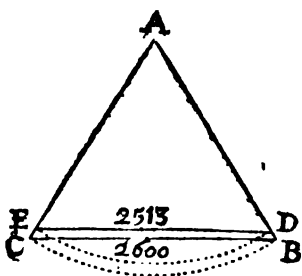
The Distance between *Martinico* and *Cape Verde* is required.

From $54^{\circ} 50'$ } Long. { *Martinico* } Lat. $14^{\circ} 50'$.
Subtr. $11\ 30$ } { *C. Verde* }

Diff. $43\ 20 = 2600$ Miles.

As Rad. to the Difference of Long. 2600 3.41497
So is the cs. of the Latitude $75^{\circ} 10'$ 9.98528

To the Distance required 2513.3 3.40025



The Delineation of this is so easy, that any one that knows how to delineate the last, may delineate this. For it is but to take the Sine of 90° , and upon A strike the Arch BC, and upon that lay the Difference of Longitude 2600, and draw the Lines AC and BA ; then with $75^{\circ} 10'$ of Sines, strike the Arch ED, and draw the Line ED, and measure it upon the Scale, and you will find it 2513.

C A S E

CASE III.

The Distance between two Places in the same Parallel given; to find the Difference of Longitude.

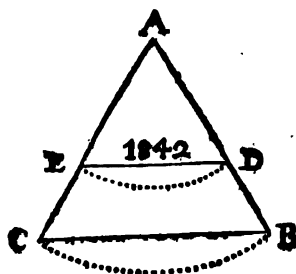
The Distance between the *Lizard* and *Penguin-Island* being 1842 Miles, and being both in the Latitude of 50° , what is the Difference of Longitude between them?

As cs. of the Latitude 40°
Is to the Distance 1842
So is Radius

9.80807
3.26529
10.

To the Difference of Long. 2866

3.45722



6,0)286,6

47 46

Answer. The Diff. of Long. $47^{\circ} 46'$

S E C T. VII.

*Of Sailing by Middle Latitude,
Or a Proportion drawn from the Middle Latitude,
nearly agreeing with MERCATOR's Sailing.*

To find the Middle Latitude.

Always add the Latitude you are come from, to the Latitude you are come to; half the Sum is the middle Latitude.

C A S E I.

*Both Latitudes, and Difference of Longitude, of
two Places given; to find the Course and Dis-
tance.*

A Ship sails from the Latitude $45^{\circ} 30'$ North, between the North and East, till she arrives in the Latitude $50^{\circ} 10'$, and then finds her Difference of Longitude $8^{\circ} 16'$; I demand her Course and Distance.

This way by middle Latitude depends upon this following Theorem.

As the Co-sine of middle Latitude
Is to the Tangent of the Course;
So is the Diff. of Latitude in Miles, or Leagues,
To the Diff. of Longitude in Miles, or Leagues.

Chap. 4. *Sailing by Middle Latitude.* 315

8° 16'

60

50° 10'

45 30

$$496 = \text{Diff. Long. Diff. of Lat. } 4 \text{ } 40 = 280$$

The Sum

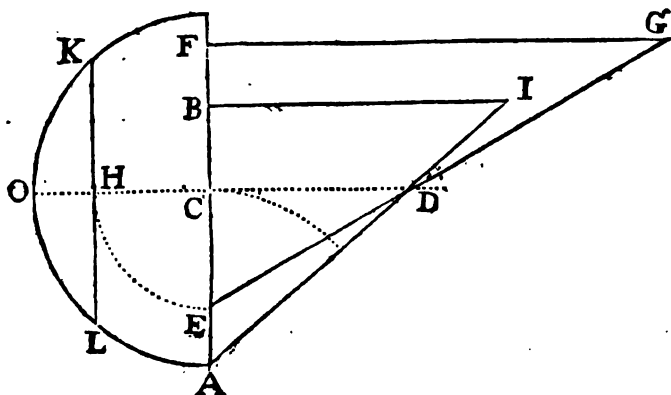
95 40

The half

47 50 = Mid. Lat.

To delineate this,

1. With the Chord of 60° , and upon the Center C, strike the Semicircle, and draw the Diameter AF; and draw OD at right Angles thereunto,



2. Set the Difference of Latitude from A to B, and draw BI Perpendicular to AF.

3. Set the middle Latitude from O to K and L; and draw KL to cut DE in H; make CE equal to CH; and make EF equal to AB, and draw FG parallel to BI, and set the Difference of Longitude given from F to G, and draw the Line EG to cut OD in the Point D.

4. Lay a Ruler upon A and the Point D, and draw the Line ADI, to cut BI in I, so shall AI (being measured

fured upon the same Scale of equal Parts, that AB and FG were laid down by) shew the Distance required; and the Angle BAI is the Course, and BI the Departure.

As the Diff. Lat. $EF=AB$ 280	7.552842
Is to the Diff. of Long. FG 496	2.695482
So is cs. of the mid. Lat. $EC=CH$ $42^{\circ} 10'$	9.826930

To the t. of the Course CD $49^{\circ} 56'$	10.075254
---	-----------

As cs. Course $40^{\circ} 04'$ AIB	9.808669
To the Diff. of Lat. 280	2.447158
So is Radius	10.

To the Distance run 435	2.638489
-------------------------	----------

As Rad. to the Diff. of Lat. 280	2.447158
So is the Tangent of the Course $49^{\circ} 56'$	10.075254

To the Departure 333 BI	2.522412
-------------------------	----------

C A S E II.

Both Latitudes, and Course given; the Distance, Difference of Longitude, and Departure required.

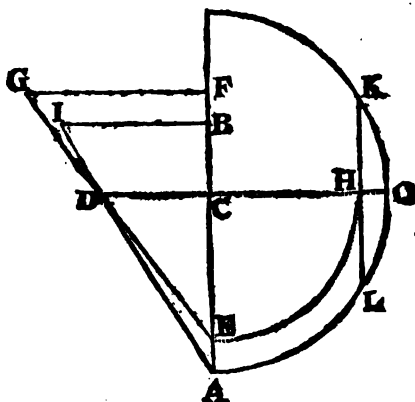
A Ship being in the Latitude of $32^{\circ} 00'$ North; sails NNW $\frac{1}{2}$ W. until she arrives in the Latitude of $32^{\circ} 39'$; I demand the Distance, Difference of Longitude, and Departure.

Lat.

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Lat. $\left\{ \begin{array}{l} 32^{\circ} 39' \\ 30 \text{ } 00 \end{array} \right\} 62^{\circ} 39'$ the Sum.
 Diff. $\begin{array}{r} 2 \text{ } 39 \\ 60 \end{array}$ middle Lat.
 159 Diff. Lat.



Ascs. of middle Lat. $31^{\circ} 19'$	0.068386
Is to the Diff. of Lat. 159	2.201397
So t. of the Course $30^{\circ} 56'$	9.777628

To the Diff. of Longitude 111.5 2.047411

'As cs. of the Course 59° 04' AIB	9.933369
Is to the Diff. of Lat. 159 AB	2.201397
So is Radius	10.

To the Distance 185.4 AU 2.268028

As

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As the Distance AI 58½ Miles	2.76716
Is to Radius	10.
So is the Diff. Lat. AB 310	2.49136

To the cs. of the Course AIB 32° 00'	9.72420
--------------------------------------	---------

So the Course is NE 6E 1° 45' Easterly.

As cs. of the middle Latitude 47° 50'	0.17307
Is to the Difference of Latitude 310	2.49136
So t. of the Course 58° 00'	10.20421

To the Difference of Longitude 739	2.86864
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As Rad. to the Diff. of Latitude 310	2.49136
So t. of the Course 58° 00'	10.20421

To the Departure 496.1	2.69557
------------------------	---------

C A S E IV.

Both Latitudes, and Departure given; the Course, Distance, and Difference of Longitude required.

A Ship in the Latitude 40° 30' North, sails between the North and West till her Departure is 450 Miles, and then finds her Latitude 51° 20': I demand her Course, Distance, and Difference of Longitude.

51° 20'	} Sum	91° 50'
40 30		

10 50=650	45 55 Mid. Lat.
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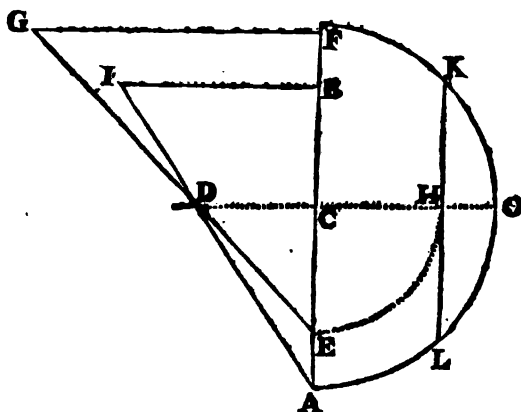
As
2

**As the Diff. Lat. AB 650
Is to the Departure BI 450
So is Radius**

2.81291
2.65321
10.

To t. of the Course $34^{\circ} 42'$

9.84030



As cs. of the middle Latitude $45^{\circ} 55'$
Is to the t. of the Course BA $34^{\circ} 42'$
So is the Diff. of Lat. AB 650

0.15758
9.84030
2.81291

To the DMF. of Long. BG 646.8

2.81079

As the cs. of the Course $55^{\circ} 18'$
Is to the Diff. of Lat. 650
So is Radius

9.91495
2.81291
10.

To the Distance-run 790.6

2.89796

The Course is NW 6N 0° 57' Westerly.

CASE

As cs. of the middle Lat. $47^{\circ} 50'$	0.13007
Is to t. of the Course BA $56^{\circ} 15'$	10.17511
So is the Diff. of Lat. AB	2.47713

To the Diff. of Long. 605.8 GR	2.78231
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As Radius to the Distance AI 540	2.73239
So s. of the Course $56^{\circ} 15'$	9.91985

To the Departure BI 449	2.65224
-------------------------	---------

C A S E VI.

One Latitude, Departure, and Course given; the Difference of Latitude, Difference of Longitude and Distance required.

A Ship from *Lisbon* in the Latitude $38^{\circ} 50'$ North, sails SW b W until her Departure be 560 Miles: I demand her Distance run, Difference of Longitude, and what Latitude she is in.

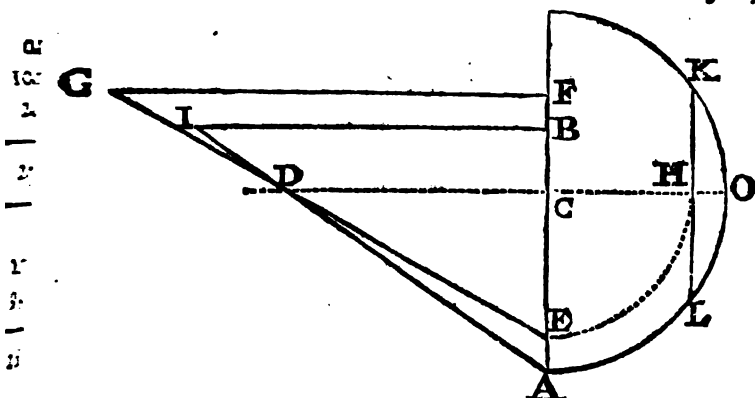
As s. of the Course $56^{\circ} 15'$	9.91985
To the Departure 560	2.74819
So is Radius	10.

To the Distance 673.5	2.82834
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As Rad. to the Distance 673.5	2.82834
So cs. of the Course $33^{\circ} 45'$	9.74474

To the Diff. Lat. 374	2.57308
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The



The Lat. come from	38° 50'	Sum of the Lat.	71° 26'
Diff. of Lat. subtr.	6 14		
	<u>32 36</u>	Middle Lat.	35 43
Lat. come into			

As cs. of the middle Lat. 54° 17'	0.09049
Is to t. of the Course 56° 15'	10.17511
So is the Diff. of Lat. 374	2.57308
	<u>2.83868</u>
To the Diff. of Long. 689.7	

C A S E VII.

*One Latitude, Distance, and Departure given;
the Difference of Latitude, Difference of Longitude, and Course required.*

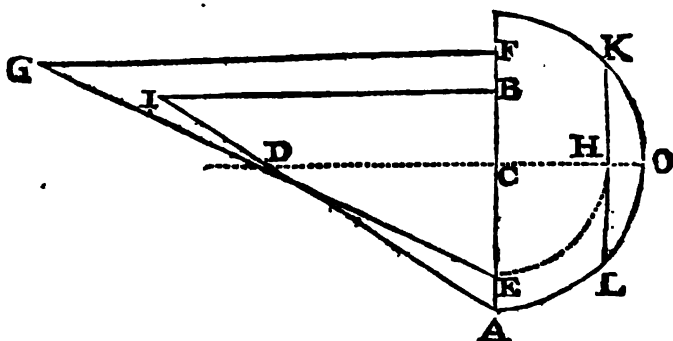
A Ship in the Latitude 38° 40' North, sails between the North and West 650 Miles, and then finds her Departure 546 Miles: I demand her Course, Difference of Latitude, and Difference of Longitude.

As the Distance 650
Is to the Departure 546
So is Radius

2.81291
2.73719
10.

To s. of the Course $57^{\circ} 08'$

9.92428



As t. of the Course $57^{\circ} 08'$
Is to the Departure 546
So is Radius

10.189697
2.737193
10.

To the Diff. of Lat. 352.8

2.547496

Lat. come from $38^{\circ} 40'$
D.f. Lat. add $5\ 53$

Sum of the Lat. $83^{\circ} 13'$

Middle Lat.

41 36

Lat. come into 44 33

As cs. of the middle Lat. $48^{\circ} 24'$
Is to the t. of the Course $57^{\circ} 08'$
So is the Diff. of Lat. 352.8

9.126216
10.189697
2.547496

To the Diff. of Long. 730.2

2.863409

The Course is NW $\frac{1}{2}$ W $0^{\circ} 53'$ Westerly.

Thus

Thus have I finished the Application of Plain or Right-lin'd Triangles so far as I design'd, tho there are many more excellent Uses which they may be, and often are applied to, which the Reader will find in his way of Practice: I shall therefore in the following Pages apply the Doctrine of Spherical Triangles to Practice. But it will be proper, before I begin to shew their Application, to shew how to project the Sphere *in Plano*.

CH A P. V.

The Projection of the Sphere in Plano.

S E C T. I.

How to project the Sphere in Plano upon the Plain of the General Meridian, by Circular Lines.

FIRST, with 60° of Chords draw the Circle BCDE upon the Center A, cross it with the two Diameters at right Angles; divide each of the four Quadrants into 90 degrees, as you see in the Figure. This Circle thus drawn and divided represents the general Meridian, C the North Pole, and E the South Pole, BAD the Equinoctial, CAE the Equinoctial Colure, and prime Vertical Circle, or Hour of six.

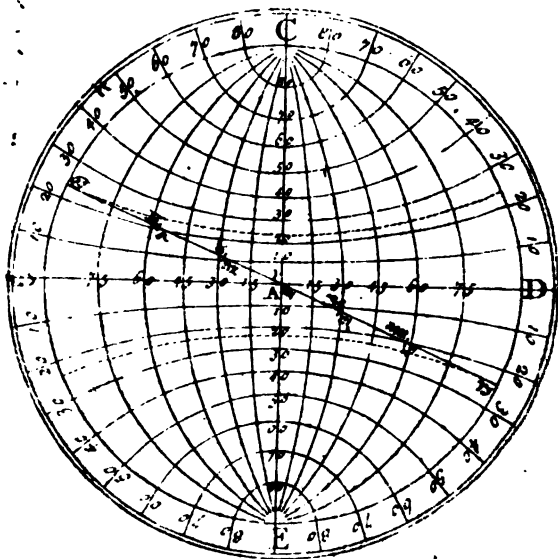
The Meridians or Hour Circles are drawn by *Prob. 5. Chap. 10.* of the first Part. Thus the half Tangent of 15, of 30, of 45, &c. set both ways from the Center, gives their Intersections with the Equinoctial; the Centers of those Circles are found by the Secant Complement, set from the Intersection upon the

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the Equinoctial (extended if occasion be;) then if your Center be found right, the Meridian which you draw will cut thro the Intersection before found, and thro the two Poles C and E.

The Intersections of the Parallels with the prime Vertical, are found also by half Tangents, that is, the half Tangents of 10, 20, 30, &c. set both ways from A; the Centers of those Parallels are found by setting the Tangents Complement of the Parallels Distance from the Equinoctial, along the prime Vertical (extended if need require.) See *Prob. 9.* of Spherical Geometry.

The Tropicks are the prick'd Parallels, and the right Line $\$ A \psi$ is the Ecliptick, and is divided by half Tangents; thus the half Tangent of 30° (that is one Sign) set both ways from A, shews the beginning of ϑ , and $\times$; and 60° set both ways from A, shews the beginning of Π , and $\equiv$: after the same manner may the intermediate Degrees be put on.



S E C T. II.

How to project the Sphere in Plano, futable to any assigned Horizon, suppose London, whose Latitude is $51^{\circ} 30'$.

Having divided the outward, or primitive Circle as in the last, extend the Meridian-line NS on both sides to B and O ; then take the half Tangent of $38^{\circ} 30'$ (the Complement of the Latitude) and set from Z the Zenith to P, so shall P be the North Pole, elevated above the North part of the Horizon NP $51^{\circ} 30'$ equal to the Latitude. ; Then take the Secant of $51^{\circ} 30'$, and set it upon the Meridian-line from P to B, so is B the Center of WPE, the Hour Circle of fix, which is also the Equinoctial Colure.

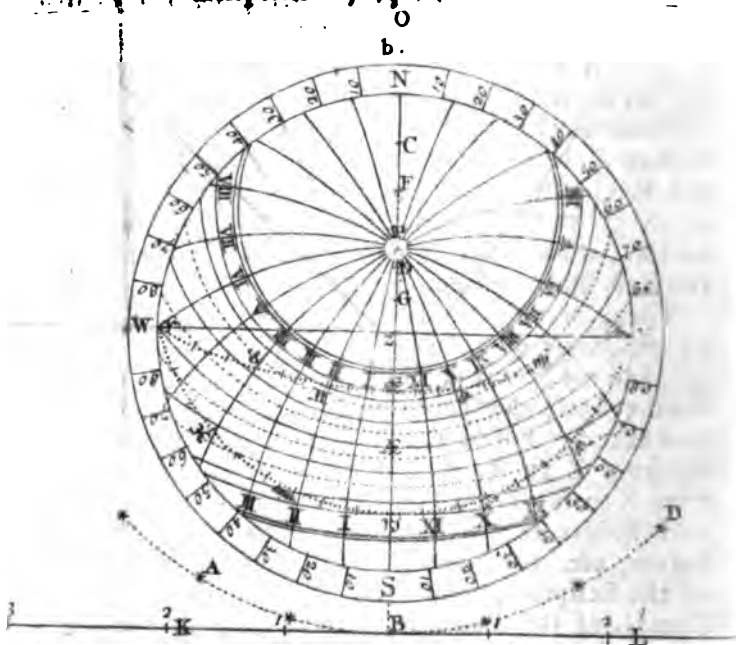
Then set the half Tangent of $51^{\circ} 30'$ from Z to Æ, so shall Æ be the Point in the Meridian thro which the Equinoctial must pass ; from Æ set the Secant of $38^{\circ} 30'$ to C, so shall C be the Center of the Equinoctial ; which being drawn, will cut the Horizon in the East and West Points thereof.

Then, because the Ecliptick cuts the Equinoctial at an Angle of $23^{\circ} 30'$, therefore add $23^{\circ} 30'$ to $51^{\circ} 30'$, and the Sum is 75 degrees : also subtract $23^{\circ} 30'$ from $51^{\circ} 30'$, and there remains 28 degrees. Wherefore take 75° out of the Scale of half Tangents, and set from Z to v, and set the half Tangent of 28° from Z to s ; so shall the two Points v and s be the two Points in the Meridian, thro which the two halves, viz. the Northern and Southern Semicircles of the Ecliptick, must pass. Now if to $38^{\circ} 30'$ the Compl. of the Latitude, you add $23^{\circ} 30'$ the Obliquity of the Ecliptick, the Sum is 62° , the Meridian Altitude of the Sun when he enters into s ; and if you subtract $23^{\circ} 30'$ from $38^{\circ} 30'$, the remainder will

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be 15° , the Sun's Meridian Altitude when he enters ϖ : Wherefore take 62° out of the Scale of Secants, and set from S to O ; so shall O be the Center of the Northern Semicircle of the Ecliptick; and the Secant of 15° set from ϖ to D , shews D to be the Center of the Southern Semicircle of the Ecliptick: both which Semicircles being drawn, will cut the Horizon in the Points of East and West, and the Meridian in the Points S and ϖ .

For dividing of the two Semicircles of the Ecliptick, set the half Tangent of 62° from Z to F ; so shall F be the Pole of the Northern Semicircle of the Ecliptick $\Upsilon, \text{♈}, \text{♊}$; and the half Tangent of 15° set from Z to G , so shall G be the Pole of the Southern half of the Ecliptick $\gamma, \text{♋}, \text{♌}$.



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From which two Poles F and G the Ecliptick is to be divided, thus: lay a Ruler to F, and to every 30° of the Primitive Circle, and it will cut the Northern Semicircle of the Ecliptick in the Points γ , Π , $\mathcal{S}$, $\mathcal{Q}$ and $\mathfrak{M}$; and being laid to G, and to every 30° of the Primitive Circle, it will cut the Southern Semicircle in the Points $\mathfrak{U}$, $\mathfrak{f}$, $\mathfrak{v}$, $\mathfrak{w}$, and $\mathfrak{x}$, for the Points of the Sun's entrance into any of the Signs: and so may every fifth, tenth, or every single Degree be divided.

The Intersection of the two Tropicks, and the other intermediate Parallels of Declination, are such as are in the following Table.

A Table of the Intersections of the Parallels of Declination with the Meridian.

Declination South.	Intersections.	Declination North	Intersections.
23	75° 00'	12°	63° 30'
23	74 30	11	62 30
22	73 30	10	61 30
21	72 30	9	60 30
20	71 30	8	59 30
19	70 30	7	58 30
18	69 30	6	57 30
17	68 30	5	56 30
16	67 30	4	55 30
15	66 30	3	54 30
14	65 30	2	53 30
13	64 30	1	52 30
		Equi.	51 30

For North Declination.

Declination South.	Intersections.	Declination North	Intersections.
23	28° 00'	12°	39° 30'
23	28 30	11	40 30
22	29 30	10	41 30
21	30 30	9	42 30
20	31 30	8	43 30
19	32 30	7	44 30
18	33 30	6	45 30
17	34 30	5	46 30
16	35 30	4	47 30
15	36 30	3	48 30
14	37 30	2	49 30
13	38 30	1	50 30

A Table of the Centers of the Tropicks and Paral. of Declination from the Zenith.

Declination South.	Centers from Z.	Declination North.	Centers from Z.
23	58° 20'	23	27° 47'
23	57 49	23	27 57
22	56 45	22	28 16
21	55 44	21	28 36
20	54 42	20	28 50
19	53 43	19	29 19
18	52 41	18	29 42
17	51 46	17	30 05
16	50 50	16	30 28
15	49 55	15	30 51
14	48 01	14	31 17
13	47 09	13	31 42
12	46 17	12	32 08
11	45 28	11	32 35
10	44 38	10	33 03
9	44 50	9	33 32
8	44 3	8	34 02
7	43 18	7	34 33
6	42 34	6	35 03
5	41 50	5	35 26
4	41 08	4	36 08
3	40 27	3	36 43
2	39 47	2	37 17
1	39 8	1	37 53
Equin.	38 30		

The Tropicks and Parallels of Declination are easily drawn by the help of the preceding Tables: For if you take the half Tangent of the Number in the Table of Intersections, and set from Z towards S; and take the Tangent of the Number in the Table of Centers, and set from Z towards N in the Meridian, there will be the Center.

Exam-

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Example. Let it be required to draw the Parallel of 15 deg. of South Declination; look in the Table of Intersections, and you will find $66^{\circ} 30'$: take the half Tangent thereof, and set from Z to d , so is d the Point of Intersection: then look in the Table of Centers, and you will find $49^{\circ} 55'$; set the Tangent of $49^{\circ} 55'$ from Z upon the North part of the Meridian, and it will fall at b , a little beyond N, which is the Center; let one Foot rest in b , extend the other to d , and draw the Parallel, and so of the rest.

Upon P (as a Center) at the distance PB, describe the Semicircle ABD, which divide into 12 equal parts in the Points ***, &c. then lay a Ruler to P, and to every one of the Points **, &c. Lines drawn from those Points will cut the Tangent Line in the Points 1, 2, 3, &c. which are the Centers of the first, second and third Hours from the Meridian: But those Points are much better and sooner found by the help of a Sector; for open the Sector to the Radius PB, and out of the Lines of Tangents you may prick down the Centers 1, 2, 3, &c. Then with one Foot of the Compasses in the said Center, open the other Foot to P, and strike the Hour Circle; and with the same Extent strike the Hour Circle, which is the same distance from the Meridian on the other side.

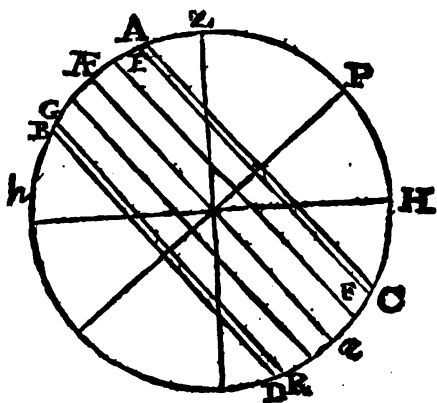
To make the Table of Intersections.

Because the Primitive Circle of the Projection is the Horizon, and the Center the Zenith, and the Diameter the Meridian, and it is required to know how far the Parallels are to lie from the Center: It is meet therefore, first to enquire the same thing in the Sphere, viz. how many degrees each Parallel lies from the Zenith of the Place, both on the South and on the North part also of the Meridian.

In this Scheme the Work will be plain, wherein the Circle represents the Meridian of any Place, as

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suppose of *London*, and in it Z the Zenith, P the Pole, *bH* the Horizon, *Ææ* the Equinoctial, AC the Tropick of ♈ , and BD the Tropick of ♏ , declining each $23^{\circ} 30'$; the question is then, how far these two Parallels are from the Zenith Point Z, both towards AB the South, and towards CD the North. For answer, consider that from Z to *Æ* Southward is the Latitude of the Place $51^{\circ} 30'$, and from Z to *æ* Northward is the Complement of that to 180° , viz. $128^{\circ} 30'$; and these are the North and South distances of the Equinoctial, or beginnings of *Aries* and *Libra*. Then *ÆA*, *ÆB*, *æC*, and *æD* are each declining $23^{\circ} 30'$ from the Equinoctial; so if we take $23^{\circ} 30'$ out of *ÆZ*, the Latitude of the Place $51^{\circ} 30'$, there will remain *AZ* $28^{\circ} 00'$, and so much is the Parallel of *Cancer* distant from the Zenith of *London* on the South part of the Meridian. Again, because *Zæ* is $128^{\circ} 30'$, and *æC* $23^{\circ} 30'$, taking *æC* out of *æZ*, there will remain $105^{\circ} 00'$; and so much is the same Parallel of *Cancer* distant from the Zenith on the North part.



Now for the Tropick of *Capricorn*, add *ZÆ* $51^{\circ} 30'$ to *ÆB* $23^{\circ} 30'$, the Sum is $75^{\circ} 00'$; and *Zæ* $128^{\circ} 30'$

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30' added to $\angle D 23^\circ 30'$, the Sum is $152^\circ 00'$: so the Distance ZB is 75° , and ZD is 152° .

Again, let us find the Distances of the Parallel of 20° of North Declination EF: from $\angle Z 51^\circ 30'$ subtract $\angle E 20^\circ 00'$, and there remains $\angle Z 31^\circ 30'$; and from $128^\circ 30'$ subtract $20^\circ 00'$, and there remains $108^\circ 30'$ ZF, the other Intersection.

Again, let us find the Distances of the Parallel of $20^\circ 00'$ of South Declination GR. To $\angle Z 51^\circ 30'$ add $\angle G 20^\circ$, and the Sum is $71^\circ 30'$: and to Z add $R 20^\circ$, and the Sum is $148^\circ 30'$ ZR, the other Intersection. Thus may you find all the Intersections to every single Degree of Declination, both upon the North and South part of the Meridian.

To make the Table of Centers.

Declin.	Archea.	Halfe.	Tangents.	Diameters and Semi-diameters.	Intersecti- from Z.	Centers from Z.
●	A $105^\circ 00'$ B $28^\circ 00'$	C $52^\circ 30'$ D $14^\circ 00'$	E 130322 F 24933	G 155255 H 77627	P 24933	S 52694
20	108 30 31 30	54 15 15 45	138908 28203	167111 83555	28203	55352
15	113 30 36 30	56 45 18 15	152525 32975	185500 92750	32975	59775
10	118 30 41 30	59 15 20 45	168085 37886	205971 102985	37886	65099
5	128 30 51 30	64 15 25 45	207321 48234	255555 127777	48234	79543
0	138 30 61 30	69 15 30 45	263946 59427	323373 161686	59427	102259
15	143 30 66 30	71 45 33 15	286358 65562	351920 175960	65562	110398
20	148 30 71 30	74 15 35 45	354576 71989	426565 213282	71989	141293
25	152 00 75 00	76 00 37 30	401078 76732	477810 238905	76732	162173

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In this Table, the Arches or Numbers in the first Column are the Distances of the Parallel from the Zenith found by the former Directions; the next Column are the halves of the first, C being the half of A, and D the half of B. The third Column contains the Tangents of those in the second, so the Tangent of C is E, and the Tangent of D is F; the first Number in the fourth Column is the Sum of the Tangents in the third, so G is the Sum of E and F, and is the Diameter of the Parallel; the lower Number in that Column is the half of the upper, so H is the half of G; the Number in the fifth Column is the same as the lower Number in the third; the Number in the sixth is the Difference between the lower Number in the third, and the lower Number in the fourth, and is the natural Tangent of the Distance of the Center from the Zenith: and if you look the said Tangent in the Table of natural Tangents, you will have the Degrees and Minutes, the same as in the Table of Centers; so if 52694 be found in the Table of natural Tangents, you will find $27^{\circ} 47'$ to answer it. But I think it is a better way to find the Degrees and Minutes answering the Semidiameter in the fourth Column; thus, find H 77627 in the Table of natural Tangents, and the Degrees and Minutes answering is $37^{\circ} 49'$; so if you take this out of your Scale of Tangents, and set it from the Intersection, that will give the Center of the Parallel, and you may draw it without altering your Compasses. But if you have a Diagonal Scale of the same Radius with your Primitive Circle, you need not find the Degrees and Minutes, but take the natural Tangent of the Intersection (as it is) from your Diagonal Scale, and set it from Z upon the Meridian; and take the Semidiameter from the same Scale, and set it upon the Meridian the other way, and that will find the Center; then without altering the Compasses draw the Parallel.

SECT.

S E C T. III.

To project the Sphere upon one of the lesser Circles, viz. the Tropick of Capricorn.

1. With 60° of Chords upon the Center P, describe the Circle $\sphericalangle$ A E γ for the Equinoctial; cross it with two Diameters at right Angles in the Center P, with $\sphericalangle$ P γ the Equinoctial, and AEP $\sphericalangle$ the Solstitial Colure, which is also the Meridian of the Place.
2. Out of your Scale of half Tangents, take $113^{\circ} 30'$, that is the distance of the Tropick of Capricorn from the North Pole of the World, and set it from P Southward to ψ .
3. Take the half Tangent of $66^{\circ} 30'$, the Complement of the Sun's greatest Declination, (or the Distance of the Tropick of Cancer from the North Pole) and set it upon the Meridian Northward from P to $\$$; and at that distance upon P describe the Circle for the Tropick of Cancer.
4. Divide the Space between $\$$ and ψ into two equal parts in the Point E; so shall E be the Center whereon to describe the Ecliptick Circle γ $\$$ $\sphericalangle$ ψ .
5. Take the half Tangent $51^{\circ} 30'$ the Latitude, out of the Scale of half Tangents, and set it from P to R in the Meridian; so shall R be the Interfection of the Horizon with the North part of the Meridian.
6. Take the Tangent of $38^{\circ} 30'$ the Complement of the Latitude, and set it upon the Meridian Southward from P to H; so shall H be the Center of the Horizon, and HR the Semidiameter thereof, whereon the Horizon ARB is to be described, cutting the Ecliptick in the Points γ and $\sphericalangle$, if there be no former Error. The next thing to be done is to divide the Ecliptick, which may be done from a Table of right Ascensions, but Geometrically thus.
7. Divide the Equinoctial into 12 equal Parts, then laying

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laying a Ruler to E (the Center of the Ecliptick) and to every of those 12 Divisions of the Equinoctial, and it will cut the Ecliptick in the Points Υ , ϑ , Π , $\varnothing$, Ω , ϖ , $\triangle$, m , $\uparrow$, v , ϖ , $\times$: and according as you divide the Equinoctial into every 10th, 5th, or every single Degree, so may you divide the Ecliptick also.

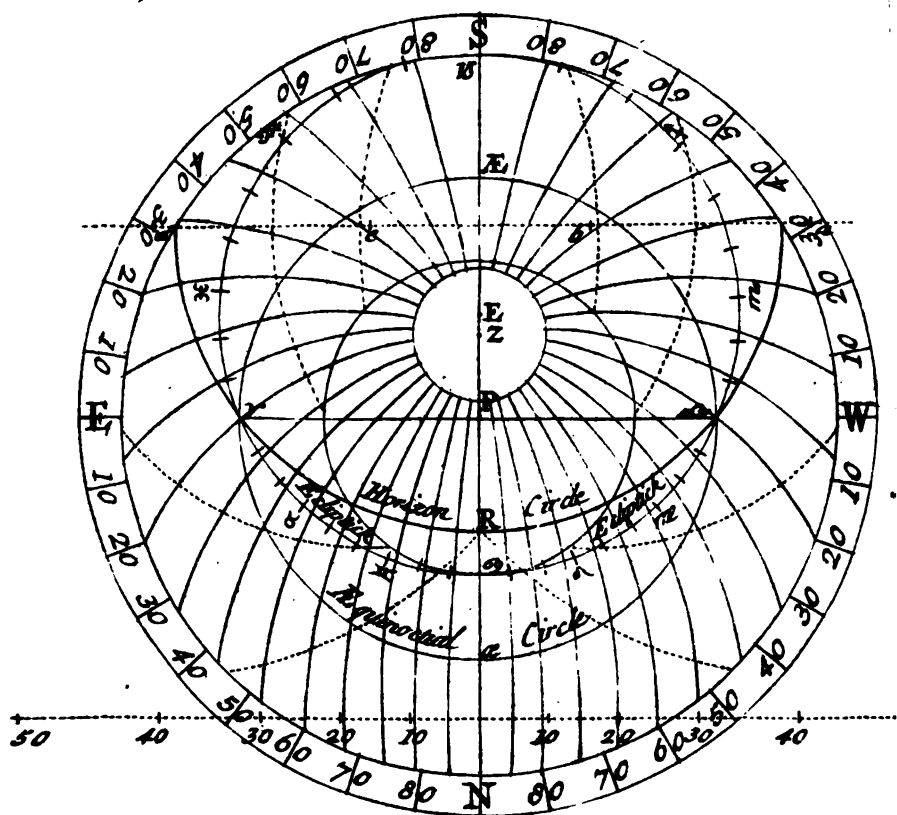
8. For the Azimuths, or Vertical Circles, to the Radius of the Equinoctial Circle, take the half Tangent of $38^{\circ} 30'$, the Complement of the Latitude, and set from P downwards (upon the South part of the Meridian) to Z; so shall Z be the Zenith where all the Azimuth Circles do concur: and how to project them, is in this manner.

9. Take $51^{\circ} 30'$ the Latitude, out of the Scale of half Tangents, and prick them down on the North part of the Meridian from P to X; and thro X draw a Tangent-line parallel to $\Upsilon P \triangle$, extending it infinitely on both sides of the Meridian.

10. Make ZX a new Radius, open your Sector to that Radius, and take out the Tangents of 10, 20, 30, &c. and set them both ways from X upon the Tangent-line. But for want of a Sector you may with any Line of Chords strike a Semicircle, and divide it into Degrees, and lay a Ruler to Z and to every 10 Degrees, and it will cut the Tangent-line in the Points 10, 20, 30, &c. which Points will be the Centers of the Azimuth-Circles; and the Semidiameter of each Azimuth will be the Distance between Z and each of the said Centers.

11. For the Circles of Position draw a right Line thro H, the Center of the Horizon parallel to $\Upsilon P \triangle$ (the Equinoctial Colure) then divide the South Semicircle of the Equinoctial into six equal parts; a Ruler laid from P the Pole, to each of those six Divisions, will cut the former Parallel-line in the Points a, b, c, d , and those shall be the Centers whereby to describe the Circles of Position: and thus is this Projection finished.

S E C T.



S E C T. IV.

How to project the Sphere upon any oblique Circle, and to find in what Latitude, and Difference of Longitude (from the Place where the oblique Circle deviates from the Meridian and Zenith of a known Latitude) it shall be an Horizontal Plain.

As great Circles of the Sphere are Horizons in some place of the World or other, so all Plains on which Sun-Dials are made, are Horizontal Dials in some one part of the World; and such a Plain (or great Circle) which I mean here, will best be represented by the Roof of a House. Now suppose in the Latitude of *London* $51^{\circ} 30'$, that the side of a House should decline from the South Westward $24^{\circ} 20'$, and the Roof thereof incline towards the North part of the Horizon 36 deg. I would know in what Latitude, and how much differing in Longitude, that part of the World is, in which this oblique Plain will be an Horizontal Plain.

Having drawn a primitive Circle $HXOD$, to represent this oblique Plain;

1. Draw the two Diameters HO and CD crossing each other at right Angles in Q .

2. Set the half Tangent of 36° , the Plain's Inclination from Q to Z ; so shall Z be the Zenith of *London*, and Q of the Place enquired.

3. Set the half Tangent of 54° , the Comp. of the Plain's Inclination from Q to B ; so shall B be one Point thro which the Horizon of *London* must pass.

4. Set the Tangent of 36° from Q to C , or the Secant thereof from B to C ; so is C the Center of the Horizon of *London* HBO .

5. Take the Tangent of 54° the Comp. of Inclination, and set from Q to F .

6. Take

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6. Take $24^{\circ} 20'$ the Declination of the Plain, from the Scale of Chords, and set from H to *c*, from D to *d*, and from O to *e*.

7. A Ruler laid from Z to *c*, *d* and *e*, will give the Points W, S, and E, for the West, South and East Points of the Horizon of *London*.

8. Draw FG perpendicular to QF, (or parallel to HO) and extend it as far as you shall see requisite.

9. Draw a Line thro the Points E and W, (which if you have committed no former Error, will pass also thro the Center Q) and extend it till it cut the Line FG last drawn; so shall the Point of Intersection be the Center of the Meridian PZS, which is the Meridian of the Place where the declining inclining Plain in *London* will be an Horizontal Plain.

10. A Ruler laid from W to Z, will cut the primitive Circle in *a*, from which Point set $38^{\circ} 30'$ the Complement of the Latitude of *London* to *b*; and a Ruler laid from W to *b*, will cut the Meridian in the Point P for the North Pole of the World.

11. Set 90° of Chords from *b* to *f*, and from *f* to *g*; a Ruler laid from W to *f*, will cut the Meridian in *Æ*, for the Point where the Equinoctial must cut the Meridian; and laid from W to *g*, will cut the Meridian (extended) in M, the South Pole; and a Right-line drawn thro P, Q and M, shall be the Axis of the World.

12. The Equinoctial Circle must be drawn thro the Points WÆE; and to find the Center thereof, divide the Right-line WE in two equal parts in R, and upon R raise the Perpendicular RT, drawing it forth till it meet the Axis of the World PQM (extended) in the Point T, which is the Center of the Equinoctial Circle: Or, if from R, thro C (the Center of the Horizon) a Right-line be drawn, and extended till it concur with the Axis of the World

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PQM extended, it will meet in T, the Center of the Equinoctial also.

13. Divide MP into two parts at right Angles in D, and draw DG, extending it infinitely.

14. Upon P, at the Distance PD (or any other Distance) with 60° of Chords, describe the Semicircle LDN; and laying a Ruler from P to G, the Center of the Meridian PZS, it will cut the Semicircle LDN in K: at which Point K begin to divide the Semicircle into 12 equal parts at the Points $\odot \odot \odot \odot$, &c. Then a Ruler laid from P to each of those 12 equal parts, shall give the Points of the Tangents of 15, 30, 45, 60, &c. in the Line DG, in which shall be the Centers of the Hour Circles, G being the Center of the first Meridian or 12 of the Clock, or the Meridian of the Place. Or having a Sector, make PD Radius, in the Lines of Tangents; and take the Tangents of 15, 30, 45, 60, &c. and setting them upon the Line GD both ways from D, shall give the Centers.

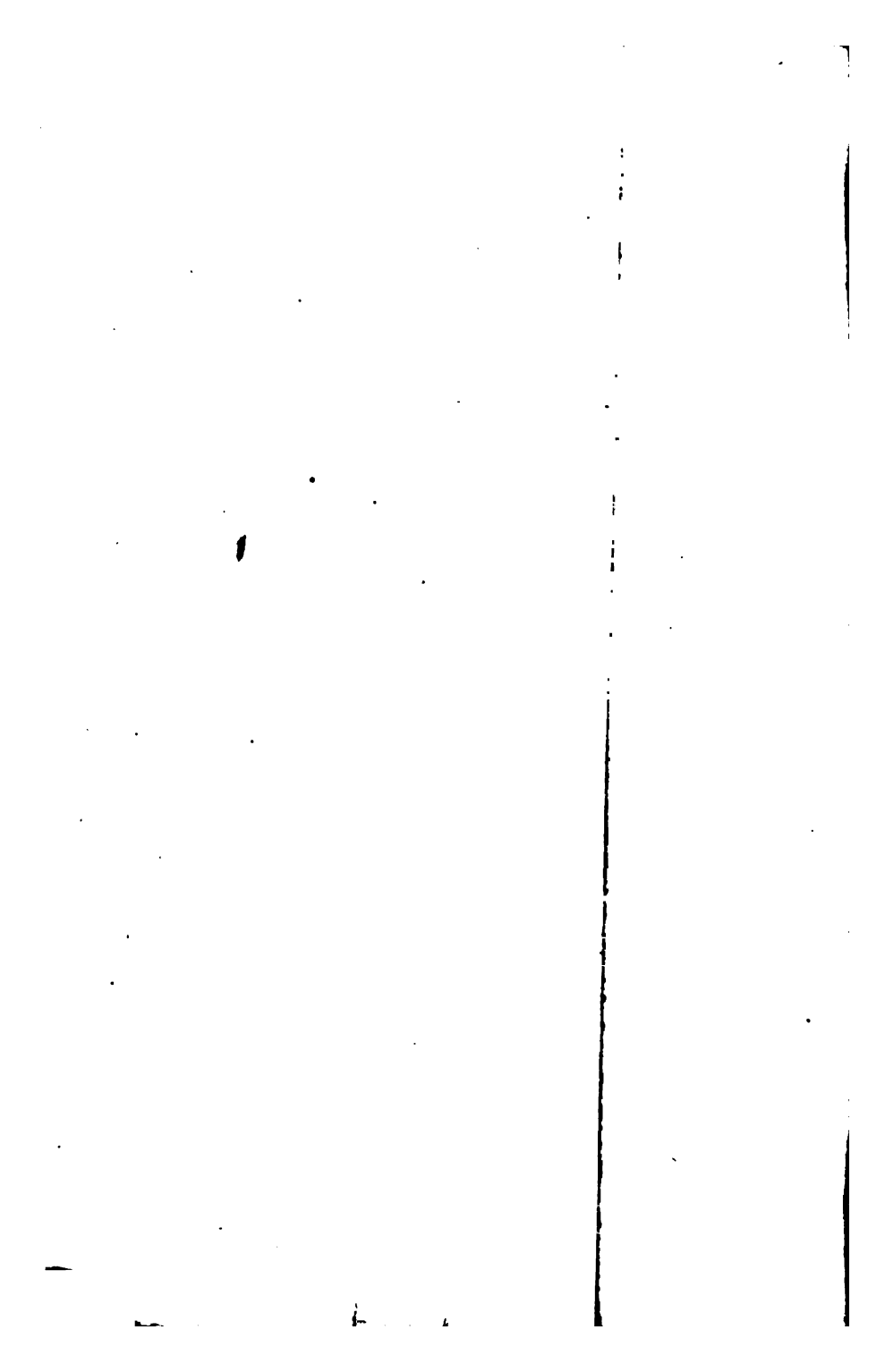
The Projection thus finished, if you take the Distance QP in your Compasses, and measure it upon the Scale of half Tangents, you will find it to contain $72^\circ 34'$, whose Complement is $PX 17^\circ 26'$; and in that Latitude will this declining inclining Plain be an Horizontal Plain.

And if you lay a Ruler from P to $\mathcal{A}$ E, it will cut the primitive Circle in b ; and the Distance Nb, measured upon the Scale of Chords, shall be $14^\circ 41'$, which is the Plain's Difference of Longitude.

So that a Plain declining $24^\circ 20'$, and inclining 36 deg. in the Latitude of *London*, will be an Horizontal Plain in the Latitude $17^\circ 16'$, and Difference of Longitude from *London* $14^\circ 41'$, which is about *Cassena* in *Negroland*.

S E C T:





S E C T. V.

How to project the Sphere upon the Plain of the Equinoctial.

1. Let the Primitive Circle $A \simeq B \gamma$ represent the Equinoctial, divide it into 24 equal Parts, and mark them with their proper Hours; the Center $\odot$ represents the Pole of the World.

2. The Diameter $A \odot B$, having XII at each end thereof, is the general Meridian, and the Solstitial Colure.

3. The Diameter $\gamma \odot \simeq$ having VI at each end, is the Hour of VI, and also the Equinoctial Colure.

4. The Circle OPFS, is a polar Circle, and is distant from $\odot$ the Pole of the World $23^{\circ} 30'$, which is thus described: Take the half Tangent of $23^{\circ} 30'$ in your Compasses, and upon $\odot$ describe the Circle OPFS.

5. The Circle $E \S W \wp$, represents either of the Tropicks, and is distant from the Equinoctial $23^{\circ} 30'$; or it is distant $66^{\circ} 30'$ from the Pole $\odot$. To project it, take $66^{\circ} 30'$ out of the Scale of half Tangents, and with that extent upon the Pole $\odot$ describe the Circle $E \wp W \S$.

And here Note, That all Parallels of Declination are described by the half Tangents of the Complement of their Distances from the Equinoctial upon the Pole $\odot$, as the Polar Circle and Tropicks were.

6. The Circle $\gamma \S \simeq$ is the Ecliptick, whose greatest Obliquity from the Equinoctial is $23^{\circ} 30'$, and it is thus described: Set the half Tangent of $66^{\circ} 30'$ from $\odot$ to $\S$, and set the Secant of $23^{\circ} 30'$ from $\S$ to G, so is G the Center: The Pole of the Ecliptick is the Intersection of the Polar Circle with the Meridian at P, thro which Point all the Circles of the Stars Longitude pass.

Z

7. The

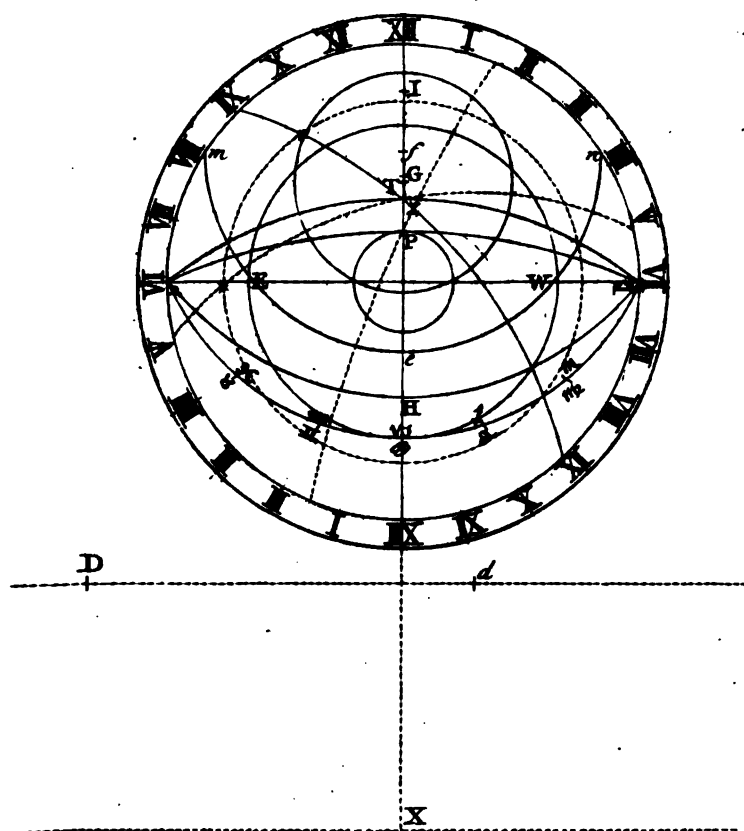
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7. The Circle $\gamma H \simeq$, represents the Horizon of London, whose Latitude is $51^{\circ} 30'$. To describe it, set the half Tangent of $38^{\circ} 30'$, from $\odot$ to H, and the Secant of $38^{\circ} 30'$ will reach from H to I the Center. For its Pole, the half Tangent of $38^{\circ} 30'$ being set from $\odot$ to Z, gives Z the Zenith, which is also the Pole of the Horizon, thro which Point all the Azimuth Circles pass.

8. The Circle KPL is a Circle of Longitude, passing thro $65^{\circ} 12'$, which is the Longitude of *Aldebaran*, or the *Bull's Eye*. To describe the Circles of Longitude: First, they must all pass thro the Point P, and thro every degree of the Ecliptick, so that the Ecliptick being divided from P the Pole thereof, as was shewed how to divide the Horizon from Z its Pole; you have three Points given, thro which they may be described. Thus the Secant of $66^{\circ} 30'$ will reach from P upon the Meridian to X the Center of 00° deg. and 180° deg. of Longitude; viz. of the Circle $\gamma P \simeq$: the Centers of the rest will be in a right Line drawn at right Angles to the Meridian, thro the Point X, and are found by their Tangents, as was before taught.

9. The Circle $* a *$ is a Parallel of the *Bull's Eye's* Declination $15^{\circ} 48'$, and is describ'd by taking the half Tangent of its Complement $74^{\circ} 12'$, and upon $\odot$ strike the Circle.

10. The Circle $\gamma Z \simeq$ is the prime Vertical, or Azimuth of East and West. To describe it, it is to pass thro Z the Zenith, as all Azimuths must do; and seeing the Zenith is distant from the Pole $38^{\circ} 30'$, set the Secant of $51^{\circ} 30'$ from Z upon the Meridian to Y the Center; and in a Line drawn at right Angles to the Meridian thro the Point Y, shall the Centers of all Azimuth Circles fall, which are found by their Tangents ZY being made Radius. So the Point D is the Center of the Azimuth $R * ZV$, an Azimuth of $39^{\circ} 42'$ from the East, and the Point *d* the
Center



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Center of the Azimuth $M * Z$ passing thro the *Bull's Eye* at 6 a-Clock, being 10 deg. from the East.

11. The Circle $m * n$, is a Circle of Altitude $20^{\circ} 20'$ passing thro the *Bull's Eye* when he is upon the West Azimuth. To describe this Circle, the Point Z the Pole of the Horizon being $38^{\circ} 30'$ distant from $\odot$, add and subtract it, to and from the Complement of $20^{\circ} 20'$, viz. $69^{\circ} 40'$, and the Sum is $108^{\circ} 10'$, and the Difference is $31^{\circ} 10'$; set the half Tangent of $108^{\circ} 10'$ from $\odot$ to g upon the Meridian, and set the half Tangent of $31^{\circ} 10'$ from $\odot$ to e , the middle between e and g is f the Center of the Parallel. After the same manner may you draw the Circle $r * u$, being a Parallel of 45° Altitude, whose Center is T .

Thus have I shewed you how to project the Sphere upon several Circles; by which, and the Directions in the 10th Chap. of the first Part, of Spherical Geometry, you will be enabled to perform any Projection required.

C H A P.

C H A P. VI.

The Doctrine of Spherical Triangles applied to Practice in Geography, shewing how to find the Distances of Places upon the Terraqueous Globe, by the Arch of a great Circle, with the Angles of Position.

S E C T. I.

Two Places differing only in Latitude, to find their Distance.

IF the two Places be both in North, or both in South Latitude, the Difference of Latitude brought to Miles is their Distance, But if one Place be in North, and the other in South Latitude, then the Sum of their Latitudes brought to Miles is their Distance.

Example 1. What is the Distance between the *Lizard* in the Latitude $50^{\circ} 12'$, and *Tangier* in the Latitude $35^{\circ} 36'$, both North?

Subtract $35^{\circ} 36'$ from $50^{\circ} 12'$, and there remains $14^{\circ} 36'$, which multiplied by 60 makes 876 Miles.

Example 2. What is the Distance between the *Lizard* in North Latitude $50^{\circ} 12'$, and *St. Helena* in South Latitude $16^{\circ} 03'$?

Add the two Latitudes together, the Sum is $66^{\circ} 15'$, which makes 3972 Miles.

S E C T. II.

Two Places differing in Longitude only, to find their Distance.

In this Section are two Varieties.

1st. Two Places in the Equator, their Longitudes being given, to find their Distance.

1. According to the old way of counting the Longitude, that is, as it is commonly number'd in Maps of the World, Globes, &c. where it begins at some particular Place, as in some it is counted from *Pico Teneriffa*, and some make the first Meridian at *Faro*; subtract the lesser Longitude from the greater, and the remainder (if less than 180°) is the Distance required: but when it is more, subtract the remainder from 360, and the last remainder is the Distance.

2. According to the new way of counting the Longitude (in the *Mariner's Compass Rectified*) the Rule is thus: If the Longitudes be both East, or both West, subtract the lesser from the greater, the remainder is the Distance: But when one is East, and the other West, add them, and the Sum (if it exceed not 180°) is their Distance; and when it exceeds 180° , subtract it from 360° , the remainder is the Distance required.

Example. What is the Distance of *Sumatra*, whose Longitude is $137^{\circ} 10'$, from the Island *St. Thoma*, whose Longitude is $33^{\circ} 10'$, both lying under the Equinoctial?

The Difference of Longitude is $104^{\circ} 00'$, which makes 6240 Miles, and so much is the Distance of the Islands.

2^d. Variety. Two Places under one Parallel, or both in one Latitude, their Longitudes being given, to find their Distance.

Exam-

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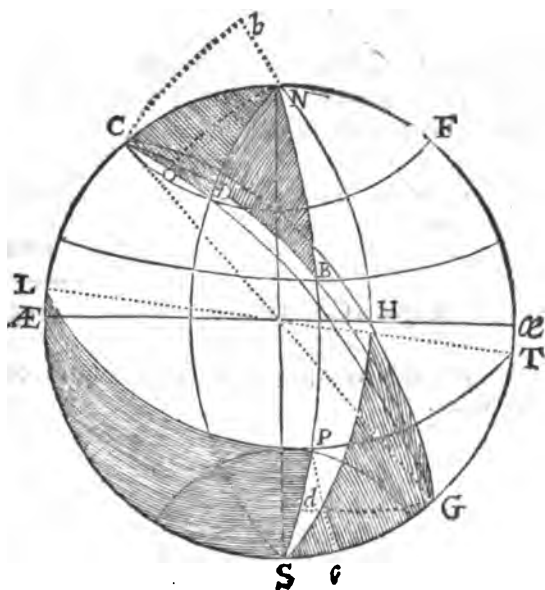
Example. I demand the Distance between the *Lizard*, and *Pengwin Island* in *Newfoundland*.

Pengwin Island } Lon. { $321^{\circ} 56'$. Or $53^{\circ} 10'$ } W. Lon.
Lizard } { $9\ 42$ Or $5\ 24$ }

$312\ 14$
 $360\ 00$

$47\ 46$ Diff. Lon.

$47\ 46$



In this Figure Ææ represents the Equator, N the North Pole, and S the South Pole; C represents the Island *Pengwin*, and D the *Lizard*, both in the Latitude of 50 deg. and CDG represents a great Circle which passes thro both Places; then in the right-angled Triangle CNø , there is given $\text{CN } 4^{\circ} 00'$ Compl. of Latitude, and the Angle $\text{CNø } 23^{\circ} 53'$ half the Difference of Longitude; to find Cø half the Distance; which being doubled, will be CD the whole Distance, Therefore it will be, As

As Radius to the s. of CN $40^{\circ} 00'$	9.808067
So is the s. of the Angle CN $23^{\circ} 53'$	9.607322

To the s. of C $15^{\circ} 05'$ half the Distance	9.415389
---	----------

So the whole Distance is $30^{\circ} 10' = 1810$ Miles.

To find the Angles of Position o CN, or o DN.

In the Triangle CN o, right-angled at o, we have given CN $40^{\circ} 00'$, and the Angle CN $23^{\circ} 53'$, to find o CN. Therefore it will be,

As ct. of CN $23^{\circ} 53'$	10.353801
Is to Radius	10.
So is cs. CN $40^{\circ} 00'$	9.884254

To ct. the Angle o CN $71^{\circ} 15'$	9.530453
--	----------

The Angle o DN is equal to o CN, being subtended by equal Sides.

S E C T.

S E C T. III.

Two Places differing both in Longitude and Latitude, to find their Distance.

In this Section are three Varieties or Cases.

C A S E I.

One Place may be in the Equator, and the other towards either of the Poles.

E X A M P L E.

Let the two Places be the *Lizard* in $50^{\circ} 00'$ North Latitude; and the Island *Gilels* under the Equinoctial, the Difference of Longitude 134 deg. what is the Distance between the two Places?

In the former Figure, C represents the *Lizard* in the Latitude $50^{\circ} 00'$, and H the Island *Gilels* in the Equator, and CHG the great Circle passing thro both Places.

Then in the spherical quadrantal Triangle CNH, or rather its opposite HSG, there is given the quadrantal Arch HS; the Side SG equal to CN $40^{\circ} 00'$, the Complement of the *Lizard's* Latitude; and the contained Angle HSG, the Complement of the Difference of Longitude $46^{\circ} 00'$, to find the Side GH. Otherwise in the right-angled Triangle GH ϵ there is given H ϵ $46^{\circ} 00'$, the measure of the Angle HSG, and G ϵ $50^{\circ} 00'$, the Complement of SG, to find GH. Then it will be,

As

As Radius to cs. H α $46^{\circ} 00'$	9.84177
So is cs. G α $50^{\circ} 00'$	9.80807
To cs. GH $63^{\circ} 29'$	9.64984

Whose Complement to 180° is CH $116^{\circ} 31'$, the Distance of the Places, which is 6991 Miles.

To find the Angles of direct Position, CHN and NCH.

In the Triangle CNH ;	
As s. of CH $116^{\circ} 31'$ (or $63^{\circ} 29'$)	0.048272
Is to s. of the Angle CNH 134° (or 46°)	9.856934
So s. of CN $40^{\circ} 00'$	9.808067

To s. of the Angle CHN $31^{\circ} 07'$	9.713273
---	----------

Again,	
As s. of CH $116^{\circ} 31'$ (or $63^{\circ} 29'$)	9.951728
Is to s. of the Angle CNH 134° (or 46°)	9.856934
So s. of NH 90°	10.

To s. of the Angle NCH $53^{\circ} 30'$	9.905206
---	----------

C A S E II.

Both Places may be on one Side of the Equator ;
that is, both in North, or both in South Latitude.

E X A M P L E.

Let one Place be the *Lizard* in $50^{\circ} 00'$ of North Latitude, and the other *Aracan* in the Bay of *Bengal* in

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in $20^{\circ}00'$ North Latitude ; the Difference of Longitude between them is $108^{\circ}00'$; what is their Distance ?

In the former Figure, C represents the *Lizard*, B represents *Aracan* ; and CBG the great Circle passing thro both Places.

Then, in the spherical Triangle CNB, there is given CN $40^{\circ}00'$, the Complement of the *Lizard's* Latitude, BN $70^{\circ}00'$, the Complement of the Latitude of *Aracan* ; and the Angle CNB 108 deg. the Difference of Longitude ; to find the Distance CB.

By Mr. Collins's Proportion. See Case the 9th of Oblique-angled Spherical Triangles.

As Radius to the cs. of the Angle in- cluded 108°	} 9.489982
So is t. of the lesser Side 40°	9.923813
To t. of a fourth Arch $14^{\circ}32'$	9.413795

Here you may observe, That when the included Angle is obtuse, you resolve the opposite Triangle, by this way. As in this Example, in the Triangle BCN the included Angle BNC is 108° , therefore you resolve the opposite Triangle BGS, wherein you have given SG equal to CN 40° , and SB 110° , the Complement of BN to 180° , and the included Angle BSG 72° , the Complement of CNB to 180° , to find BG, whose Complement to 180° will be the Distance required BC. In the first place, the Perpendicular G d being let fall ; in the right-angled Triangle G d S, there is given SG 40° , and the Angle GS d 72° , to find S d the 4th Arch, which is $14^{\circ}32'$; this being subtracted from BS 110° , the residual Arch is $95^{\circ}28'$. Then,

As

As cs. of S d the 4th Arch $14^{\circ} 32'$ 0.014124
 Is to cs. of B d the residual Arch $95^{\circ} 28'$ } 8.978941
 (or $84^{\circ} 32'$) }
 So cs. of SG equal to the lesser Side 40° } 9.884254
 00'

To cs. of BG, whose Comp. to 180° is } 8.877319
 BC $85^{\circ} 41'$ }

Here the contained Angle being obtuse, (in the given Triangle) and the residual Arch more than 90° , therefore the Side BC is less than 90° , as plainly appears by the Figure.

If the included Angle be acute, the Operation differs nothing from the common Method, by letting fall a Perpendicular.

This may be otherwise resolved by Mr. Collins's other Proportion, explained in the said 9th Case.

The Log. Sine of $40^{\circ} 00'$ the lesser Side 9.8080678
 The Log. Sine of $70^{\circ} 00'$ the greater Side 9.9729858
 The Log. Sine of 54° (half the Ang.) } 19.8159152
 doubled }
 The Log. of 2 0.3010300

The natural Sine against 9.8979985
 Is 7906896

Add 1339746 } = the natural versed Sine of $30^{\circ} 00'$
 the Diff. of the Legs.

Sum 9246642 } = the natural versed Sine of $85^{\circ} 41'$
 the Distance required.

C A S E

C A S E III.

*The two Places may one lie in North Latitude,
and the other in South.*

E X A M P L E.

Let one of the Places be *Cape Horn*; that is, the farthest South Point of Land in *America*, in $57^{\circ} 54'$ South Latitude, and the other the Island of *Barbudos* in the great *South-Sea*, whose Latitude is $7^{\circ} 00'$ North; the Difference of Longitude between them is 108° , what is their Distance?

In the former Figure, P represents *Cape Horn*, and L the Island *Barbudos*; and SPN represents the great Circle passing thro both Places.

In the Triangle LPS, there is given the Side SP, the Complement of the Latitude of *Cape Horn* $32^{\circ} 06'$, the Side LS the Latitude of *Barbudos*, with 90° added $97^{\circ} 00'$; and the contained Angle LSP 108° , the difference of Longitude; or rather the opposite Triangle SPT, in which is given the Side SP $32^{\circ} 06'$, the Side ST the Complement of SL to 180° , and the Angle PST the Complement of the difference of Longitude to 180° , to find the Side PT, whose Compl. to 180 is LP required.

Having let fall the Perpendicular Pc; in the right-angled Triangle cPS there is given PS $32^{\circ} 06'$, and the Angle PS c 72° , to find Sc. Thus,

As Radius to cs. of PS c 72°	9.489982
To t. of PS $32^{\circ} 06'$	9.797474

To t. of the fourth Arch Sc $10^{\circ} 58'$	9.287456
--	----------

Which subtracted from ST 83° , there remains T c $72^{\circ} 02'$. Then,

Aa

As

As the cs. of the fourth Arch S $10^{\circ} 58'$	0.008004
Is to cs. of the residual Arch T $72^{\circ} 02'$	9.489204
So cs. of the lesser Side PS $32^{\circ} 06'$	<u>9.927946</u>

To the cs. of PT $74^{\circ} 34'$	9.425154
-----------------------------------	----------

Or thus,

The Log. Sine of $32^{\circ} 06'$ the lesser Side	9.7254204
The Log. Sine of 97° the greater Side	9.9967507
The Log. Sine of 54° (half the Ang.) } doubled	19.8159152
The Log. of 2	<u>0.3010300</u>

The natural Sine against	9.8391163
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Is = 6904231

Add 5758006

12662237	} = the natural versed Sine of 64° 54', the diff. of the Legs.
7337763	

= the Arithmetical Complement.

Which is the natural versed Sine of $74^{\circ} 34'$, whose Complement to 180° is $105^{\circ} 26'$, the Distance sought.

C H A P. VII.

Spherical Trigonometry applied in Astronomical Problems.

P R O B L E M I.

The Sun's Place, and greatest Declination being given; to find the present Declination.

Definition.

THE Declination of the Sun, or of a Star, is a part of the Meridian intercepted between the Equinoctial Circle, and the Center of the Sun or Star.

In the Triangle A B Π (in the following Figure) we have given, the Angle B A Π the Obliquity of the Ecliptick $23^{\circ} 29'$, and A Π the Distance of the Sun from γ 60° ; to find B Π .

As Radius to the Distance of the Sun } 9.93753
from γ $60^{\circ} 00'$ }

So is the Sine of the greatest Declina- } 9.60070
tion $23^{\circ} 29'$ }

To Sine of the present Declinat. $20^{\circ} 12'$ 9.43823

Note. That the Sun's Distance is always counted from the nearest of the Equinoctial Points γ or ω ; therefore if the Sun be in the Northern Signs γ , δ , or Π , or in the Southern Signs ν , ω , or κ , his Distance is counted from γ . But if his Place be in the Northern Signs ϵ , η , or θ , or in the Southern Signs α , μ , or ϕ , it is reckoned from ω .

A a 2

P R O-

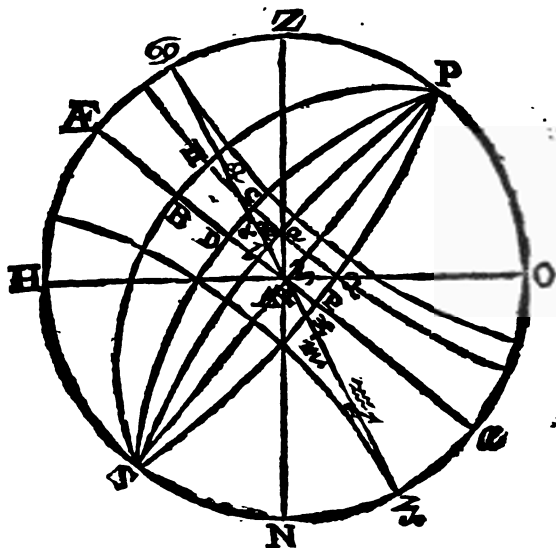
PROBLEM II.

The Sun being entring into Gemini, what is the right Ascension?

Definition.

The right Ascension is an Arch of the Equinoctial intercepted between the point *Aries* and the Meridian that passes thro the Center of the Sun or Star.

In the Triangle AB Π , we have A Π the Sun's Distance from γ 60° , and the Angle B A Π $23^\circ 29'$; we find the Side AB.



As ct. of A Π 60°

To Radius

So is ct. of $23^\circ 29'$

To t. of AB $57^\circ 49'$

9.761439

10.

9.962453

10.201014

Or

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Or thus,

As Radius to t. of the Sun's Distance } 10.238561
from γ 60°

So is cs. of the greatest Declination } 9.962453
 $23^\circ 29'$

To t. of right Ascension AB $57^\circ 49'$ 10.201014

PROBLEM III.

If the Sun's Declination be $12^\circ 30'$, what is the Sun's Place, or Longitude from γ ; and what his right Ascension?

In the right-angled Triangle ADC, there is given the Side DC the present Declination, and the Angle DAC the greatest Declination; to find AC the Sun's Distance from γ , and also his right Ascension AD.

As s. of the greatest Declination $23^\circ 29'$ 9.60070
Is to the s. of the present Declinat. $12^\circ 30'$ 9.33534
So is Radius 10.

To s. of the Sun's Distance from γ $32^\circ 52'$ 9.73464
So the Sun is $2^\circ 52'$ in γ .

As Radius to ct. of greatest Declination } 10.36204
 $23^\circ 29'$ }
So t. of the present Declination $12^\circ 30'$ 9.34575

To the s. of right Ascension AD $30^\circ 41'$ 9.70779

If the Sun's Declination be North, and increasing, this Proportion finds the Sun's Distance from *Aries*; if decreasing, from *Libra* in Northern Signs, & *contra*.

PROBLEM IV.

The Latitude of a Place, and the Sun's Declination being given, to find the Ascensional Difference, and Amplitude.

EXAMPLE.

In the Latitude $51^{\circ} 32'$, the Sun's Declination being $15^{\circ} 29'$, what is the Ascensional Difference, and Amplitude?

Defn. 1. The Latitude of a Place upon Earth, is an Arch of the Meridian of that Place, contained between the Zenith and the Equinoctial, as ZE ; equal to the Height of the Pole above the Horizon, as OP .

2. The Ascensional Difference, is the Difference between the right and oblique Ascension or Descension; or it is the space of Time the Sun rises or sets before or after six a-Clock.

3. Oblique $\left\{ \begin{array}{l} \text{Ascension} \\ \text{Descension} \end{array} \right\}$ is an Arch of the Equinoctial, between the beginning of γ , and that part of the Equinoctial that $\left\{ \begin{array}{l} \text{riseth} \\ \text{setteth} \end{array} \right\}$ with the Center of the Sun, or Star, or with any Point of the Ecliptick, in an oblique Sphere.

4. The Amplitude is an Arch of the Horizon, contained between the Ecliptick and the Equinoctial, and shews how far the Sun rises or sets from the East or West Points of the Horizon.

In the Triangle APQ right-angled at P , there is given the Angle PAQ $38^{\circ} 38'$ the Complement of the Latitude, and PQ the Sun's Declination; to find AP the Ascensional Difference, and AQ the Amplitude.

As Rad. to ct. of PAQ $38^{\circ} 28'$ Comp. of $\left. \begin{array}{l} \text{Lat.} \\ \text{So is t. of PQ } 15^{\circ} 29' \text{ the Declination} \end{array} \right\}$	10.09991
	9.44249
Tos. of AP the Ascensional Diff. $20^{\circ} 24'$	9.54240

Which reduced to Time makes 1 hour $21\frac{1}{2}$ min. and so much the Sun rises before Six, and sets after Six at Night if the Declination be North, as is supposed in this Example; so the Sun rises at $38\frac{1}{2}$ min. after 4, and sets $21\frac{1}{2}$ min. after 7.

If the Sun's setting be doubled, it gives the length of the Day; and the Sun's rising doubled gives the length of the Night.

As s. PAQ $38^{\circ} 28'$ the Comp. of the Lat.	9.79383
Tos. PQ $15^{\circ} 29'$ the Declination	9.42644
So is Radius	$10.$
To s. of the Amplitude AQ $25^{\circ} 25'$	9.63261

And so many Degrees the Sun rises Northerly from the East, and sets so much from the West.

PROBLEM V.

To find the oblique Ascension or Descension.

First, Find the right Ascension, by the second Problem; and the Ascensional Difference by the last.

Secondly, If the Sun's Declination be Northerly, the Ascensional Difference subtracted from the right Ascension, leaves the oblique Ascension; and added to the right Ascension, gives the oblique Descension.

Thirdly, If the Sun's Declination be Southerly, the Ascensional Difference added to the right Ascension, gives the oblique Ascension; and subtracted therefrom, leaves the oblique Descension.

P R O B L E M VI.

The Latitude of the Place and Declination of the Sun being given, to find at what time the Sun will be due East or West.

E X A M P L E.

In the Latitude $51^{\circ} 32'$, and Sun's Declination $15^{\circ} 29'$ North, to find at what time the Sun will be due East or West.

In the right-angled Triangle $A b a$ in the former Figure, there is given the Side $a b$ $15^{\circ} 29'$, and the opposite Angle $a A b$ $51^{\circ} 32'$; to find the Side $b A$ the time from Six. Then,

As Radius to ct of $a A b$ $51^{\circ} 32'$ the Lat.	9.90009
So t. of $a b$ $15^{\circ} 29'$ the Declination	9.44249
To s. $b A$ $12^{\circ} 43'$ the time from Six	9.34258

Which being reduced to Time, makes 51 min. *scilicet*, which added to 6, gives 6 h. 51 m. at which time the Sun comes to the East, and being subtracted from 6, the remainder is 5 h. 9 m. at which time the Sun is due West.

PROBLEM VII.

The Latitude of the Place, and Sun's Declination being given, to find the Sun's Altitude, being due East or West.

EXAMPLE.

In the Latitude $51^{\circ} 32'$, and Declination $15^{\circ} 29'$ North, to find the Sun's Altitude, being due East or West.

In the right-angled Triangle Aba in the former Figure, there is given the Side ab $15^{\circ} 29'$, and the opposite Angle aAb $51^{\circ} 32'$, to find the Hypotenuse aA . Then,

Ass. of aAb $51^{\circ} 32'$ the Latitude	9.89374
To Radius	10.
So s. of ab $15^{\circ} 29'$ the Declination	9.42644
To s. of aA $19^{\circ} 56'$ the Sun's Altitude	9.53270
at East or West	

PROBLEM VIII.

The Latitude of the Place, and the Sun's Declination being given, to find his Altitude at Six of the Clock.

EXAMPLE.

In the Latitude $51^{\circ} 32'$, and Declination $15^{\circ} 29'$, to find the Sun's Altitude at Six.

In the right-angled Triangle Aab , in the following Figure, there is given the Hypotenuse aA $15^{\circ} 29'$, and the Angle bAa $51^{\circ} 32'$, to find the Side ab .

2

As

As Radius

10.

To s. A $15^{\circ} 29'$

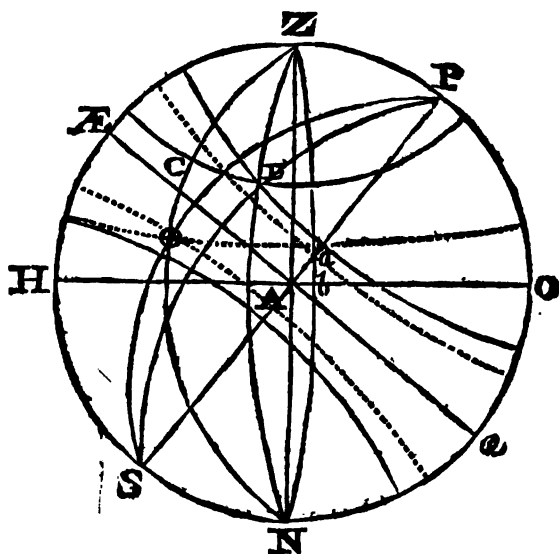
9.42644

So s. $a b$ $51^{\circ} 32'$

9.89374

To s. $a b$ $12^{\circ} 04'$

9.32018

So the Altitude of the Sun at Six is $12^{\circ} 04'$.

PROBLEM IX.

*The Latitude of the Place, and Sun's Declination
being given, to find his Azimuth at Six.*

EXAMPLE.

In the Latitude $51^{\circ} 32'$, and Declination $15^{\circ} 29'$
North, to find the Sun's Azimuth at Six.

In

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In the right-angled Triangle Aba , there is given the Hypotenuse Aa the Sun's Declination, and the Angle aAb $51^{\circ} 32'$; to find the Side Ab .

As Radius to the t. of Aa $15^{\circ} 29'$ the	}	9.44249
Declination		
So cs. of aAb $51^{\circ} 32'$		9.79383
To t. of Ab $9^{\circ} 55'$, the Azimuth from	}	9.23632
the East		

PROBLEM X.

The Latitude of the Place, the Sun's Altitude, and Declination being given, to find his Azimuth.

E X A M P L E 1.

In the Latitude $51^{\circ} 32'$ North, the Declination $23^{\circ} 30'$ North, and the Altitude $44^{\circ} 40'$, to find the Sun's Azimuth.

In the oblique-angled Triangle DPZ , there are given, the three Sides ZP $38^{\circ} 28'$, the Complement of the Latitude DP $66^{\circ} 30'$, the Complement of the Declination, or the Sun's Distance from the elevated Pole, and DZ $45^{\circ} 20'$ the Complement of the Altitude; to find the Angle DZP , the Sun's Azimuth from the North.

By the first Proportion demonstrated, Page 76, 77.

DZ $45^{\circ} 20'$	}	DP opp. to $Z=66^{\circ} 30'$, the half is $33^{\circ} 15'$	
ZP $38^{\circ} 28'$			
		The half Diff. add and subtract	3 26
Diff. 6 52	}	Sum	36 41
Half 3 26		Diff.	29 49
			DZ

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Part II.

DZ $45^{\circ} 20'$	} Comp. Arithm.	{ Sine	0.148003
ZP $38^{\circ} 28'$			{ Sine
The Sum $36^{\circ} 41'$ Sine			9.776259
The Diff. $29^{\circ} 49'$ Sine			9.696554
The Sum			19.826984

The half Sum is the Sine of $55^{\circ} 01'$ 9.913492

Doubled is = $110^{\circ} 02'$ = the Angle Z,
which is the Azimuth from the North part of the
Meridian.

E X A M P L E 2.

In the Latitude $51^{\circ} 32'$ North, the Sun's Declina-
tion $15^{\circ} 30'$ South, and the Sun's Altitude $15^{\circ} 40'$;
to find the Azimuth from the North.

In the Triangle $\odot P Z$, there is given $PZ 38^{\circ} 28'$,
 $\odot Z 74^{\circ} 20'$, and $\odot P 105^{\circ} 30'$, the Sun's Distance
from the North Pole; to find the Angle $\odot Z P$.

By the second Proportion demonstrated, Page 79.

	PZ $38^{\circ} 28'$	}	$\odot Z 74^{\circ} 20'$	Ar. Com.	0.016442	
	$\odot Z 74^{\circ} 20'$		PZ $38^{\circ} 28'$	Ar. Com.	0.206168	
Base	$\odot P 105^{\circ} 30'$		Half Sum $109^{\circ} 9'$	Sine	9.975277	
			Diff. $3^{\circ} 39'$	Sine	8.803876	
Sum	<u>218 18</u>	}			<u>19.001763</u>	
Half	<u>109 9</u>					
			$\frac{1}{2}$ Sum is the S. of $18^{\circ} 28'$			<u>9.500881</u>
Diff.	<u>3 39</u>		Which doubl. is $36^{\circ} 56'$			

Which $36^{\circ} 56'$ is the Azimuth from the South; and
the Complement to 180° is the Azimuth from the
North $143^{\circ} 04'$.

E X A M P L E

E X A M P L E 3.

In the Latitude $51^{\circ} 32'$ South, the Sun's Declination $23^{\circ} 30'$ South, and his Altitude $44^{\circ} 40'$; to find his Azimuth from the South.

In the Triangle DZP, in the former Figure, let P represent the South Pole; then there is given PZ $38^{\circ} 28'$ the Complement of the Latitude, DZ $45^{\circ} 20'$ the Complement of the Altitude, and DP $66^{\circ} 30'$ the Sun's Distance from the elevated Pole; to find PZD, the Sun's Azimuth from the South.

The Operation in this will be the same as in the first Example, and the Azimuth from the South will be $110^{\circ} 02'$, the same as before.

E X A M P L E 4.

In the Latitude $51^{\circ} 32'$ South, the Sun's Declination $15^{\circ} 30'$ North, and his Altitude $15^{\circ} 40'$; to find the Azimuth from the South.

In the Triangle $\odot ZP$, let P represent the South Pole; then there is given PZ $38^{\circ} 28'$, $\odot Z 74^{\circ} 20'$, and $\odot P 105^{\circ} 30'$, and $\odot ZP$ required.

The Operation is the same with the second Example, and the Azimuth from the South will be $143^{\circ} 04'$, the same as in that.

If the Sun be in the Equinoctial, you may find the Azimuth by the following Proportion. In the Triangle $C\hat{E}Z$, $\hat{E}Z$ is the Latitude, and CZ the Complement of the Sun's Altitude; therefore it will be,

As Radius to t. of the Lat. $\hat{E}Z$; so t. of the Altitude, to cs. of $\hat{E}ZC$, the Azimuth from the South.

P R O-

PROBLEM XL

The Latitude of the Place, the Sun's Declination and Altitude being given; to find the Hour of the Day.

EXAMPLE I.

In the Latitude $51^{\circ} 32'$ North, the Sun's Declination $23^{\circ} 30'$ North, and his Altitude $44^{\circ} 40'$; to find the Hour from Noon.

In the Triangle DZP, in the former Figure, there is given PZ $38^{\circ} 28'$, DZ $45^{\circ} 20'$, and DP $66^{\circ} 30'$; and DPZ the Hour from Noon required. The Work,

DP $66^{\circ} 30'$	} DZ opp. to P = $45^{\circ} 20'$, the $\frac{1}{2}$ is $22^{\circ} 40'$	
PZ $38 28$		The $\frac{1}{2}$ Diff add and subtract $14 01$
Diff. $28 02$		The Sum $36 41$
		The Diff. $8 39$
Half $14 01$		

DP $66^{\circ} 30'$	} The Arith. Comp. of	Sine	0.037608
PZ $38 28$		Sine	0.206168
	The Sum $36^{\circ} 41'$	Sine	9.776259
	The Diff. $8^{\circ} 39'$	Sine	9.177242

The Sum 19.197277

The half Sum is the Sine of $23^{\circ} 23'$ 9.598638

Which doubled is $46 46$ = the Ang. DPZ.

Which in time is 3 Hours 7 Minutes from Noon.

E X A M-

E X A M P L E 2.

In the Latitude $51^{\circ} 32'$, the Sun's Declination $15^{\circ} 30'$ South, and his Altitude $15^{\circ} 40'$; to find the Hour from Noon.

In the Triangle $\odot ZP$, in the former Figure, is given $PZ 38^{\circ} 28'$, $\odot Z 74^{\circ} 20'$, and $\odot P 105^{\circ} 30'$; and $\odot PZ$ the Hour from Noon is required.

OP	105° 30'	}	OP 105° 30' Sine Ar. Com.	0.016089
PZ	38 28		PZ 38 28 Sine Ar. Com.	0.206168
Base	74 20		Half Sum 109° 09' Sine	9.975277
			Diff.	34 49 Sine 9.756600
Sum	218 18			
			Sum	19.954134
Half	109 09			
Diff.	34 49		The $\frac{1}{2}$ S. is the s. of 71° 33'	9.977067
			Whose Compl. is 18 27	

Which doubled is $36^{\circ} 54' = \text{An. } \odot PZ$

Which in Time is 2 hours 27 min. 36 seconds from Noon.

P R O B L E M XII.

The Latitude of the Place, the Sun's Declination, and the Hour of the Day given, to find the Sun's Altitude.

E X A M P L E.

In the Latitude $51^{\circ} 32'$ North, the Sun's Declination $23^{\circ} 30'$, and the Time Afternoon 3 h. 7 min. equal to $46^{\circ} 45'$.

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I shall here make use of Mr. *Collins's* Proportion mention'd in *Case 9.* of oblique-angled spherical Triangles. See an Example, *Page 146.*

The Compl. Lat. $38^{\circ} 28'$	Sine	9.7938317
The Comp. of Declin. $66^{\circ} 30'$	Sine	9.9623978
Log. of 2		0.3010300

The fixed Logarithm	20.0572595
Log. Sine of $23^{\circ} 23'$ (half the hour)	} 19.1973204
doubled add	

The natural Sine against it is 1796607 39.2545799

The Summer Meridian Alt. is $61^{\circ} 58'$ } 8826743
its nat. s. }

From which subtract 1796607

Remains 7030136

Which Remainder is the natural Sine of $44^{\circ} 46'$, the Summer Altitude for the hour and minute proposed.

The Winter Meridian Alt. is $14^{\circ} 58'$ } 2588190
and its natural Sine }

Subtract the former Diff. 1796607

Remains the natural Sine of } Rem. 791583
 $4^{\circ} 32'$, the Winter Altitude }
for the h. and min. proposed. }

P R O:

P R O B L E M X I I .

*The Latitude of the Place, the Sun's Declination
and the Hour given, to find the Sun's Azimuth.*

E X A M P L E .

In the Latitude $51^{\circ} 32'$ North, the Sun's Declination $15^{\circ} 30'$ South, and the time from Noon 2 h. 27' 36" equal to $36^{\circ} 54'$, to find the Sun's Azimuth.

In the Triangle $\odot PZ$ (in the former Figure,) there are given the two Sides $PZ\ 38^{\circ} 28'$, and $\odot P\ 105^{\circ} 30'$, and the contained Angle $\odot PZ\ 36^{\circ} 54'$, to find the Angle $\odot ZP$.

$\odot P\ 105^{\circ} 30'$
 $PZ\ 38\ 28$

Sum $143\ 58$ half Sum $= 71^{\circ} 59'$ } $\odot PZ\ 36^{\circ} 54'$

Diff. $67\ 02$ half Diff. $= 33\ 31$ } half $18\ 27$ |

As s. of the half Sum of the Sides $71^{\circ} 59'$ 0.021835
To s. of half their Diff. $33\ 31$ 9.742080
So ct. of half the cont. Ang. $18\ 27$ 10.476741

To t. of half the Diff. of the opposite }
Angles $60^{\circ} 07'$ } 10.240656

As cs. of half the Sum of the Sides $71^{\circ} 59'$ 0.509629
To cs. of half their Difference $33\ 31$ 9.921023
So ct. of half the contained Ang. $18\ 27$ 10.476741

To t. of half the Sum of the opposite }
Angles $82^{\circ} 57'$ } 10.907392

The $\frac{1}{2}$ Sum of the opp. Angles $82^{\circ} 57'$ The $\frac{1}{2}$ Diff. add $60 \ 07$ The Sum $143 \ 04 = \odot ZD$ required.

P R O B L E M X I I I .

The Latitude of the Place, the Sun's Altitude, and Azimuth given; to find the Hour.

E X A M P L E .

In the Latitude $51^{\circ} 32'$, the Sun's Altitude $44^{\circ} 40'$, and his Azimuth from the North $110^{\circ} 02'$; to find the Hour from Noon.

In the Triangle DZP, there are given the two Sides DZ $45^{\circ} 20'$, the Compl. of the Altitude ZP $38^{\circ} 28'$, and the contained Angle DZP $110^{\circ} 02'$; and the opposite Angle required.

DZ $45^{\circ} 20'$ PZ $38 \ 28$ Sum $83 \ 48 \ \frac{1}{2} \text{ Sum} = 41^{\circ} 54'$ } DZP $110^{\circ} 02'$ Diff. $6 \ 52 \ \frac{1}{2} \text{ Diff.} = 3 \ 26$ } the $\frac{1}{2} \ 55 \ 01$ As s. of the half Sum of the Sides $41^{\circ} 54'$ 0.175332To s. of half their Diff. $3 \ 26$ 8.777333So ct. of half the Angle DZP $55 \ 01$ 9.844958To t. of half the Diff. of the opposite }
Angles $3^{\circ} 35'$ } 8.797623

As

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As cs. of the half Sum of the Sides $41^{\circ} 54'$ 0.128245

To cs. of half their Diff. 3 26 9.999220

So ct. of half the Angle DZP 55 01 9.844958

To t. of $\frac{1}{2}$ the Sum of the opp. Angles } 9.972423

The half Diff. add $43^{\circ} 11'$
03 35

The Sum $46^{\circ} 46' =$ Ang. DPZ

So the Time from Noon is 3 h. 07 min.

PROBLEM XIV.

The Latitude and Longitude of a fixed Star being given, to find the right Ascension and Declination.

EXAMPLE.

The Longitude of *Pollux* is $19^{\circ} 14'$ of *Cancer*, and his Latitude $6^{\circ} 40'$ North; to find his right Ascension and Declination.

In the oblique-angled Triangle SPE, there are given the two Sides PE $23^{\circ} 30'$, the distance between the Pole of the Equinoctial, and the Pole of the Ecliptick; SE $83^{\circ} 20'$, the Complement of the Star's Latitude; and the contained Angle $19^{\circ} 14'$, the Longitude from *Cancer*; to find the Angle SPE, the Complement to 180 of the right Ascension from *Cancer*, and the Side SP, the Complement of Declination.

SE $83^{\circ} 20'$

PE 23 30

Sum 106 50 $\frac{1}{2}$ Sum $53^{\circ} 25'$ } The Angle SEP $19^{\circ} 14'$

Diff. 59 50 $\frac{1}{2}$ Diff. 29 55 } The half 9 37

Diff. 59 50

B b 2

Aa

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Ascension of Cancer, the Sum is $111^{\circ} 55'$, the right Ascension from Aries.

For the Declination:

Ass. of SPE $158^{\circ} 05'$ (or $21^{\circ} 55'$)	0.427991
Tos. of SE $83^{\circ} 20'$ the Comp. of Lat.	9.997053
So s. of SEP $19^{\circ} 14'$ the Long. from S	9.517745

To s. of SP $61^{\circ} 14'$ the Comp. of Declin. 9.942789

Whose Complement $28^{\circ} 46'$ is the Declination Northerly.

PROBLEM XV.

The right Ascension and Declination of a fixed Star being given; to find the Longitude and Latitude thereof.

EXAMPLE.

The right Ascension of Pollux being $111^{\circ} 55'$, and his Declination $28^{\circ} 46'$ North; to find his Longitude and Latitude.

In the oblique Triangle SPE, there are given the two Sides PE $23^{\circ} 30'$, SP $61^{\circ} 14'$, and the contained Angle SPE $158^{\circ} 05'$; to find the Angle SEP, and the third Side SE.

SP $61^{\circ} 14'$
PE $23 \quad 30$

Sum $84 \quad 44$ the $\frac{1}{2}$ Sum $42^{\circ} 22'$ } Ang. SPE $158^{\circ} 05'$

Diff. $37 \quad 44$ $\frac{1}{2}$ Diff. $18 \quad 52$ } The half $79 \quad 02 \frac{1}{2}$

B b 3

As.

As s. of $\frac{1}{2}$ the Sum of the Sides	42° 22'	0.171422
To s. of $\frac{1}{2}$ their Diff.	18 52	9.509696
So ct. of $\frac{1}{2}$ the Angle SPE	79° 02' $\frac{1}{2}$	9.286963

To t. of $\frac{1}{2}$ the Diff. of the opp. Ang.	5° 19'	8.968081
---	--------	----------

As cs. of $\frac{1}{2}$ the Sum of the Sides	42° 22'	0.131445
To cs. of $\frac{1}{2}$ their Diff.	18 52	9.976017
So ct. of $\frac{1}{2}$ the contained Ang. SPE	79 02 $\frac{1}{2}$	9.286963

To t. of $\frac{1}{2}$ the Sum of the opp. Ang.	13 55	9.394425
The half Diff. add	5 19	

The Sum	19 14	= SEP the Long. from S.
---------	-------	-------------------------------

To find the Latitude.

As s. SEP 19° 15' the Long. from <i>Cancer</i>	0.482255
To s. SP 61° 14' the Comp. of Declination	9.942795
So s. SPE 158° 05' the Comp. of right Ascen.	9.572009

To s. SE 83° 20' the Comp. of the Lat.	9.997059
--	----------

Hence the Latitude is 6° 40' North.

PROBLEM XVI.

The Distance of a Planet, Comet, or new Star, from two known fixed Stars, being given; to find the unknown Star's Longitude and Latitude.

EXAMPLE.

The unknown Star's Distance from the *Swan's Beak* is 49° 05', and from *Perseus's Side* 88° 57'; to find the Longitude and Latitude thereof.

Lon,

Lon. $\left. \begin{array}{l} \text{of the Swan's Beak } \text{v} \text{ } 27^{\circ} 07' \\ \text{of Perseus's Side } \text{x} \text{ } 27 \text{ } 43 \end{array} \right\}$ Lat. $\left. \begin{array}{l} 549^{\circ} 02' \text{ N} \\ 30 \text{ } 09 \text{ N} \end{array} \right\}$

1. In the Triangle ADE, there are given two Sides AE $40^{\circ} 58'$, the Comp. of the Latitude of the *Swan's Beak*; DE $59^{\circ} 51'$, the Comp. Lat. of *Perseus's Side*; and the contained Angle AED $120^{\circ} 36'$, the Difference of Longitude between the two Stars; the Angle DAE, and the Side AD required.

DE $59^{\circ} 51'$
AE $40 \text{ } 58$

$\frac{100 \text{ } 49 \text{ half Sum } 50^{\circ} 24' \frac{1}{2}}{18 \text{ } 53 \text{ half Diff. } 9 \text{ } 26' \frac{1}{2}} \left. \begin{array}{l} \\ \end{array} \right\} \text{ Ang. AED } 120^{\circ} 36'$
half $60 \text{ } 18$

As s. of the $\frac{1}{2}$ Sum of the Sides $50^{\circ} 24' \frac{1}{2}$ 0.113168
To s. of $\frac{1}{2}$ their Diff. $9 \text{ } 26' \frac{1}{2}$ 9.214958
So ct. of $\frac{1}{2}$ the Angle AED $60 \text{ } 18$ 9.756172

To t. of $\frac{1}{2}$ Diff. of opp. Angles $6 \text{ } 55$ 9.084298

As cs. of $\frac{1}{2}$ Sum $50^{\circ} 24' \frac{1}{2}$ 0.195648
To cs. of $\frac{1}{2}$ Diff. $9 \text{ } 26' \frac{1}{2}$ 9.994077
So ct. of $\frac{1}{2}$ the Ang. $60 \text{ } 18$ 9.756172

To t. of $\frac{1}{2}$ the Sum $41 \text{ } 26$ 9.945897
Half Diff. add $6 \text{ } 55$

The Ang. DAE $48 \text{ } 21$

As s. DAE $48^{\circ} 21'$ 0.126552
To s. DE $59 \text{ } 51$ 9.936872
Sos. AED $120 \text{ } 36$ 9.934873

To s. AD $84^{\circ} 56'$ 9.998297
2. In

3. In the Triangle ABE, there are given the two Sides AE $40^{\circ} 58'$, AB $49^{\circ} 05'$, and the contained Angle BAE $141^{\circ} 21'$, and the Angle AEB, the Difference of Longitude between the unknown Star and the *Swan's Beak*; the Side BE, the Complement of the unknown Star's Latitude, is required.

$$\begin{array}{r} AB\ 49^{\circ} 05' \\ AE\ 40\ 58 \end{array}$$

$$\text{Sum } 90\ 03\ \frac{1}{2} \text{ Sum } 45\ 01\ \frac{1}{2} \text{ Ang. BAE } 141^{\circ} 21'$$

$$\text{Diff. } 8\ 07\ \frac{1}{2} \text{ Diff. } 4\ 03\ \frac{1}{2} \text{ half } 70\ 40\ \frac{1}{2}$$

$$\begin{array}{rcl} \text{As s. of the half Sum of the Sides } 45^{\circ} 01' \frac{1}{2} & 0.150326 \\ \text{To s. of half their Diff.} & 4\ 03\ \frac{1}{2} & 8.849861 \\ \text{So ct. of half the Angle BAE } 70\ 40\ \frac{1}{2} & 9.544917 \end{array}$$

$$\text{Tot. of } \frac{1}{2} \text{ the Diff. of opp. Angles } 2^{\circ} 01' \quad 8.545104$$

$$\begin{array}{rcl} \text{As cs. of the half Sum of the Sides } 45^{\circ} 01' \frac{1}{2} & 0.150704 \\ \text{To cs. of the half Diff.} & 4\ 03\ \frac{1}{2} & 9.998910 \\ \text{So ct. of half the Angle BAE } 70\ 40\ \frac{1}{2} & 9.544917 \end{array}$$

$$\begin{array}{rcl} \text{To t. of } \frac{1}{2} \text{ the Diff. of opp. Angles } 26^{\circ} 20' & 9.694531 \\ \text{The half Diff, add} & 2\ 01 & \end{array}$$

$$\text{Sum } 28\ 21 = \text{the Ang. AEB.}$$

Which added to the Longitude of the *Swan's Beak*, $27^{\circ} 07'$, makes the unknown Star's Longitude to be $25^{\circ} 28'$,

As

As s. AEB $28^{\circ} 21'$	0.323438
To s. of AB $49^{\circ} 05'$	9.878328
So s. of BAE $141^{\circ} 21'$	9.795575
To s. of BE $83^{\circ} 40'$	9.997341

Whose Complement $6^{\circ} 20'$ is the unknown Star's Latitude Northerly.

PROBLEM XVII.

The Meridian Altitude of an unknown Star or Planet, and the Distance from a known fixed Star being given; to find the unknown Star's Latitude and Longitude.

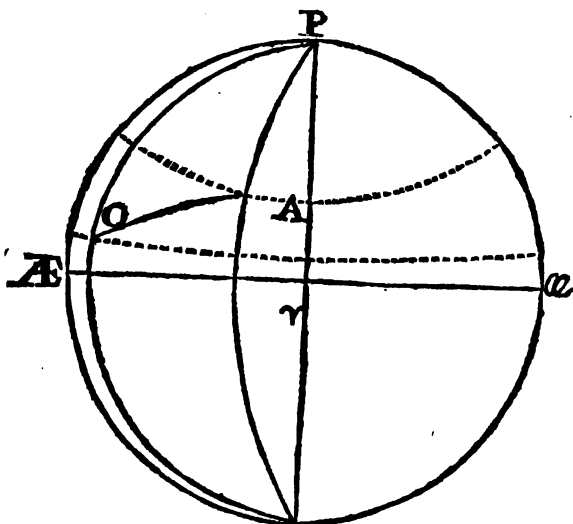
EXAMPLE.

In the Latitude $51^{\circ} 32'$ North, the Meridian Altitude of an unknown Star is $75^{\circ} 49'$, and his Distance from the right Shoulder of *Orien* $54^{\circ} 30'$; to find his Longitude and Latitude.

If you subtract the Complement of the Latitude $38^{\circ} 28'$ from $75^{\circ} 49'$, the Star's Meridian Altitude, there will remain $37^{\circ} 21'$, the Star's Declination North.

In the Triangle AOP, there are given the three Sides, OP $82^{\circ} 39'$, the known Star's Distance from the North Pole; AP $52^{\circ} 39'$, the unknown Star's Distance from the Pole; and OA the Distance between the two Stars; the Angle APO required, being the Difference of right Ascension of the two Stars.

OP



OP 82° 39'	Sine	Ar. Com.	0.003583
AP 52 39	Sine	Ar. Com.	0.099663
OA 54 30	Half Sum 94° 54'	Sine	9.998410
	Diff. 40 24	Sine	9.811655
Sum 189 48			
		Sum	19.913311
$\frac{1}{2}$ Sum 94 54	Co-line of 25° 19'	$\frac{1}{2}$ Sum	9.956655
Diff. 40 24			
Doubled is 50 20—the Angle APO.			

Which subtracted from 84° 53' the right Ascension of *Orien's* Shoulder (because the Star was Westward from *Orien*) there will remain 34° 33', the right Ascension of the unknown Star.

Then having the unknown Star's right Ascension and Declination, you may find his Longitude and Latitude by *Problem* the 15th.

P R Q.

PROBLEM XVIII.

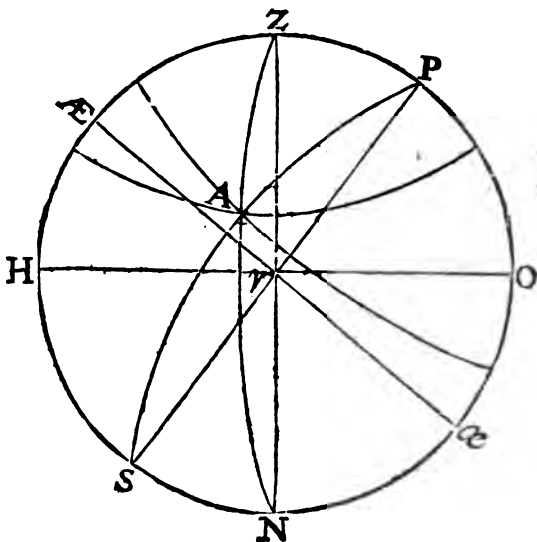
Having the Latitude of a Place, the Sun's right Ascension, and the Altitude of a known fixed Star; to find the Hour of the Night.

EXAMPLE.

In the Latitude $51^{\circ} 32'$ the Sun's right Ascension $228^{\circ} 45'$, and the Altitude of *Aldebaran*, or the Southern Eye of the *Bull*, $32^{\circ} 30'$, to the Eastward of the Meridian; to find the Hour of the Night.

The right Ascension of *Aldebaran* is $64^{\circ} 51'$, and Declination $15^{\circ} 55'$.

In the Triangle APZ , there are given the three Sides, PZ $38^{\circ} 28'$, the Comp. of the Lat. AZ $57^{\circ} 30'$, the Comp. of the Star's Altitude; AP $74^{\circ} 05'$, the Complement of his Declination; to find the Angle APZ , the Difference of right Ascension between the *Medium Cæli* and *Aldebaran*.



AP

AP 74° 05'	Sine	Ar. Com.	0.016978
ZP 38 28	Sine	Ar. Com.	0.206168
AZ 57 30	$\frac{1}{2}$ Sum 85° 01' $\frac{1}{2}$	Sine	9.998360
	Diff. 27 31 $\frac{1}{2}$	Sine	9.664769
Sum 170 03			
			19.886275
$\frac{1}{2}$ Sum 85 01 $\frac{1}{2}$			
	Co-sine of 28° 41'		9.943137
Diff. 27 31 $\frac{1}{2}$			

If 28° 41' be doubled, it makes 57° 22', the Angle APZ; which being subtracted from 64° 51', the right Ascension of *Aldebaran*, there remains 7° 29', the right Ascension of *Medium Cæli*. Add 360 to 7° 29', and from the Sum subtract the right Ascension of the Sun 228° 45', the remainder is 138° 44', which reduced to Time makes 9 hours 15 min. *ferè*, the hour of the Night.

P R O B L E M XIX.

The Elevation of the Pole, and the Declination of the Sun being given; to find the just quantity of Twilight.

E X A M P L E.

In the Latitude 51° 32'; and Declination of the Sun 11° 30' North, what time is it break of Day?

In the Triangle $\odot$ PZ, there is given ZP the Complement of the Latitude, $\odot$ P the Complement of the Sun's Declination, and $\odot$ Z which is 90° with 18° added, (18 deg. being the Depression the Sun is reckoned to have below the Horizon when Day-light first of all breaks forth;) to find the Angle $\odot$ PZ.

ZP

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Thus if the Sun be entring *Scorpio*, and so have $11^{\circ} 30'$ South Declination; then in the Triangle BSN, we shall have the three Sides SN $38^{\circ} 28'$, the Complement of Latitude; BS the Sun's Distance from the South Pole, or Complement of his Declination; and BN $72^{\circ} 00'$, the Sun's Distance from the *Nadir* given; to find the Angle BSN.

SN	$38^{\circ} 28'$	Sine	Ar. Com.	0.206168
SB	$78^{\circ} 30'$	Sine	Ar. Com.	0.008807
BN	$72^{\circ} 00'$	$\frac{1}{2}$ Sum $94^{\circ} 29'$	Sine	9.998669
		Diff. $22^{\circ} 29'$	Sine	9.582534
Sum	$188^{\circ} 58'$			<u>19.796178</u>
$\frac{1}{2}$ Sum	$94^{\circ} 29'$			<u>9.898089</u>
Diff.	$22^{\circ} 29'$	Co-sine of $37^{\circ} 44'$		<u>9.898089</u>
Doubled is $75^{\circ} 28' =$ the Ang. BSN.				

Which reduced into Time, makes 5 h. 2 min. *feré*, for the time of Day-breaking in the Morning; and being subtracted from 12 h. there remains 6 h. 58 min. for the end of Twilight in the Evening.

P R O.

PROBLEM XX.

The Latitude of a Place, the Sun's place in the Ecliptick, and the Time of the Day given; to find what Point of the Ecliptick culminates in the Meridian, the highest Point in the Ecliptick (called the Nonagesima Degree, or 90th Degree of the Ecliptick) the Distance of each of these from the Vertex, and the Parallactick Angle, or Angle which the Vertical Circle makes with the Ecliptick.

EXAMPLE.

In the Latitude $51^{\circ} 32'$, the Sun's Place Π 0.00, and the Hour 9 in the Morning, being given; to find the Point of the Ecliptick culminating, the Nonagesima Degree, and the Parallactick Angle.

To delineate this Figure.

Having described the Primitive Circle representing the Ecliptick, and drawn the two Diameters $\Upsilon K \simeq$, which is the Equinoctial Colure, and $S K \simeq$ the Solstitial Colure; next draw the Meridian $\Pi P \simeq$ thro the three Points, Π the Sun's place, P the Pole, and $\simeq$ the opposite Point to Π : this is called the proper Meridian.

Then draw the Parallel $ABED$ called the Path of the Vertex, thus: Find the Sum and Difference of the Complement of the Latitude $38^{\circ} 28'$, and $23^{\circ} 30'$ the Distance between the Pole of the Equinoctial P , and K the Pole of the Ecliptick; so the Sum is $61^{\circ} 58'$, and the Difference is $14^{\circ} 58'$; set the half Tangent of $61^{\circ} 58'$ from K to A , and the half Tangent of $14^{\circ} 58'$ from K to E , the middle of that Distance is the Center.

Then

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Then at 45 deg. (or 3 hours) Distance (the Time from Noon) draw the Meridian CBP, cutting the Path of the Vertex in B, and the Ecliptick in C and in E.

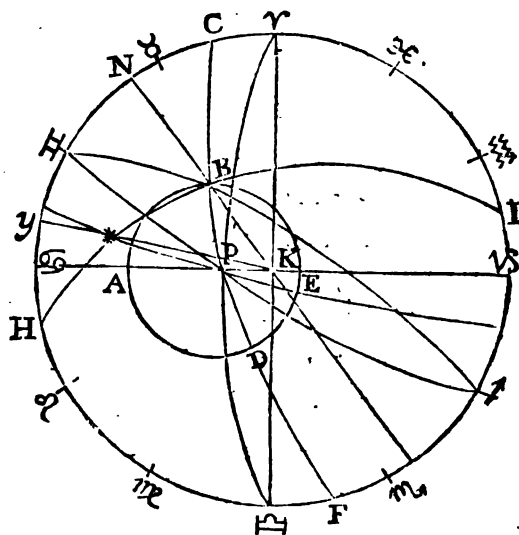
Thro Π B and γ draw the Vertical Circle; and thro K and B draw the Circle of Longitude, to cut the Ecliptick in N; and it is done.

1. C is the Point of the Ecliptick culminating, or in the Meridian of the Place, at the given time.

2. N is the Nonagesima Degree, or highest Point of the Ecliptick, at the same time.

3. CB, and NB, are their respective Distances from the Vertex of *London*, at that time.

4. The Angle N Π B is the Parallactic Angle, or Angle which the Vertical Circle makes with the Ecliptick, at the same time.



Cc

Then

Then in the right-angled Triangle $P \odot C$,

1. The Angle $CP \odot$ is the Complement of the right Ascension of the *Medium Celi*, or the Point of the Ecliptick in the Meridian of the Place at the proposed Time, and being the Time from Noon (when Afternoon, added to the Sun's right Ascension, found by *Problem 2.* but when the proposed Time is in the Forenoon, it is subtracted) in this Case is $77^{\circ} 11'$, the Complement of $12^{\circ} 49'$, found by subtracting (the Time) 45° from $57^{\circ} 49'$, the Sun's right Ascension.

2. The Side $P \odot$ is the Complement of the Distance of the Poles of the Ecliptick, and Equinoctial, $66^{\circ} 30'$.

3. The Side $C \odot$ is the Complement of $C \gamma$, the Longitude of the Mid-heaven, or the Point of the Ecliptick culminating in the Meridian at the proposed time.

4. The Hypothenufe CP , the Distance of the Mid-heaven from the next nearest Pole of the Globe, at the same time.

Then to find the Leg $C \odot$, the Point culminating, the Proportion is,

As Rad. to s. $P \odot 66^{\circ} 30'$	9.962398
So t. of the Angle $CP \odot 77^{\circ} 11'$	10.643018
To t. of $C \odot 76^{\circ} 04'$	10.605416

Whose Complement $13^{\circ} 56'$ is γC , the place of *Medium Celi* in the Ecliptick.

Then to find the Hypothenufe CP ;

As Rad. to cs. of the Ang. $CP \odot 77^{\circ} 11'$	9.346024
So ct. $P \odot 66^{\circ} 30'$	9.638302
To ct. of the Hypothenufe $CP 84^{\circ} 28'$	8.984326

From

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From which take PB the Distance of the Pole from the Vertex $38^{\circ} 28'$, the remainder is BC, the Distance of the *Medium Caeli* from the Vertex 46° .

Then in the oblique-angled Triangle PBK, there are given, the Side PK, the Distance of the two Poles $23^{\circ} 30'$; the Side PB, the Complement of the Latitude of the Place $38^{\circ} 28'$; and the Angle KPB, the Complement of the *Medium Caeli* to 180 , or its Distance from ν , in this Case being $102^{\circ} 49'$: the Angle PKB, the Longitude of Nonagesima Degree from the first Point of $\odot$, and the Side KB, the Complement of the Nonagesima Degree's Distance from the Vertex, are required.

First, For the Angle PKB, it will be thus :

PB	$38^{\circ} 28'$		
PK	$23 \quad 30$		
Sum	<u>$61 \quad 58$</u>	$\frac{1}{2}$ is $= 30^{\circ} 59'$	The Ang. KPB $102^{\circ} 49'$
Diff.	$14 \quad 58$	$\frac{1}{2}$ is $= 7 \quad 29$	The half is <u>$51 \quad 24 \frac{1}{2}$</u>
As s. of the half Sum of the Sides	$30^{\circ} 59'$	0.288371	
To s. of half their Diff.	$7 \quad 29$	9.114737	
So ct. of half the Angle KPB	$51 \quad 24$	<u>9.902160</u>	
To t. of half the Diff. of the opposite		$\} 9.305268$	
Angles	$11^{\circ} 25'$		

As cs. of half the Sum of the Sides	30° 59'	0.066858
To cs. of half the Diff.	7 29	9.996285
So cr. of half the Angle KPB	51 24	9.902160

To t. of half the Sum of the opposite		
Angles	42° 43'	9.965303
The half Diff. add	11 25	

The Sum is the Ang. PKB = 54° 08', or ☉ N.
 Subtract it from 90 00

Remain. is the Nonagesima 35 52 from ♑.

That is, the Nonagesima is 5 52 in 8.

Secondly, To find the Side KB.

As s. of the Angle PKB	54° 08'	0.091310
To s. of the Side PB	38° 28'	9.793832
So s. of the Angle KPB	102° 49'	9.989042
To s. of the Side KB	48 27	9.874184

The Distance of the Vertex from the Pole of the Ecliptick, equal to which is the Altitude of the Nonagesima Degree, whose Complement is 41° 33', is NB, its Distance from the Vertex.

Again, in the right-angled Triangle Π N B.

The Leg N Π 24° 08', the Sun's Distance from the Nonagesima Degree, found by subtracting ♑ N 35° 52' from ♑ Π 60 deg. and the Leg NB, the Distance of the Nonagesima Degree from the Vertex, found as above, 41° 33' given; the Hypothensuse Π B, the Sun's Distance from the Vertex, and the Parallaxick Angle N Π B is required.

To

To find the Paralla ϕ tick Angle.

As Radius to s. of N Π $24^{\circ} 08'$	9.611576
So ct. NB $41^{\circ} 33'$	10.052428

To ct. of the Angle N Π B $65^{\circ} 14'$	9.664004
--	----------

Which $65^{\circ} 14'$ is the Paralla ϕ tick Angle at the Sun.

To find the Sun's Distance from the Vertex Π B.

As Radius to cs. Π N $24^{\circ} 08'$	9.960279
So cs. NB $41^{\circ} 33'$	9.874120

To cs. Π B $46^{\circ} 55'$, the Sun's Distance } from the Vertex,	9.834399
--	----------

But to calculate the Paralla ϕ tick Angle at the Moon, or at a Star, that has Latitude from the Eccli ϕ tick, will require a little more Labour; as in the next Problem.

PROBLEM XXI.

The Latitude of the Place, the Sun's Place in the Ecliptick, the Time of the Day or Night, and the Longitude and Latitude of the Moon, or of a Star, being given; to find the Parallax Angle at the Moon, or at a Star, and its Distance from the Vertex.

EXAMPLE.

In the Latitude $51^{\circ} 32'$ North, the Sun's Place Π 0. 00; the Hour 9 in the Morning; the Star *Capella*, whose Longitude is $17^{\circ} 49'$ in *Gemini*; his Latitude $22^{\circ} 52'$ North, being all given: the Parallax Angle at the Star, and its Distance from the Vertex are required.

First, Find the Place of the Star in the last Scheme, and through the Star and the Vertex at B describe a great Circle.

For $\ast B$ is the Star's Distance from the Vertex; and the Angle $K \ast B$ the Complement of the Parallax Angle required,

In order to the finding them by Calculation. First, Find the Place of the Nonagesima Degree by the last Problem, which is $8^{\circ} 5' 52''$, or γ N $35^{\circ} 52'$. Secondly, The Distance of the Nonagesima Degree from the Vertex, which will be $NB 41^{\circ} 33'$.

Then in the oblique-angled Triangle $\ast K B$, the Side $K \ast$; the Star's Distance from the next Pole of the Ecliptick, or the Complement of its Latitude, which in this Example is $67^{\circ} 08'$; the Side $K B$, the Distance of the Vertex from the said Pole, which is equal to the Altitude of the Nonagesima Degree, and is here $48^{\circ} 27'$, found by the last

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last Problem; the Angle * K B, the Difference of the Longitude of the given Star, and the Nonagesima Degree, which in this Example is $41^{\circ} 57'$, and is the Difference of $\gamma N 35^{\circ} 52'$, and $\gamma y 77^{\circ} 49'$, being all given: to find the Angle K * B, the Complement of the Parallaſtick Angle at the given Star, and the Side * B, the Star's Distance from the Vertex.

First, to find the Angle K * B.

$$\begin{array}{r} K * 67^{\circ} 08' \\ KB \quad 48 \quad 27 \\ \hline \end{array}$$

$$\begin{array}{rcl} \text{Sum} & 115 \quad 35 \quad \frac{1}{2} \text{ is } 57^{\circ} 47' \frac{1}{2} & \text{The A. * KB } 41^{\circ} 57' \\ \text{Diff.} & 18 \quad 41 \quad \frac{1}{2} \text{ is } 9 \quad 20 \frac{1}{2} & \text{The half is } 20 \quad 58 \frac{1}{2} \end{array}$$

$$\begin{array}{rcl} \text{As s. of the half Sum of the Sides } 57^{\circ} 47' \frac{1}{2} & 0.072571 & \\ \text{To s. of half their Diff.} & 9 \quad 20 \frac{1}{2} & 9.210376 \\ \text{So ct. of half the Angle * K B } 20 \quad 58 \frac{1}{2} & 10.416389 & \end{array}$$

$$\begin{array}{rcl} \text{To t. of half the Diff. of the opposite} & & \\ \text{Angles } 26^{\circ} 35' & \} & 9.699336 \end{array}$$

$$\begin{array}{rcl} \text{As cs. of half the Sum of the Sides } 57^{\circ} 47' \frac{1}{2} & 0.273274 & \\ \text{To cs. of half their Diff.} & 9 \quad 20 \frac{1}{2} & 9.994201 \\ \text{So ct. of half the Ang. * K B } 20 \quad 58 \frac{1}{2} & 10.416389 & \end{array}$$

$$\begin{array}{rcl} \text{To t. of half the Sum of the opposite} & & \\ \text{Angles } 78^{\circ} 18' & \} & 10.683864 \\ \text{The half Diff. subtract } 26 \quad 35 & & \end{array}$$

$$\text{The Diff. } 51 \quad 43 = \text{the An. K * B.}$$

Whose Complement $38^{\circ} 17'$ is the Parallaſtick Angle at *Capella*.

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Secondly, To find the Side * B, the vertical Distance.

As s. of the Angle K * B	51° 43'	0.105154
To s. of the Side KB	48 27	9.874120
So s. of the Angle * KB	41 57	9.825090

To s. of * B, the Star's Distance from the
Vertex 39° 36' } 9.804364

Thus have I gone thro the most Practical *Problems* in Astronomy, tho there might be a great many others formed; for the whole Course of the heavenly Bodies, and all their Motions and Eclipses are solved by Trigonometry: so I shall conclude this Chapter, and refer my Reader for further Satisfaction to Books treating of Astronomy.

C H A P.

C H A P. VIII.

Arithmetical Dialling

DEMONSTRATED;

O R,

TRIGONOMETRY

Applied to

PRACTICE in DIALLING.

DIALS may be made upon any plain Superficies, and all plain Superficies have one or other of these three Positions, *viz.* either Parallel, Perpendicular, or Oblique to the Horizon of the Place wherein the Plain is seated; and all the Hour-Lines drawn upon any Plain are great Circles of the Sphere, which being projected upon a plain Superficies, become straight Lines.

Now the Art of Dialling consisteth chiefly in the finding out of these Lines, and their true Distances from each other, which do continually vary, according as the Plains upon which they are described, or projected, are situated in respect of the Horizon of the Place,

How

How to find the Declination of a Plain.

Definition. The Declination of a Plain is the quantity of that Arch of the Horizon, which is comprehended between the true North or South Points, and a Line drawn perpendicular to the Plain.

By your Quadrant draw an Horizontal Line upon the Plain, and apply one Side of your Quadrant to the said Line, so that the Limb of the Quadrant may be towards the Sun, holding the whole Quadrant as level as you can; and at the same time hold up a Line and Plummets at full Liberty by the Limb, guiding it so as that the Shadow of the Thread may pass directly thro the Center: then mind what Degree of the Limb the Thread cuts, being counted from the Side of the Quadrant that is perpendicular to the Plain, which Degrees are the Horizontal Distance.

At the same time (or as near as possible) find the Altitude of the Sun by your Quadrant, and thereby find the Azimuth by *Problem* the 10th of the 7th *Chapter*. Or if it be *Collins's* Quadrant, you may find the Azimuth near enough the Truth by that.

When you make your Observation of the Sun's Horizontal Distance from the Perpendicular of the Plain, mark whether the Shadow of the Thread do fall between the South Point of the Horizon, and that Side of the Quadrant which is perpendicular to the Plain. Then observe these Rules.

I. If the Shadow fall between them, then the Horizontal Distance and Azimuth added together, make the Declination. And in this Case, the Declination of the Plain is towards the same Coast whereon the Sun's Azimuth is.

II. If

II. If the Shadow fall not between the South Point and that Side of the Quadrant which is perpendicular to the Plain; then the Difference between the Horizontal Distance and the Azimuth is the Declination of the Plain. And if the Azimuth be the greater of the two, then the Plain declines to the same Coast the Sun is upon; if it be the least, then it declines to the contrary Coast.

How to find the Reclination of a Plain.

Definition. The Reclination of a Plain, is the quantity of an Arch of the Vertical Circle, which passeth through the Zenith, and cutteth the reclining Plain at right Angles.

To find the quantity of Reclination: First, by the help of a Level draw an Horizontal Line. cross the Plain, and about the middle draw another Line at right Angles thereunto; then let some Assistant hold a long Ruler upon this last Line, and over the top of the Plain, till you hold a Quadrant to the under Side of the Rule, letting the Thread and Plummets hang at free Liberty, and observe the Degrees cut by the Thread, which is the Reclination of the Plain.

How to know which of the Poles, whether the North or South, is to be elevated above any Dial-Plain.

The Axis, or Stile of every Dial, lies parallel to the Axis of the World; and therefore the two ends of the Stile of every Dial directly respect the two Poles of the World: And consequently if the South Pole be elevated upon any Dial Plain, a Dial made on the back-side of that Plain, will have the North Pole elevated.

General Rules.

1. Upon the Horizontal Plain, in North Latitude, the North Pole is elevated, but in South Latitude the South Pole.

2. Upon all erect Plains, whether direct or declining, if the Plain lie open to the South, the South Pole is elevated; but if it behold the North, the North Pole must be elevated.

3. Upon all direct East or West Plains reclining, (how far soever) the North Pole is elevated; and upon all East and West Incliners opposite to them, the South Pole.

4. Over all North reclining Plains, whether direct or declining, the North Pole is elevated; and over the inclining Plains opposite to them, the South Pole.

Lastly, Over all South reclining Plains, whether direct or declining, if the Plain pass between the Zenith and the Pole, the Axis of the Stile must have respect to the South Pole; and on the inclining Plains opposite to them, the North Pole: but, if the Plain pass between the Horizon and the Pole, the North Pole; and on the Incliners opposite to them, the South Pole,

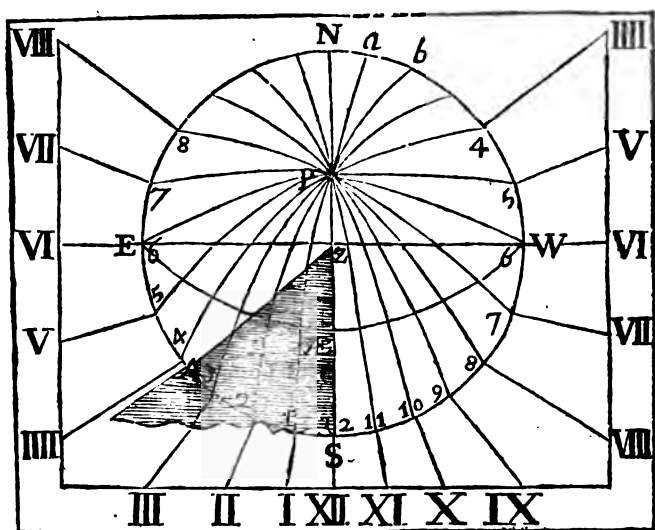
S E C T. I.

To draw the Hour Lines upon a Vertical (commonly called Horizontal) Plain.

First, Draw a right Line NS for the Meridian, and Hour-Line of 12, and cross it with another at right Angles EW, for the Hour-Line of six, cutting each other in Z; then upon Z as a Center describe

scribe the Circle ENWS, representing the Horizon of *London* : Within this Circle project the Sphere, as has been taught.

Then shall the several Hour-Circles touching the Plain of the Horizon, give you Points to draw the Hour-Lines upon your Dial-Plain by : So that a Ruler laid to Z, and every of the Points 1, 2, 3, &c. 11, 10, 9, &c. where the Hour-Circles touch the Horizon, if you draw straight Lines thereby, they shall be the true Hour-Lines.



For the making of an Horizontal Dial, there is nothing required to be known, but the Latitude of the Place, equal to which must the height of the Stile be ; wherefore take $51^{\circ} 32'$ out of your Scale of Chords, and set it upon the Horizon from S to A, and draw a Line ZA for the Stile. The Sub-stile (or Line upon which the Stile standeth) is the Meridian, or Hour-Line of 12 ; and so is the Dial finished.

By Trigonometrical Calculation.

There is nothing required to be found in this Dial by Calculation but the Hour-Distances from the Meridian: In the Triangle $NP a$ right-angled at N , there is given NP the Latitude of $51^{\circ} 32'$, and the Angle $NP a$, the Angle the Hour-Circle makes at the Pole; to find $N a$ the Hour-Distance. Wherefore the Proportion will be,

As Rad. to the s. of the Latitude $51^{\circ} 32'$ 9.89374
 So is t. of the Angle $NP a$ $15^{\circ} 00'$ for } 9.42805
 one hour }

To t. of the Hour Distance $N a$ $11^{\circ} 51'$ 9.32179

Again,

As Rad. to s. of the Lat. $51^{\circ} 32'$ 9.89374
 So t. of the Ang. $NP b$ $30^{\circ} 00'$ for 2 hours 9.76144

To t. of the 2d hour Distance $N b$ $24^{\circ} 19'$ 9.65518

After the same method you may calculate a Table to every quarter of an Hour for any Latitude, making the Angles at the Pole $3^{\circ} 45'$ for the first quarter, $7^{\circ} 30'$ for the half hour, $11^{\circ} 15'$ for three quarters, &c.

Hours from Noon.		Angles at the Pole.		Hour-Distances on the Plain.	
12		00	00	00	00
11	1	15	00	11	51
10	2	30	00	24	19
9	3	45	00	38	03
8	4	60	00	53	36
7	5	75	00	71	06
6		90	00	90	00

After you have calculated your Table, upon the Center of your Dial with 60° of Chords describe a Semicircle, as ESW; then by the same Scale of Chords set off the Degrees and Minutes in the third Column of the Table both ways from S. Thus take $11^\circ 51'$ from the Chords, and set from S to 11 and 1; then take $24^\circ 19'$, and set from S to 10 and 2, and so of the rest: then lay a Ruler to the Center Z, and to each of those Points, and draw the Hour-Lines; and set $51^\circ 32'$ from S to A, and draw ZA for the Stile.

Thus have you two ways for performing the same thing, *viz.* by Geometrical Projection, and by Arithmetical Calculation, whereby you may see not only an agreement of one with the other, but also a plain Demonstration of one by the other.

S E C T. II.

How to describe Hour-Lines upon an erect direct South or North Plain.

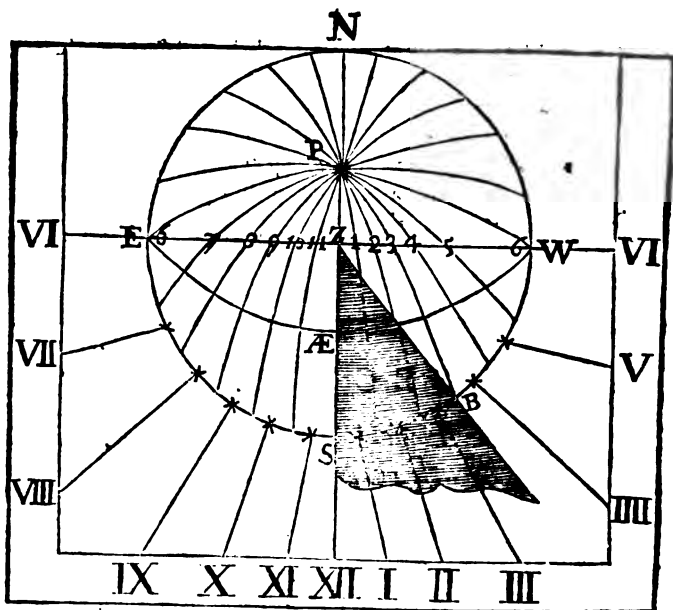
First, Upon a Circle representing the Horizon of London, project the Sphere, as in the foregoing Section. The next thing is to draw a Line upon your Projection, which shall represent your Plain.

And here Note, That all upright Plains are represented upon the Projection by straight Lines.

Now, an erect direct Plain, which beholdeth the South, must needs lie in the Azimuth Circle of East and West; therefore a right Line drawn from E to W, shall represent your Plain.

And the Point N is the Pole thereof, being 90° distant from it.

If you lay a Ruler to N, and to the several Points, 1, 2, 3, &c. 11, 10, 9, &c. where the Hour-Lines of the Projection cut the Plain, and where the Ruler cuts the Primitive Circle, make small Marks, as ***. Then Lines drawn from the Center Z thro those Marks, shall be the true Hour-Lines.



The height of the Pole above an erect direct North or South Plain, is always equal to the Complement of the Latitude: therefore if you take $38^{\circ} 28'$ from the Scale of Chords, and set from S to B, and draw the Line ZB, it shall be the Stile, which must stand upon the Meridian; and in the South Plain must point downwards to the South Pole, but in the North Plain it must point upwards to the North Pole.

By

By Trigonometrical Calculation.

There is nothing but the Hour-Distances (as in the Horizontal) to be calculated. Thus in the right-angled Triangle $Z P_1$, there is given $ZP\ 38^\circ 28'$, and the Angle $ZP_1\ 15^\circ$, the Angle at the Pole; to find Z_1 , thus.

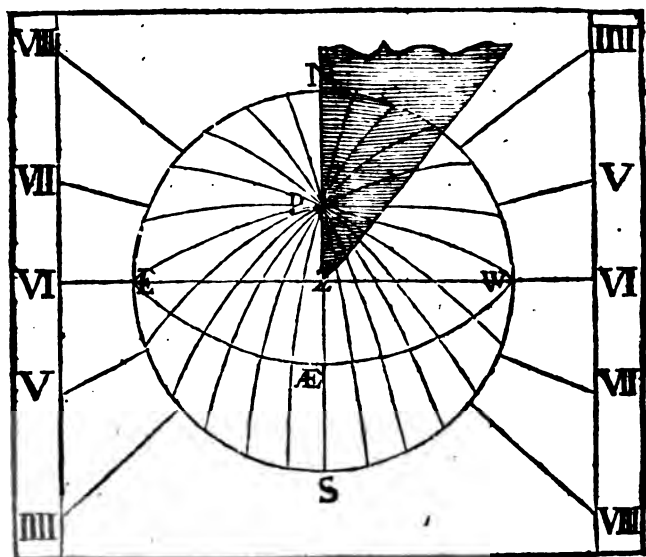
As Rad. to s. of $ZP\ 38^\circ 32'$, the height	}	9.79383
of the Pole or Stile above the Plain		
So t. of $ZP_1\ 15^\circ 00'$, the Angle at the	}	9.42805
Pole		

To t. of $Z_1\ 9^\circ 28'$, the first hour Dist.	9.22188
--	---------

The drawing of this Dial is the same as the Horizontal. For first, with 60° of Chords you strike the Semicircle ESW ; then from S set off the Degrees and Minutes in the third Column of the Table both ways; then lay a Ruler to Z the Center, and to each of those Pricks, and draw Lines, which shall be the true Hour-Lines. Then from the Line of Chords take $38^\circ 28'$, the height of the Stile above the Plain, and set from S to B , and draw ZB for the Stile.

Hours from Noon.	Angles at the Pole.	Hour-Distances on the Plain.
12	00° 00'	00° 00'
11 1	15 00	9 28
10 2	30 00	19 45
9 3	45 00	31 53
8 4	60 00	47 08
7 5	75 00	66 42
6	90 00	90 00

The making of the North Dial is in all respects the same as the making of the South; and the Stile has the same Elevation above the Plain: all the Difference is, that whereas that points downwards to the South Pole, this must point upwards towards the South Pole. The Hours next the Meridian are omitted, because the Sun can never come upon the Dial at those Hours.



S E C T. III,

To describe Hour-Lines upon an erect direct East or West Plain.

An East or West erect Plain lies in the Azimuth Circle of North and South, and so passes directly thro the Poles of the World, neither of the Poles having

having any Elevation above those Plains; and therefore the Hour-Lines will be all parallel one to another.

To draw an East Dial for the Latitude $51^{\circ} 32'$.

1. Near the bottom of the Plain draw an Horizontal Line DC; then upon C, as a Center, with 60° of Chords describe the Arch EF, and set the Chord of $38^{\circ} 28'$, the Complement of the Latitude from E to F; and draw the Line CF quite thro the Plain: which Line will be elevated exactly to the Equinoctial Circle in the Heavens, and therefore it is called the Equinoctial Line.

2. That you may the better proportion your Stile to your Plain, and that all the Hours may come on, assume two Points in the Equinoctial Line, one towards the end C, for the Hour of 11, and the other towards the other end thereof for the Hour 6, as the Point H; and thro these two Points G and H, draw two Lines at right Angles to the Equinoctial Line, for the Hour-Lines of 11 and 6.

3. Upon the Point G with 60° of Chords describe the Arch IK below the Equinoctial Line, setting thereon the Chord of 15 deg. from I to K; and draw the obscure Line GK, extending it till it cut the Hour-Line of 6 in the Point L: so shall LH be the height of the perpendicular Stile proportioned to this Plain.

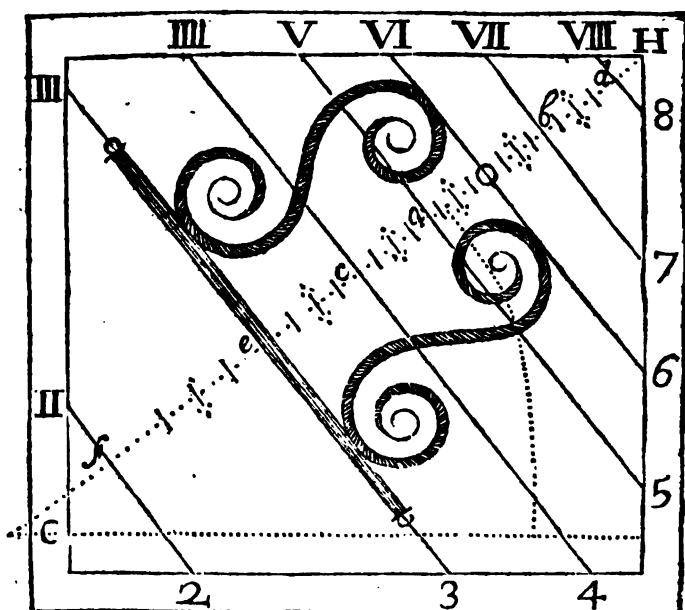
4. Open your Compasses to 60° of Chords, and upon L as a Center describe the obscure Arch of a Circle MN, between the Hour-Line of 6 and the Line GL; and with the Chord of 15 deg. divide it into 5 equal parts, in the Points $\odot \odot \odot, \&c.$ and lay a Ruler from L to each of these Points, and the Ruler shall cut the Equinoctial Line in the Points *****: thro which Points draw Lines parallel to the Hour-Line of 6, and they shall be the true Hour-

The making of a West Dial is in all respects the same as this, only the Line CH called the Equinoctial-Line, is drawn from the left-hand Corner, and so all the Hour-Lines the other way; and they must be numbered the other way too, the Hour of 11 will be 1, and 10 will be 2, &c. Or, when you have drawn an East Dial upon Paper, if you draw the Lines as they appear thro on the back-side, you'll have a West-Dial.

When I make any of these Dials, my way is to leave out the Hour of 11 or 1, and then I have more room for the rest of the Hour-Lines; for that one Hour-space takes up almost half the Dial-Plain: so that when it is left out, there is room for the other Hour-spaces to be divided into halves and quarters; but when you put that Hour on, there is no room for that, nor scarce to set the Figures, but they must be made very small.

This West Dial I made by help of the Sector, thus: I open my Sector to the Radius of the Stile's height, then take off the Tangent of 15° , and set from O to *a* and *b*, and take the Tangent of 30° , and set from O to *c* and *d*; and the Tangent of 45° will reach from O to *e*; and the Tangent of 60° will reach from O to *f*: then draw Lines thro these Points at right-angles to the Equinoctial Line, and they shall be the true Hour-Lines.

After the same manner you may set off the halves and quarters, $3^{\circ} 45'$ for a quarter, $7^{\circ} 30'$ the half-hour, $11^{\circ} 15'$ for three quarters; and by the continual addition of $3^{\circ} 45'$ you will have the Degrees and Minutes for every quarter.



S E C T. IV.

How to draw Hour-Lines upon an erect North or South Plain, declining East or West,

By Projection.

Our Example shall be of an upright Plain, declining from South, Westward 30 degrees.

First, Draw a Right-Line AB, representing your declining Plain, crossing it with another Right-Line CD, at right-angles in Z, making Z for the Zenith of the Place, and Center of your Dial: upon Z describe the Circle ABCD; then from C towards B (because the Plain declines Westward, but from C towards

towards A if it had declined Eastward) set 30 deg. the Declination, from C to N, and draw the Line NZS for the Meridian of the Place; upon which, from Z to P, set off the Pole of the World, and finish your Projection, as is before taught.

Secondly, Your Projection being finished, and AB being the Line representing the Plain, the Points C and D will be the Poles thereof, being 90° distant therefrom; then thro the three Points C, P, and D, describe the Meridian CPRD passing thro the Pole of the World, and the Poles of the Plain, whose Center will be in the Line of the Plain extended: Then find the Pole of this Circle thus, Lay a Ruler upon D and R, and it will cut the primitive Circle in *a*; then from *a* set 90° to *e*, then a Ruler laid from D to *e* will cut the Plain in Q, so is Q the Pole of the Meridian CPD.

There are three Requisites to be found, before we can draw the Hour-Lines; which three things are all comprised in the little spherical Triangle PZR. The three things to be found are,

1. The height of the Pole above the Plain PR.
2. The Deflection, or Sub-stile's Distance from the Meridian ZP.
3. The Plain's difference of Longitude; or, the Angle between the Meridian of the Place NZS and the Meridian of the Plain CRD, that is, the Angle ZPR.

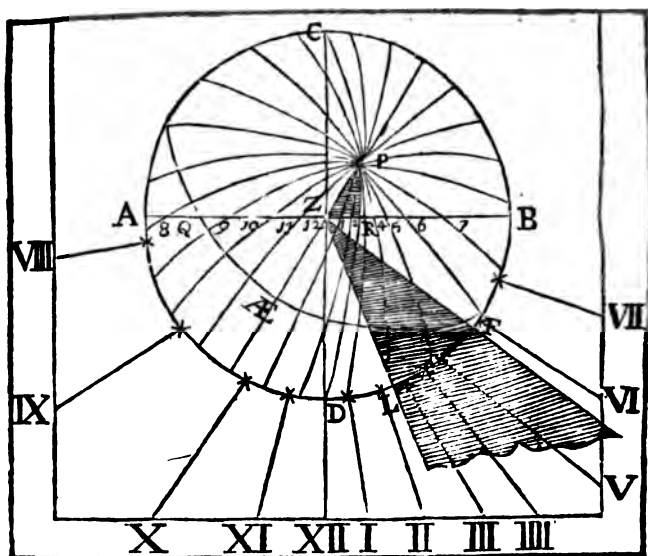
All these Requisites may be found by the 7th and 8th Problems of Chap. 10. Part 1.

And for drawing the Hour-Lines, lay a Ruler to C, and to the several Points 8, 9, 10, 11. and 1, 2, 3, &c. where the Hour-Circles cross the Line of the Plain AB; and where the Ruler cuts the primitive Circle, make small marks **, &c. and Lines drawn

D d 4

from

from the Center Z, and thro these marks, shall be the true Hour-Lines.



A Ruler laid from C to R, will give the Point L, whereby to draw the Sub-stile ZL. And $32^{\circ} 36'$ set from L to F, will give the Point where to draw the Stile ZF, and your Dial is finished.

By Trigonometrical Calculation.

The three fore-mentioned Requisites, viz. the Height of the Stile, the Distance of the Sub-stile from the Meridian, and the Plain's difference of Longitude, are all to be found in the Triangle ZPR; in which there is given the Side PZ $38^{\circ} 28'$, the Complement of the Latitude, and the Angle PZR $60^{\circ} 00'$, the Complement of the Declination, to find

1. The Stile's height above the Plain, PR.

As Rad. to s. of ZP $38^{\circ} 28'$ 9.79383
 So s. of PZR $60^{\circ} 00'$ the Comp. of Declin. 9.93753

To s. RP $32^{\circ} 36'$ the height of the Stile 9.73136

2. The Distance of the Sub-stile from the Meridian, ZR.

As Rad. to cs. PZR $60^{\circ} 00'$ the Compl. } 9.69897
 of Declination }
 So t. ZP $38^{\circ} 28'$ Compl. of Latitude 9.90008

Tot. of ZR $21^{\circ} 40'$, the Sub-stile's dist. } 9.59905
 from the Meridian }

3. The Plain's Difference of Longitude ZPR.

As s. of $38^{\circ} 28'$ ZP, the Compl. of the Lat. 9.79383
 To Radius 10.
 So s. ZR $21^{\circ} 40'$, the dist. of the Sub-stile } 9.56727
 from the Meridian }

To s. ZPR $36^{\circ} 25'$, the Diff. of Long. 9.77344

From the Plain's Difference of Longitude thus found, allowing 15 deg. of the Equinoctial to an Hour, and one deg. for four min. of Time, you may perceive that the Sub-stile will fall between the Hours of 2 and 3, for the Difference of Longitude being more than 30° , and less than 45° ; wherefore from the Difference of Longitude $36^{\circ} 25'$, subtract 30° , and there remains $6^{\circ} 25'$ the Distance of the Sub-stile from 2 a-Clock: and subtracting $36^{\circ} 25'$, the Plain's Difference of Longitude, from $45^{\circ} 00'$, there remains $8^{\circ} 25'$, the Equinoctial Distance of the 3 a-Clock from the Sub-stile: then by adding 15 deg. to each of the Remainders, you have the next Equinoctial

noctial Distances of the Hours 1 and 4; and so by a continual addition of 15 deg. you get all the Equinoctial Distances or Angles at the Pole, as you see in the second Column of the Table. Then to find the Hour-Distances from the Sub-stile, in the Triangle RP 2, there is given RP $32^{\circ} 36'$, the height of the Pole or Stile above the Plain; and the Angle at the Pole 2 PR $60^{\circ} 25'$; to find the Distance of the Hour-Line from the Sub-stile 2 R .

As Rad. to s. of the height of the Stile }
 above the Plain RP $32^{\circ} 36'$ } 9.73140
 So t. of 2 PR $60^{\circ} 25'$ the Angle at the Pole 9.05101

To t. of 2 R $3^{\circ} 28'$, the dist. of 2 a-Clock }
 from the Sub-stile, } 8.78241

Then in the Triangle 1 PR , having PR $32^{\circ} 36'$, and the Angle 1 PR $21^{\circ} 25'$, the Angle at the Pole, you may find 1 R $11^{\circ} 56'$ by the same Proportion: and so you may find all the Hour-Distances from the Sub-stile, as you see in the third Column of the Table.

If you lay the Chords of the Numbers in the third Column from the Sub-stile, you will find them to fall in the Points of the Hour-Distances * * *, &c.

And the Chord of $21^{\circ} 40'$ being set from D to L , gives the Point to draw the Sub-stile by, and the Line ZL is the Sub-stile.

And the Chord of $32^{\circ} 40'$ (the Stile's height) set from L to F , gives the Point to draw the Stile by, and the Line ZF shall be the Stile,

Hours

Hours for the		The Angles at the Pole or the E- quinoctial Distances.		Hour Dif- ferences on the Plain, from the Sub-stile.	
East	West				
4	8	83°	35'	78°	12'
5	7	68	35	53	57
	6	53	35	36	08
7	5	38	35	23	16
8	4	23	35	13	14
9	3	8	35	4	36
The Sub-stile.					
10	2	6	25	3	28
11	1	21	25	11	56
	12	36	25	21	41
1	11	51	25	34	03
2	10	66	25	51	00
3	9	81	25	74	21

S E C T. V.

How to draw Hour-Lines upon a South or North Plain, which declines many Degrees towards the East or West.

If a Plain declines many Degrees from the North or South towards either East or West; altho the Requisites may be found, and the Dial made in all respects as the former Dial in the last Section; yet by reason that the Pole of the World will have but small Elevation above such a Plain, the Hour-Lines will fall so close together, that there will be
no

no competent Distance between them, without extending them out to a great length, as the Antients used to do; who used to draw such a Dial upon the Floor of a Room, and then extend out the Hour-Lines, Stile, and Sub-stile, till they spread themselves to a competent Distance; and then cut the Dial off, Stile and all, and transfer it to the Plain. But this is no artificial way, and therefore I will shew how to draw such a Dial Geometrically, having no regard to the Center of the Dial.

You must first find the Requisites Trigonometrically, as was shewed in the last *Section*, viz. The height of the Stile above the Plain; the Distance of the Sub-stile from the Meridian; and the Difference of Longitude.

Suppose an upright Plain in the Latitude $53^{\circ} 15'$ declines from the North, Westward $81^{\circ} 45'$.

1. For the Stile's height.

As Rad. to cs. of the Latitude $53^{\circ} 15'$	9.77694
So cs. of the Plain's Declinat. $81^{\circ} 45'$	9.15683
To s. of the Stile's height	4 56 8.93377

2. For the Sub-stile's Distance from the Meridian.

As Rad. to s. of Declinat. $81^{\circ} 45'$	9.99548
So ct. of the Latitude $53^{\circ} 15'$	9.87317
To t. of $36^{\circ} 28'$ the Deflection	9.86865

3. For the Plain's Difference of Longitude.

As s. of the Latitude $53^{\circ} 15'$	9.90377
To Radius	10.
So t. of Declination $81^{\circ} 45'$	10.83865
To t. of the Diff. of Long. $83^{\circ} 22'$	10.93488
The	

The next thing we should do is to calculate the Hour-Distances: but because the Stile has so little Elevation, the Hours near the Sub-stile will be very near together; wherefore we shall shew a Geometrical way to draw this Dial, and that without having any regard to the Center of the Dial, as followeth.

1. Draw a Right-line AB perpendicular to one Side of your Plain, which must be towards the Right, if the Plain's Declination be South-Easterly, or North-Westerly; but the contrary Declination towards Left-hand: then with 60 deg. of Chords upon A as a Center (which must be towards the top, if the Declination be from the South, but towards the bottom when it declines from the North) describe the Arch CE; and upon it set CD $36^{\circ} 28'$ (the Distance of the Sub-stile from the Meridian) and set the height of the Stile $4^{\circ} 56'$ from D to E, and draw the Lines AD for the Sub-stile, and AE the Stile.

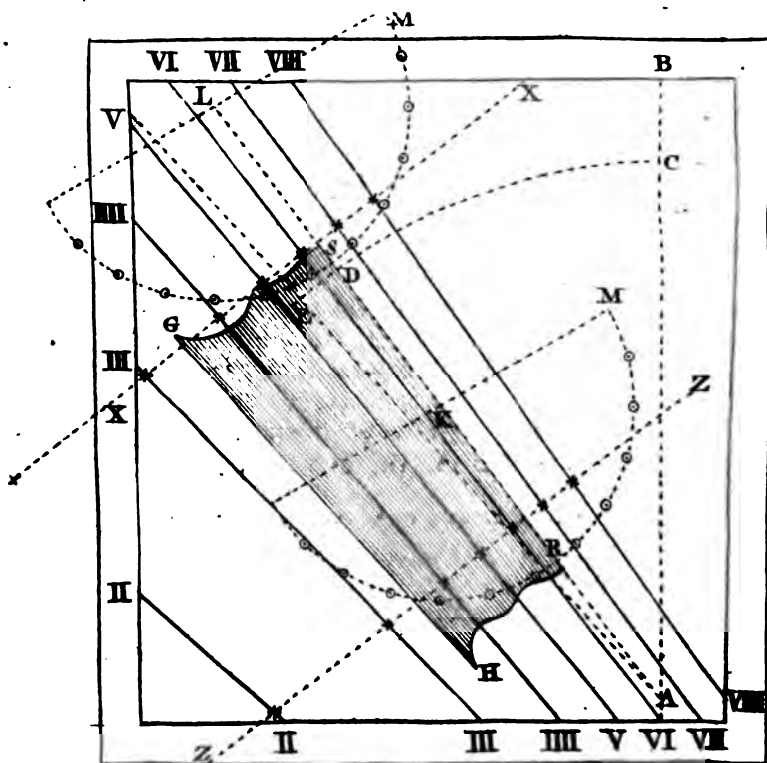
2. Then (because the Stile has but small Elevation) draw another Line GH, parallel to the Line of the Stile AE, at such convenient distance as you think fit; which shall be the augmented Stile.

3. Assume any two Points in the sub-stilar Line AD, at some convenient Distance from each other, as R and S; and thro those two Points draw two Right-Lines, both of them at Right-angles to the sub-stilar Line AD, as the Lines ZZ and XX.

4. From the Point R, with your Compasses take the nearest Distance to the new augmented Stile GH, and set that Distance upon the Sub-stile from R to K. Also from the Point S take the nearest Distance to the new augmented Stile GH, and set that Distance also upon the Sub-stile from S to L.

5. Upon these two Points K and L with 60 deg. of Chords describe two Semicircles, and in each of them set off $83^{\circ} 22'$, the Plain's Difference of Longitude;

gitude; as from R to M, and also from S to M; both of them on the same Side of the sub-stilar Line, on which the first perpendicular Line AB was drawn.



6. Divide each of those Semicircles into 12 equal Parts, beginning at the Point M, with 15 deg. of the Line of Chords, as the Points @ @ @, &c.

7. Lay a Ruler to the Point L, and the respective Points $\odot \odot \odot$, &c. in that Semicircle, and the Ruler will cut the Line XX in the Points $***$, &c. Also lay a Ruler to K, and the several Points $\odot \odot \odot$, &c.

☉, &c. and the Ruler will cut the Line ZZ in the several Points * * *, &c.

Lastly, Lines drawn from the first Point * in the Line ZZ, to the first Point * in the other Line XX, (which the Sub-ſtile will direct you how to do) thoſe Lines ſo drawn ſhall be the true Hour-Lines proper for the Plain, and be at a competent Diſtance one from another, without having any relation at all to the Center.

Now in making of this Dial, you have made four Dials, viz.

A { South declining Weſt
South declining Eaſt } $81^{\circ} 45'$
North declining Weſt
North declining Eaſt }

only by changing the Names of the Hours, and placing the Style on the contrary Side of the Line AB, for the North declining Eaſt; and by turning of the Dial upſide downwards, for the two South decliners; ſo that the Style may point downwards to the South Pole.

S E C T. VI.

How to draw the Hour-Lines upon a direct Eaſt, or Weſt reclining Plain.

1. By Projection.

Suppoſe an Eaſt Plain recline from the Zenith 38 deg. in the Latitude $51^{\circ} 32'$. As in all upright Plains whether direct, or declining, the Meridian of the Place and Hour-Line of 12, is always perpendicular to the Horizon; ſo in all direct Eaſt and Weſt

West reclining, or inclining Plains, the Meridian of the Place, and Hour-Line of 12, is parallel to the Horizon.

To make the Dial.

First, Draw a Right-line NZS representing the Base of your reclining Plain, and Hour-Line of 12. Upon Z describe a Circle, and draw the Diameter WE, to cut the Line NS at Right-angles in Z. Upon Z describe a Circle; and upon the Meridian-Line NS, set off the Pole of the World from Z to P, answerable to the Complement of the Latitude of the Place, and finish your Projection as you have been taught.

Secondly, Because the Plain reclines $38^{\circ} 00'$, take 38 deg. out of half Tangents, and set from Z to O; and now have you three Points N, O, and S, whereby to draw your reclining Plain, the Secant Complement of 38° is the Center; the Circle NOS being drawn, you may find the Pole by setting the half Tangent of the Complement of $38^{\circ} 00'$ from Z to Q, so is Q the Pole of the Plain; then thro P the Pole of the World, and Q the Pole of the Plain, draw an Arch of a great Circle, which will cut the reclining Plain at right-Angles in R. Now part of an Arch of the Circle RP, part of the Meridian NP, and part of the reclining Plain NR, do constitute a spherical Triangle NRP, right-angled at R. And out of this Triangle may all the Requisites belonging to this Plain be found, viz,

1. The height of the Pole above the Plain PR.
2. The Distance of the Sub-style from the Meridian NR.
3. The Plain's Difference of Longitude, the Angle NPR.

First, You must find the Pole of the Meridian of the Plain, which will always be where the Circle of the
Plain

Plain cuts the Equinoctial Circle, as it doth in A; and then,

You may find all these Requisites by the 7th and 8th *Problems* of *Chap. 10. Part 1.*

The Hour-Lines in this are drawn after the same manner as in the former Dials; for lay a Ruler from Q the Pole of the Plain, and the several Points where the Hour-Circles cut the Plain, and make marks upon the primitive Circle; then lay your Ruler to Z and the several marks, and draw the Hour-Lines.

2. *By Trigonometrical Calculation.*

First, Find the Requisites from the Triangle NRP.

1. For the height of the Pole above the Plain PR.

As Rad. to s. of the Latitude	$51^{\circ} 32'$	9.89374
So s. of the Plain's Reclin. PNR	$38^{\circ} 00'$	9.78934

To s. of RP $28^{\circ} 49'$, the height of the Stile, 9.68308

2. For the Sub-stile's Distance from the Meridian NR.

As Rad. to t. of NP	$51^{\circ} 32'$ the Lat.	10.099913
So is cs. of Reclination PNR	$38^{\circ} 00'$	9.896532

To the t. of NR $44^{\circ} 46'$ the Deflection 9.996445

3. To find the Plain's Difference of Longitude NPR.

As s. of the Latitude NP	$51^{\circ} 38'$	9.89374
To Radius		10.
So s. of the Deflection NR	$44^{\circ} 46'$	9.84771

To s. of the Ang. NPR $64^{\circ} 05'$ Diff. Long. 9.95397
E c The

The next thing is to find the Hour-Distances, thus: The Difference of Longitude being $64^{\circ} 05'$, it is four Hours (allowing 15° deg. to an hour) and $4^{\circ} 5'$ remains; wherefore the Sub-File must stand between the Hour-Lines of 4 and 5, between which Hours write Sub-file; and because the remainder is $4^{\circ} 5'$, when four Hours are subtracted, write $4^{\circ} 5'$ above Sub-file in the second Column, against 4 and 8; and if $4^{\circ} 5'$ be subtracted from 15° deg. the remainder will be $10^{\circ} 55'$, which write under Sub-file against 5 and 7: then by a continual addition of 15° deg. to each of these Numbers you will have the Equinoctial Distances, or Angles at the Pole belonging to each Hour, as in the second Column of the Table.

East Reclining		$38^{\circ} 00'$	
Latitude		$51^{\circ} 32'$	
Stile's Height		$28. 49$	
Declination		$44. 46$	
Diff. of Longitude		$64. 05$	
Hours from the Sub- file.	Angles at the Pole.	Hour Dist. from the Sub-file.	
11	1	$79^{\circ} 05'$	$68^{\circ} 11'$
12		$64. 05$	$44. 46$
1	11	$49. 05$	$29. 05$
2	10	$34. 05$	$18. 04$
3	9	$19. 05$	$9. 28$
4	8	$4. 05$	$1. 58$
The	Sub-	file.	
5	7	$10. 55$	$5. 19$
6		$25. 55$	$13. 11$
7	5	$40. 55$	$22. 41$
8	4	$55. 55$	$35. 28$
9	3	$70. 55$	$54. 20$
10	2	$85. 55$	$81. 35$

Then

Chap. 8. *Of Dialling.*

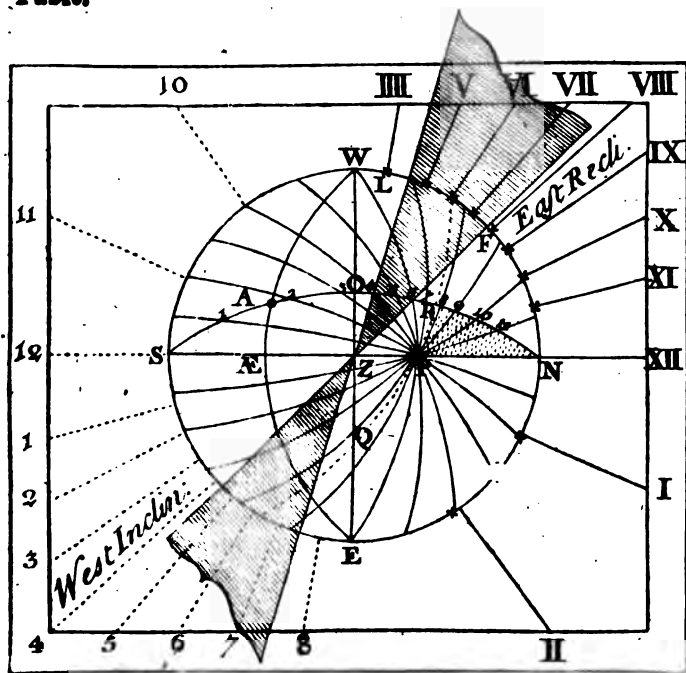
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Then to find the Hour-Distances: The Arch PR with the Hour-Circles, and Hour-Distances upon the Arch of the Plain, make several right-angled Triangles, wherein there is given PR $28^{\circ} 49'$, the height of the Pole above the Plain, and the Angle at the Pole, as in the Triangle 8 R P. the Angle at the Pole is $4^{\circ} 05'$; to find the Hour-Distance 8 R, say,

As Rad. to s. PR $28^{\circ} 49'$ 9.683055
So t. 8 P R $4^{\circ} 05'$, the Angle at the Pole 8.853628

Tot. 8 R 1 58 the Hour-Dist. from } 8.436683
the SS.

After the same manner you may find all the rest of the Hour-Distances, as in the third Column of the Table.



Ec 2

The

The drawing of this Dial by the Numbers in the Table is the same as in the former Dials; for you must set off the Distance of the Sub-*stile* $44^{\circ} 46'$ from N to F, and the height of the *Stile* $28^{\circ} 49'$ from F to L; and draw ZF for the Sub-*stile*, and ZL the *Stile*.

Then set the Hour-distances from F the Sub-*stile* to the several Marks ****, &c.* and lay a Ruler from Z the Center to each of those Points, and draw the Hour-Lines.

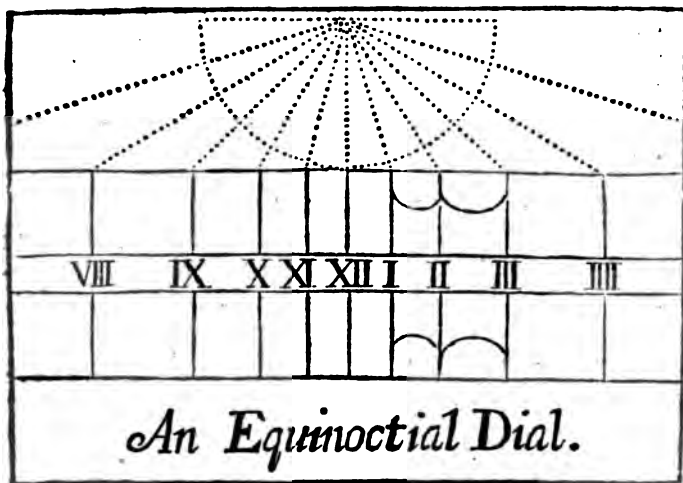
If those Hour-Lines, *Stile* and Sub-*stile* be drawn thro the Center, (as you see the prick'd Hour-Lines are) you'll have a West Incliner; and if Lines be drawn upon the back-side of the Paper answerable to those on the fore-side, you'll have a West Recliner and an East Incliner.

S E C T. VII.

How to draw Hour-Lines upon direct South reclining Plains.

The first Variety, reclining equal to the Pole.

Of these Plains, there are three Varieties: For, first, the South Plain may so recline, that it may fall just into the Pole, and that is when the Reclination of the Plain is equal to the Complement of the Latitude of the Place. Over such a Plain the Pole hath no Elevation, and therefore the Dial has no Center, and all the Hour-Lines must be parallel; and the making of this is the same as the East or West erect Dials were; only, the *Stile* in those Dials stands upon the Hour 6, in this it must stand in the Hour-Line of 12. And the Equinoctial-Line in this is an Horizontal-Line.



The second Variety, South reclining less than the Pole.

Having drawn a Circle, and crossed it with two Diameters, SN for the Meridian, and WE for the prime Vertical Circle, and Hour of six, within this Circle project the Sphere, and finish the Dial as before taught.

Suppose in the Latitude $51^{\circ} 30'$, a Plain reclines from the Zenith 25 deg.

1. By Projection.

Set the half Tangent of 25 deg. the Reclination from Z to R, and draw the reclining Plain thro W, R, and E, and find Q the Pole thereof: The Plain you may see in the Projection passes between P the Pole, and Z the Zenith, therefore the South Pole is

elevated above the Plain; and to find how much, subtract ZR 25° the Reclination from ZP the Complement of the Latitude $38^\circ 28'$, and there remains PR $13^\circ 28'$, the height of the South Pole above the Plain. Draw the Hour-Lines from the Projection, as in former Dials was done.

2. By Trigonometrical Calculation.

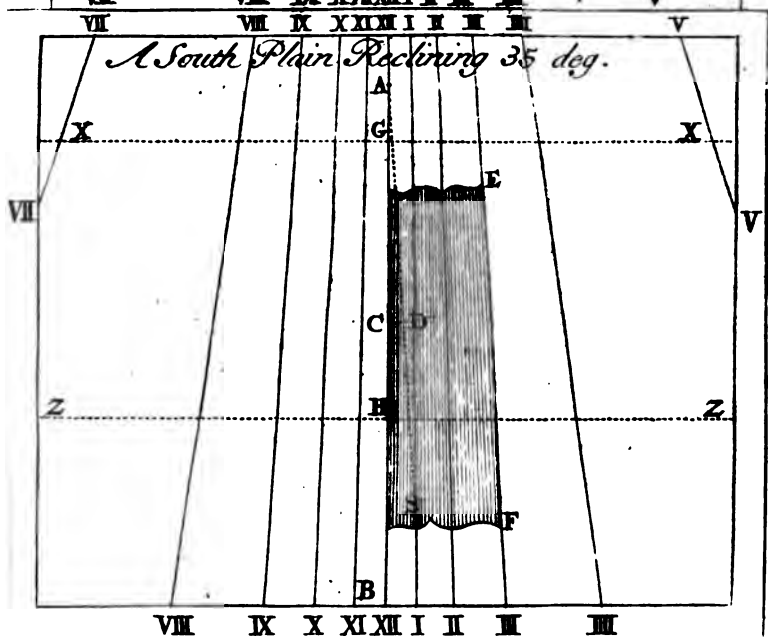
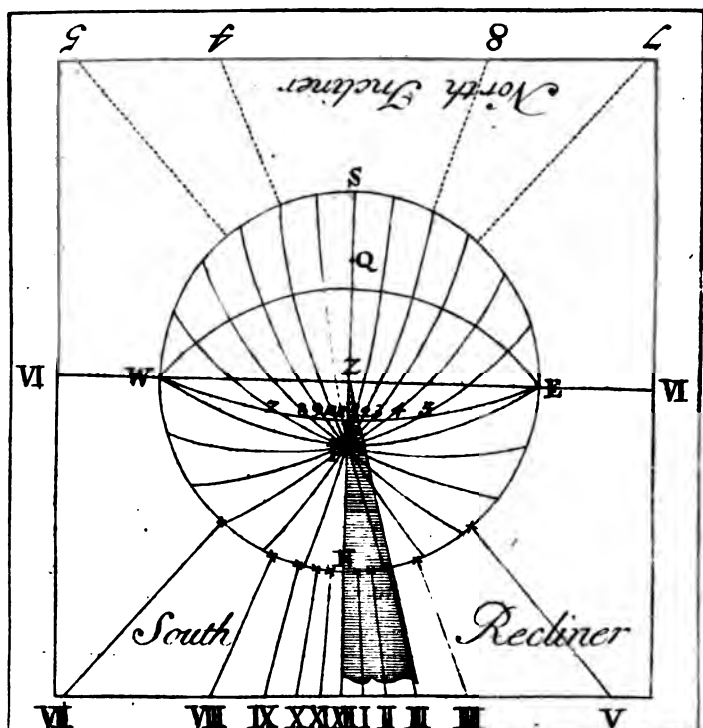
There is nothing besides the Hour-Distances to be found, which are found by the common Proportion used in former Dials, thus,

As Rad. to the s. of PR $13^\circ 28'$, the	5	9.36713
Stile's height		
So ist. of 15° , the Angle at the Pole		9.42805
To t. of the Hour-Distance $3^\circ 34'$		8.79518

And by repeating the Work you may find the rest of the Hour-Distances, as in the third Column of the Table.

Stile's Height $13^\circ 28'$		
Hours from Noon.	Angles at the Pole.	Hour-Distances.
12	$0^\circ 00'$	$0^\circ 00'$
11	$15^\circ 00'$	3 34
10	$30^\circ 00'$	7 39
9	$45^\circ 00'$	13 07
8	$60^\circ 00'$	21 58
7	$75^\circ 00'$	41 00
6	$90^\circ 00'$	90 00

If the Reclination be such that the Pole has but small elevation above the Plain, it will be the best way to draw the Dial without a Center, as is shewed in the 5th Section; or thus, draw the Line AB for the Hour-



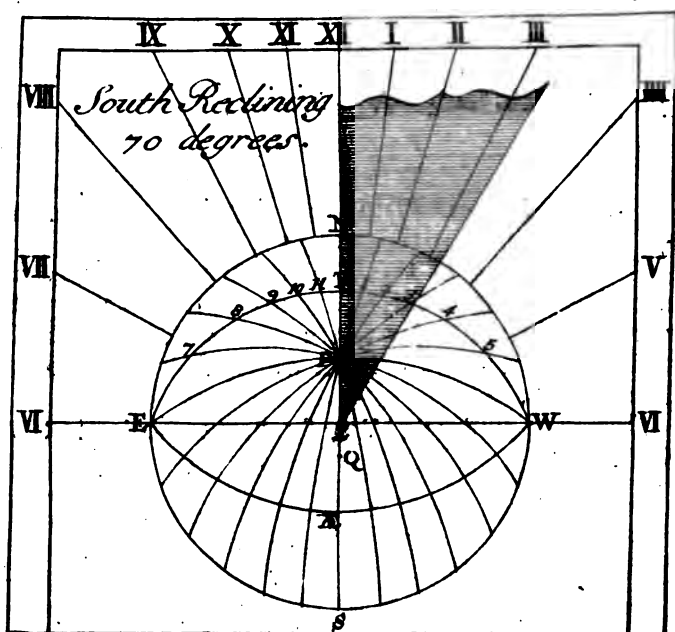
Hour-line of 12; then upon A, with 60 deg. of Chords, describe the Arch CD, and from C set $3^{\circ} 28'$, the height of the Pole above the Plain, and draw AS, and parallel to it draw EF the augmented Stile; then draw the Lines XX and ZZ at right Angles to AB; then take the nearest Distance from G to the augmented Stile, and open the Sector to that Radius, and take out the Tangents of $15^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}$, and 75° , and set them both ways from the point G upon the Line XX; then take the nearest Distance from the point H to the augmented Stile; and open the Sector to that Radius, and take out the same Tangents, and set both ways from the point H upon the Line ZZ; then draw Lines from each point in one Line thro its correspondent point in the other Line, and those Lines shall be the true Hour-Lines.

The third Variety, South reclining more than the Pole.

Having drawn the Projection, and in that the Circle ERW representing the reclining Plain, which you may perceive falls between the Horizon and the Pole; therefore to find how much the Pole is elevated above the Plain, you must subtract ZP, the Complement of the Latitude $38^{\circ} 28'$, from ZR 70 deg. the Reclination of the Plain, and the remainder is PR $31^{\circ} 32'$, the elevation of the Pole above the Plain.

The drawing of this being the very same as the former, I shall say no more about it: Only set down the Table, and the Figure, which will be sufficient to the ingenious.

The Stile's Height $31^{\circ} 32'$					
Hours from Noon.		Angles at the Pole.		Hour-Distances on the Plain.	
	12		$00^{\circ} 00'$		$00^{\circ} 00'$
11	1		$15^{\circ} 00'$		$07^{\circ} 59'$
10	2		$30^{\circ} 00'$		$16^{\circ} 48'$
9	3		$45^{\circ} 00'$		$27^{\circ} 36'$
8	4		$60^{\circ} 00'$		$42^{\circ} 10'$
7	5		$75^{\circ} 00'$		$62^{\circ} 52'$
	6		$90^{\circ} 00'$		$90^{\circ} 00'$



SECT.

S E C T. VIII.

How to draw Hour-Lines upon direct North reclining Plains.

Of these kind of Plains there are three Varieties, as there were of South Recliners: For, (1.) The Reclination may be such that the Plain may just lie in the Equinoctial Circle. Or (2.) It may recline so, that the Plain may fall between the Zenith and the Equinoctial Circle. Or (3.) The Plain may recline so, as to fall between the Equinoctial and the Horizon.

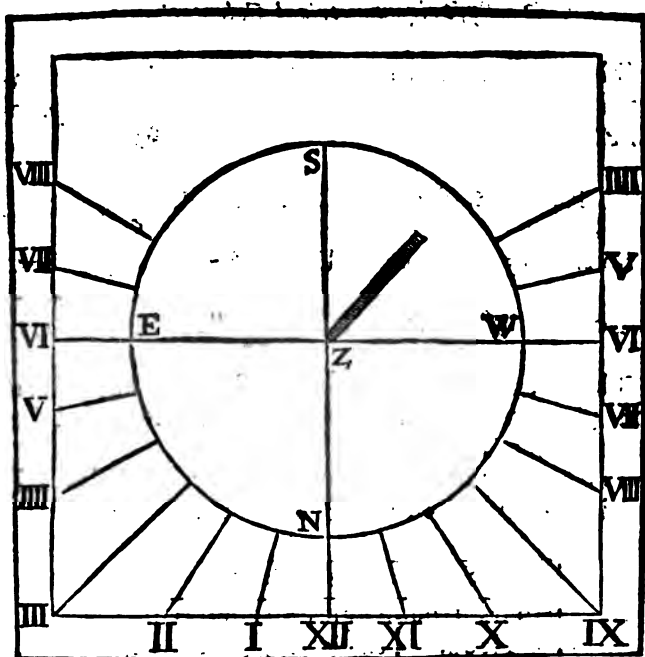
The first Variety; North reclining equal to the Equinoctial.

Suppose a North Plain, reclining from the Zenith $51^{\circ} 32'$, in the Latitude $51^{\circ} 32'$.

The drawing of this Dial is very easy; for you need but describe a Circle, and divide it into 24 equal parts, and lay a Ruler to the Center Z, and each of those parts, and draw Lines which shall be the true Hour-Lines.

The Stile must be a Wire set perpendicular in the Center Z.

The



The second Variety; North reclining less than the Equinoctial.

1. By Projection.

Let there be a North Plain, reclining from the Zenith 25 deg. Having projected the Sphere, and described the Circle ERW representing the reclining Plain, which you may see passes between the Zenith and the Equinoctial; then the height of the Pole above the Plain is the Sum of $PZ\ 38^{\circ} 28'$, the Complement of the Latitude; and $ZR\ 25^{\circ}$ deg. the Reclination, which is $63^{\circ} 28'$, equal to RP.

2. By

2. By Trigonometrical Calculation.

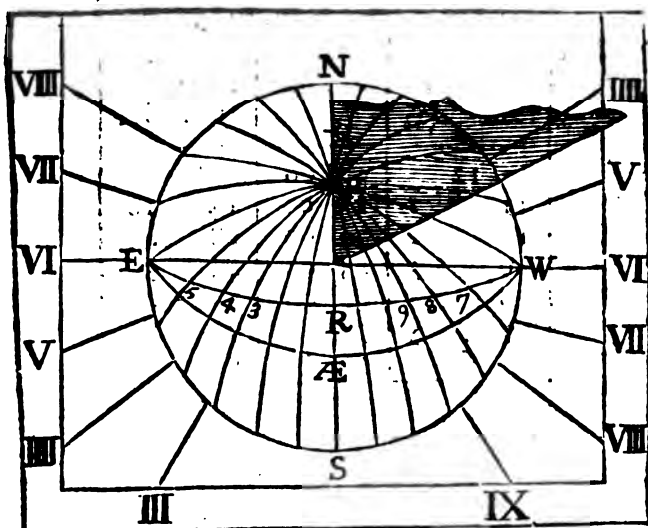
There is nothing to calculate but the Hour-Distances from the Meridian, which are found by the general Proportion, thus.

As Rad. to s. of PR $63^{\circ} 28'$, the Stile's height } 9.951665
 So t. of 15 deg. the Angle at the Pole } 9.428052
 To t. of $13^{\circ} 29'$ the first Hour dist. } 9.379717

The like must be done for all the rest, and so shall you produce such Numbers as are in the third Column of the Table.

The Stile must stand upon the Meridian Line NZ, making an Angle of $63^{\circ} 28'$ therewith.

Stile's Height $63^{\circ} 28'$		
Hours from Noon.	Angles at the Poles.	Hour-Distances on the Plain.
12	$00^{\circ} 00'$	$00^{\circ} 00'$
11	$15^{\circ} 00'$	$13^{\circ} 29'$
10	$30^{\circ} 00'$	$27^{\circ} 19'$
9	$45^{\circ} 00'$	$41^{\circ} 49'$
8	$60^{\circ} 00'$	$57^{\circ} 10'$
7	$75^{\circ} 00'$	$73^{\circ} 20'$
6	$90^{\circ} 00'$	$90^{\circ} 00'$



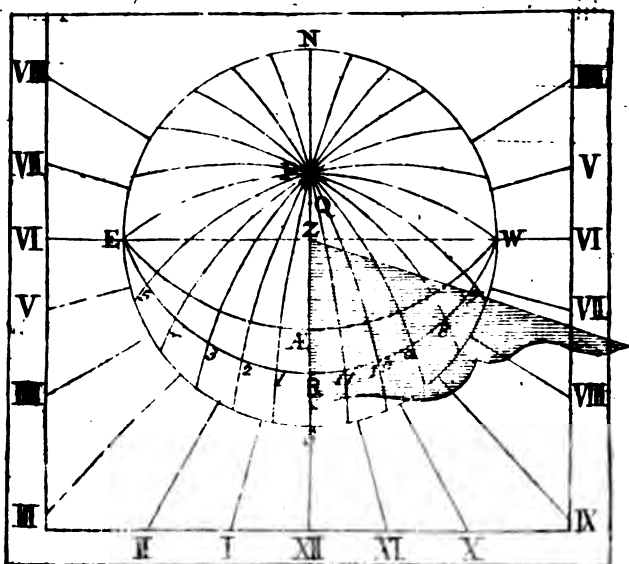
The third Variety ; North reclining more than the Equinoctial.

Suppose a North Plain in the Latitude $51^{\circ} 32'$, should recline from the Zenith 70° .

Having projected the Sphere, and therein drawn the Circle representing the reclining Plain ERW, whose Pole is Q, you may perceive in the Projection that the Plain passes between $\text{\AA}$ the Equinoctial, and S the South part of the Horizon; and to find the height of the Pole above the Plain, we must add $\text{ZP } 38^{\circ} 28'$, the Complement of the Latitude, and ZR the Reclination 70° . together, whose Sum is $108^{\circ} 28'$. But the height of the Pole above the Plain can never exceed 90° . wherefore the Complement of $108^{\circ} 28'$ to 180° is $71^{\circ} 32'$, the true height of the Pole, or Stile above the Plain. The rest of the Work is the same as in the last.

The

The Stile's Height $71^{\circ} 32'$				
Hours from Noon..		Angles at the Poles.	Hour-Distances on the Plain.	
	12	$00^{\circ} 00'$	00°	$00'$
11	1	$15^{\circ} 00'$	14	16
10	2	$30^{\circ} 00'$	28	43
9	3	$45^{\circ} 00'$	43	30
8	4	$60^{\circ} 00'$	58	41
7	5	$75^{\circ} 00'$	74	14
	6	$90^{\circ} 00'$	90	00



S E C T. IX.

How to draw the Hour-Lines upon South declining reclining Plains.

Of South direct reclining Plains, there were three Varieties, so are there as many of South Recliners which decline also; for to any Declination, the Plain may recline so, that it may pass thro the Pole of the World. Or Secondly, It may recline so, that the Plain shall fall above the Pole, and so fall between the Zenith and the Pole. Also, Thirdly, It may so recline, that the Plain shall pass between the Pole and the Horizon. Of all which Varieties I shall give a particular Example.

The first Variety ; Of South declining reclining Plains. The Plain passing thro the Pole.

1. By Projection.

An Example shall be of a South Plain declining 35 deg. Eastward, and reclining from the Zenith $33^{\circ} 03'$.

First, Draw a right Line AB, representing the Basis of your reclining Plain, and cross it with another Line CD, at right Angles in the point Z.

Secondly, Upon Z describe a Circle, and in it from C towards E (because the Plain declines Eastward) set 35 deg. the Declination of the Plain to N, and draw the Line NZS for the Meridian of the Place, upon which from Z to P, set the half Tangent $38^{\circ} 28'$, the Complement of the Latitude, and finish the Projection.

Thirdly,

Thirdly, Take the half Tangent of the Reclination of the Plain $33^{\circ} 03'$, and set it from Z to R upon the Line CD, so have you three Points A, R, and B, by which to draw your Plain; and set the Secant $56^{\circ} 57'$ (the Complement of Reclination) from R upon the Line CD, and it will give the Center; and the half Tangent of $56^{\circ} 57'$ set from Z will give you the point Q, the Pole of the Plain; thro Q the Pole of the Plain, and P the Pole of the World, draw the Arch of a Circle (by *Problem 4. Chap. 10. Part 1.*) And now seeing the Plain passes just thro the Pole of the World, therefore the Pole hath no Elevation above it, and consequently the Hour-Lines must be parallel one to the other. But before you can draw them, two things must be found.

1. The Distance of the Meridian from the Horizon PB.
2. The Plain's Difference of Longitude, the Angle ZPQ.

Both which by the Projection is thus found:

A Ruler laid from Q to P, will cut the Circle in *a*; and the distance *a* B measured upon the Chords will give $70^{\circ} 10'$, for the Distance of the Meridian from the Horizon; and the Arch S *b* measured upon the Chords will be $28^{\circ} 45'$, for the Plain's Difference of Longitude.

These things being found, the Dial must be drawn Geometrically, not much differing from the direct East and West, and Equinoctial Dials.

2. By Trigonometrical Calculation.

1. To find the Distance of the Meridian from the Horizon PB. In the right-angled spherical Triangle PRZ right-angled at R, there is given, ZP $38^{\circ} 28'$ the Complement of the Latitude, and the Angle RZP

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RZP 35 deg. the Plain's Declination, and if you will RZ 33° 03' the Plain's Reclinations; to find RP, the Complement of the Meridian's Distance from the Horizon. Thus,

As Rad. to s. PZ 38° 28'	9.79383
So s. RZP 35 deg. the Plain's Declinat.	9.75859
To s. RP 20° 54'	9.55242

The Complement whereof 69° 06' is the Arch PB, the Distance of the Meridian from the Horizon.

2. For the Plain's Difference of Longitude ZPQ.

As s. of PZ 38° 28' the Compl. of Lat.	9.79383
Is to Radius	10.
So s. of ZR 33° 03' the Plain's Reclinat.	9.73669
To s. of ZPR 61° 15'	9.94286

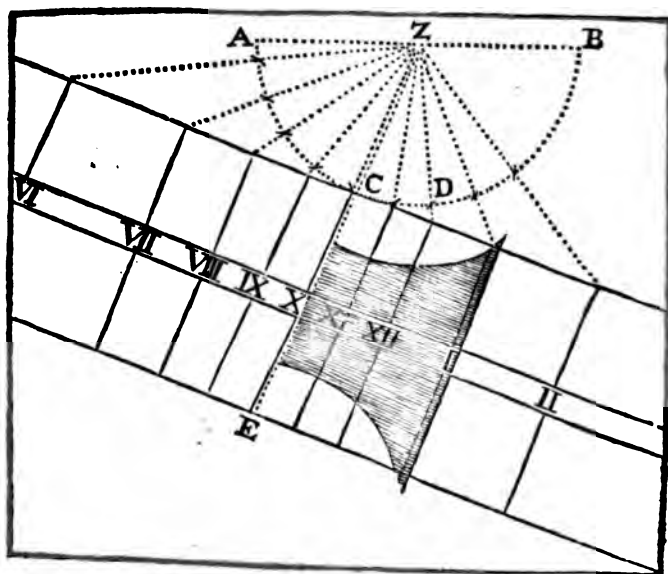
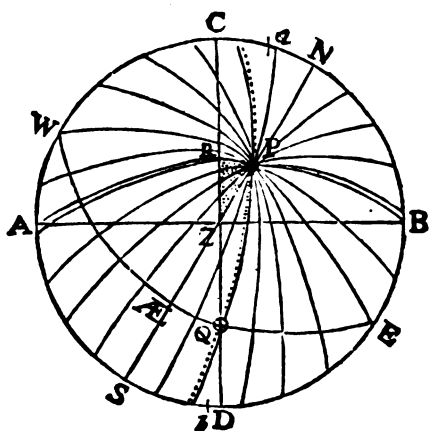
Whose Complement 28° 45' is the Angle ZPQ, the Plain's Difference of Longitude.

The Geometrical Projection of this Dial.

First, Draw a Line at pleasure, as AZB, representing the Horizontal Line of the Plain. Then considering what length you would have your Stile to be, answerable to the bigness of your Plain; suppose ZD, take that Distance in your Compasses, and setting one foot in Z, with the other describe the Semicircle ACDB, upon which, with a Line of Chords, set 69° 06', the Distance of the Meridian from the Horizon, from A to C, and draw the Line ZCE for the Sub-stile.

Secondly, Take 28° 45', the Plain's Difference of Longitude, and set from C to D, and thro the Point
F f C

C draw the Tangent Line FG at right-angles to the Sub-ſtile.



Thirdly,

Thirdly, With 15 deg. of Chords divide the Semicircle ADB into 12 equal parts; and lay a Ruler to Z and each of those Points, and make Marks by the edge of the Ruler upon the Tangent Line, and thro those Marks draw Lines parallel to the Sub-stile, which shall be the true Hour-Lines.

The second Variety : Of South declining reclining Plains; the Plain passing between the Pole and the Zenith.

1. *By Projection.*

Let us suppose a Plain to decline from the South Eastward 35 deg. and to recline from the Zenith $18^{\circ} 30'$.

First, Draw a Line AB, for the Base of the reclining Plain, and another CD, perpendicular thereunto, for the Vertical Line of the Plain, crossing one another at Right-angles in Z.

Secondly, Upon Z describe a Circle, and set 35 deg. the Declination from A to E, and from C to S, and draw EW for the prime Vertical Circle, and SZN for the Meridian of the Place, and finish the Projection as before taught.

Thirdly, Out of the Scale of half Tangents take $18^{\circ} 30'$, the Reclination of the Plain, and set it from Z to H, upon the Line CD; and set the Secant of $71^{\circ} 30'$, the Compl. of Reclination downwards from H, and it will give the Center whereby to describe the reclining Plain AHB; and the half Tangent of $71^{\circ} 30'$ will reach from Z to Q the Pole of the Plain; then thro Q the Pole of the Plain, and P the Pole of the World, draw the Arch of a great Circle QRP, by *Prob. 4. Chap. 10. Part 1.*

The next thing is to find the Requisites belonging to the Plain, which in all declining reclining Plains (excepting such as pass thro the Pole of the World, or by the intersection of the Meridian and Equinoctial) are Four, viz.

1. The Dist. of the Mer. and Horizon	} represent- ed by	OB	77° 28'
2. The Height of the Pole or Stile above the Plain		PR	13 24
3. The Distance of the Sub-stile from the Mer.		OR	9 00
4. The Plain's Diff. of Longitude		OPR	34 02

And all these may be found by the Projection, by *Prob. 7, 8. Chap. 10. Part I.*

Having found the Requisites, the Hour-Lines are drawn after the same manner as in other Plains.

2. By Trigonometrical Calculation.

By the Intersection of the Circles of the Sphere with the great Circles of the reclining Plain, there are constituted several spherical Triangles; but especially two: By the resolving of which all the foremention'd Requisites may be found: The two Triangles are HOZ, right-angled at H, and OPR right-angled at R.

1. For the Distance of the Meridian from the Horizon BO.

This will be found in the right-angled Triangle HOZ, in which is given HZ $18^{\circ} 30'$, the Plain's Reclination; and the Angle HZQ 35° deg. the Plain's Declination; to find the Side HO.

As

As Rad. to s. ZH $18^{\circ} 30'$, the Reclination 9.50148
 So t. of HZO $35^{\circ} 00'$, the Declination 9.84523

Tot. of HO $12^{\circ} 32'$ 9.34671

Whose Complement $77^{\circ} 28'$, is the Distance of the Meridian from the Horizon BO.

2. For the height of the Pole or Stile above the Plain PR.

This must be found in the Triangle POR, but in it there is not yet enough given; wherefore ZO must be first found in the former Triangle HOZ, thus.

Ass. of HZO $35^{\circ} 00'$, the Declinat. 9.75859
 To s. of HO $12^{\circ} 32'$ 9.33647
 So s. of ZHO 90° 10.

To s. of OZ $22^{\circ} 14'$ 9.57788

Which subtracted from ZP $38^{\circ} 28'$, there remains $16^{\circ} 14'$ for the Side OP, in the other Triangle OPR: And now seeing the two Triangles have the same Angles at the Bases, *viz.* the Angle ZOH, and ROP, and therefore are similar, and the Sines of the Hypothenufes and Perpendiculars are proportional (by *Axiom 1. Chap. 7. Part 1.*) you may work with both Triangles together to find PR, thus.

As s. of ZO $22^{\circ} 14'$ Hypoth.	} in Tri. HOZ	{	0.42212
To s. of ZH $18^{\circ} 30'$ Perpend.			9.50148
So s. of PO $16^{\circ} 14'$ Hypoth.	} in Tri. POR	{	9.44646
To s. of PR $13^{\circ} 34'$ Perpend.			9.37006

3. For the Distance of the Sub-stile from the Meridian OR.

In the Triangle POR, there is given the Side PO $16^{\circ} 14'$, and the Side PR $13^{\circ} 34'$; to find OR.

As cs. of PR $13^{\circ} 34'$	9.98771
To Radius	10.
So cs. of PO $16^{\circ} 14'$	9.98233

To cs. of OR $9^{\circ} 00'$, the Dist. of the	} 9.99462
Sub-stile from the Merid.	

4. For the Plain's Difference of Longitude OPR.

As s. of PO $16^{\circ} 14'$	9.44646
To s. of PRO 90°	10.
So s. of OR $9^{\circ} 00'$	9.19433

To s. of the Angle OPR $34^{\circ} 02'$	9.74787
---	---------

Which is the Plain's Difference of Longitude.

The next thing you seek is the Hour-Distances upon the Plain.

The Plain's Difference of Longitude being $34^{\circ} 02'$, which is two Hours, and $4^{\circ} 02'$ remains; wherefore the Plain declining towards the East, the Sub-stile falls between the Hours of 9 and 10 in the Forenoon: wherefore write Sub-stile in the Table between the Hours of 9 and 10; and below the Sub-stile against the Hour 10, write $4^{\circ} 02'$ in the second Column; subtract $4^{\circ} 02'$ from 15° , and there remains $10^{\circ} 58'$, which write above the Sub-stile against the Hour 9: then by a continual addition of 15° , you will produce the Numbers in the second Column of the Table, which are the Equinoctial Distances or Angles at the Pole.

N. B.

N. B. Observe in making up the Numbers in the second Column, that the Number which falls against the Hour 12, is always the same as the Plain's Difference of Longitude, if you have committed no Error.

Then by the general Proportion.

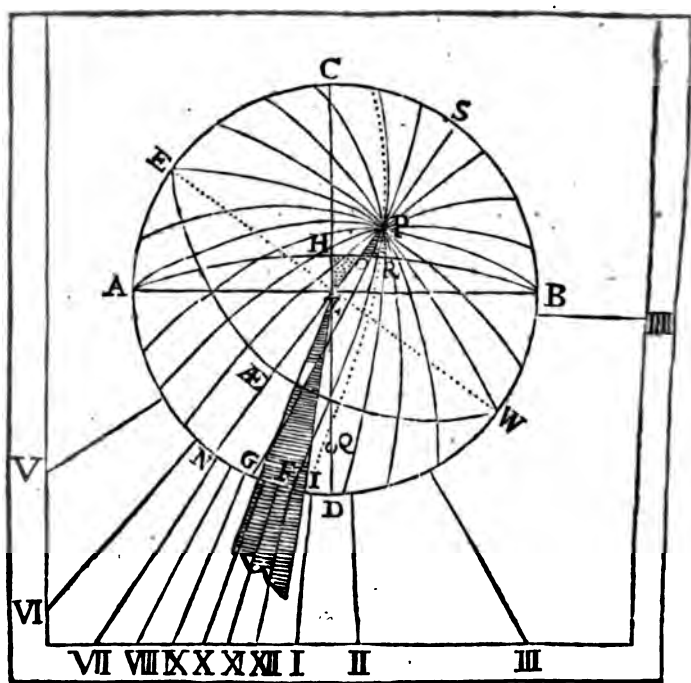
As Rad. to s. of PR $13^{\circ} 34'$, the Stile's } height } 9.37006

So t. of $4^{\circ} 02'$, the Angle at the Pole 8.84826

To t. of the Hour-Distance $0^{\circ} 57'$	8.21832
--	---------

Latitude	51° 32'
Declination	35 00
Reclination	18 30
Dist. Merid. and Hor.	77 28
Stile's Height	13 34
Deflection	9 00
Diff. of Longitude	34 02

Hours from Noon.	Angles at the Pole.	Hour-Distances.
4 8	85° 58'	73° 16'
5 7	70 58	34 12
6	55 58	19 09
7 5	40 58	11 30
8 4	25 58	6 31
9 3	10 58	2 36
The	Sub-	stile.
10 2	4 02	0 57
11 1	19 02	4 37
12	34 02	9 00
1 11	49 02	15 07
2 10	64 02	25 42
3 9	79 02	50 26



Set $77^{\circ} 28'$, the Distance of the Meridian and Horizon, from A to F (because the Plain declines Eastward) and draw ZF the Hour-Line of 12; then set $9^{\circ} 00'$ from F to G, and draw ZG the Sub-stile; and set $13^{\circ} 34'$ from G to I, and draw ZI the Stile; then set the Hour-Distances both ways from the Sub-stile.

If the Plain reclines so that the Pole shall have but small Elevation above the Plain, it will be the best way to draw it without a Center.

Suppose a Plain declines 35 deg. Eastward, and reclines from the Zenith $30^{\circ} 00'$.

First, find the Requisites, as in the last Example.

As

As Rad. to s. ZH $30^{\circ} 00'$, the Reclinat. 9.69897
 So t. of HZO $35^{\circ} 00'$, the Declinat. 9.84523

To t. of HO $19^{\circ} 18'$ 9.54420

Whose Complement $70^{\circ} 42'$ is the Arch BO, the Distance of the Meridian from the Horizon.

Secondly, For the height of the Stile, PR.
 But first find ZO, thus.

As s. HZO $35^{\circ} 00'$, the Declinat. 9.75859
 To s. HO $19^{\circ} 18'$ 9.51919
 So is Radius 10.

To s. of OZ $35^{\circ} 11'$ 9.76060

Subtract $35^{\circ} 11'$ from $38^{\circ} 28'$, and there remains $3^{\circ} 21'$: then,

As s. OZ $35^{\circ} 11'$ 9.23940
 To s. ZH $30^{\circ} 00'$ 9.69897
 So s. OP $3^{\circ} 21'$ 8.76667

To s. PR $2^{\circ} 54'$, the Stile's height 8.70504

Thirdly, For the Deflection OR.

As cs. PR $2^{\circ} 54'$ 9.999443
 To Radius 10.
 So cs. PO $3^{\circ} 21'$ 9.999257

To cs. OR $1^{\circ} 41'$, the Deflection 9.999814

For the Plain's Difference of Longitude OPR.

As s. PO $3^{\circ} 21'$

8.766675

To Radius

10.

So s. RO $1^{\circ} 41'$

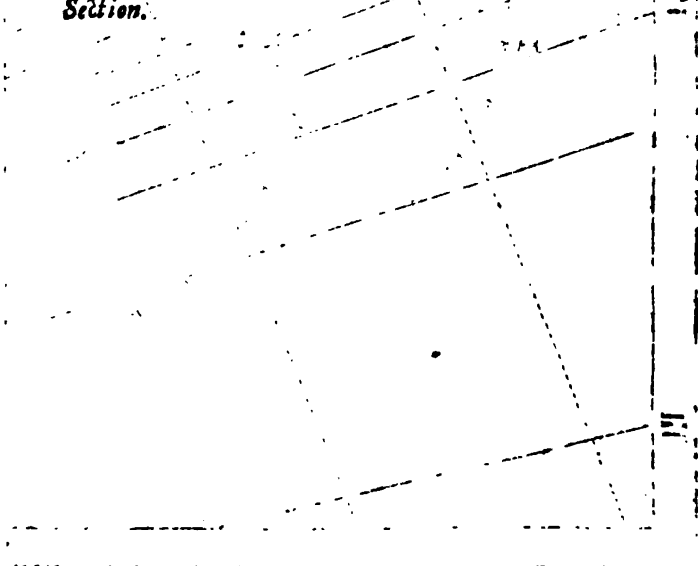
8.467985

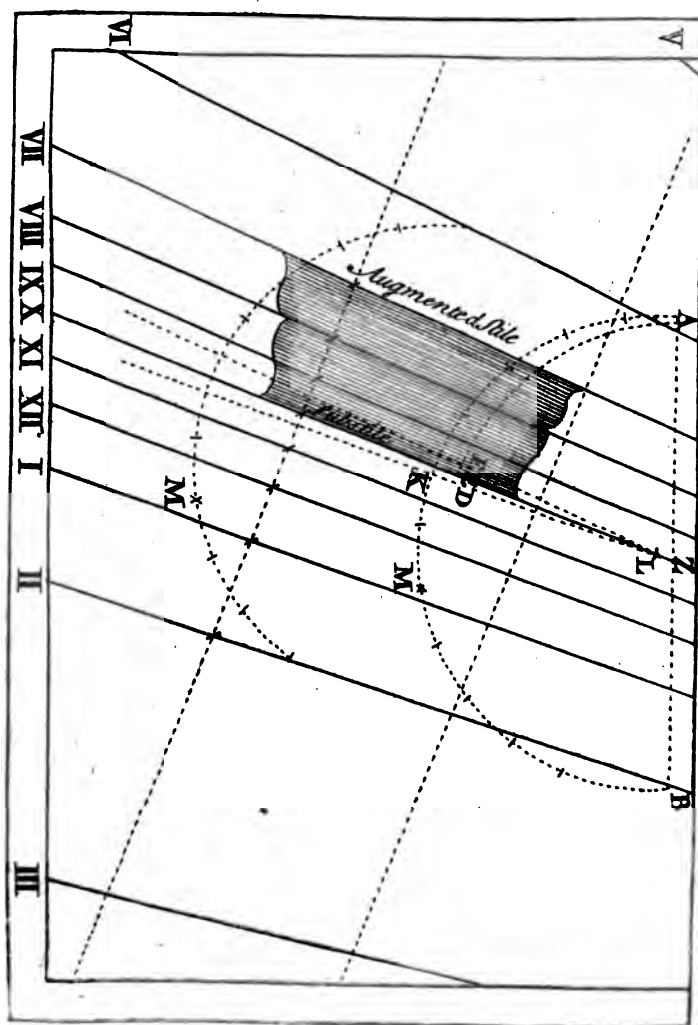
Tos. OPR $30^{\circ} 11'$, the Plain's Diff. of
Longitude

9.701310

Having found the Requisites, we shall proceed to draw the Dial.

And First, Draw AB parallel to the Horizon, and upon Z describe the Arch AD. Set $70^{\circ} 42'$, the Distance of the Meridian from the Horizon, from A to D, and draw ZD for the Meridian of the Plain; and set $1^{\circ} 41'$, the Distance of the Sub-stile from the Meridian, from D to E, and draw ZE the Sub-stile; then set $2^{\circ} 54'$, the Stile's height, from E to F, and draw ZF the Stile; finish it as is shewed in the 5th Section.





The third Variety, of South declining reclining Plains: The Plain passing between the Pole and the Horizon.

1. *By Projection.*

We will suppose a South Plain to decline from the South Westward $35^{\circ} 00'$, and reclining from the Zenith $53^{\circ} 30'$.

First, Draw a Right-line AB for the Base of the Plain, and cross it with another Line CD, at Right-angles in Z, for the Vertical Line of the Plain.

Secondly, Upon Z describe a Circle, and upon the Periphery thereof set $35^{\circ} 00'$, the Declination from C to N, and draw the Line NZS for the Meridian of the Place, projecting the Sphere within the Circle.

Thirdly, Take $53^{\circ} 30'$, the Reclination, from the Scale of half Tangents, and set from Z to F, and set the Complement thereof from Z to Q the Pole of the Plain; and the Secant Complement of Reclination set from F downward, will give the Center of the Circle of the Plain, which draw thro the three Points A, F, and B.

Fourthly, Thro P the Pole of the World, and Q the Pole of the Plain, draw the Arch of the great Circle RPQ, and find the Requisites, *viz.*

1. The Dist. of the Merid. from the Horizon,	} represent- ed by	OB
2. The height of the Pole or Stile above the Plain,		RP
3. The Dist. of the Sub-stile from the Meridian,		RO
4. The Plain's Diff. of Longitude RPO, or		EPQ.
		ALL

All which may be found by the Projection, by
Prob. 7, 8. Chap. 10.

2. By Trigonometrical Calculation.

The Triangles wherein the Requisites are to be found, are the Triangles OFZ, ROP, and ONB.

1. For the Distance of the Meridian from the Horizon BO. In the Triangle OFZ, you have given the Side FZ, the Reclination $53^{\circ} 30'$, and the Angle FZO, the Plain's Declination; to find FO.

As Rad. to s. FZ $53^{\circ} 30'$, the Reclinat.	9.90518
So t. of FZO $35^{\circ} 00'$, the Declination	9.84523
To t. FO $29^{\circ} 22'$	9.75041

Whose Compl. $60^{\circ} 38'$ is BO, the Distance of the Meridian from the Horizon.

2. For the height of the Pole above the Plain PR.

This should be found in the Triangle ROP, but there is not enough given; wherefore in the Triangle ONB, you must seek the Side NO, thereby to gain the Side OP. Thus,

As Rad. to the Sine of BO $60^{\circ} 38'$	9.94027
So s. of NBO $36^{\circ} 30'$, the Compl. of Reclin.	9.77439
To s. of NO $31^{\circ} 13'$	9.71466

This NO $31^{\circ} 13'$ taken out of NP $51^{\circ} 32'$, leaves OP $20^{\circ} 19'$. And then because the Hypothenuses and Perpendiculars of the Triangles ONB and RPO are proportional, it will be,

As s. of BO 60° 38' Hypoth. γ in Triang.	$\{ 0.059733$
To s. of NB 55 00 Perpend. $\S$ NOB.	$\{ 9.913364$
So s. of PO 20 19 Hypoth. γ in Triang.	$\{ 9.540931$
To s. of RP 19 04 Perpend. $\S$ ROP.	$\{ 9.514028$

3. For the Distance of the Sub-stile from the Meridian RO.

As the cs. of RP 19° 04'	9.975496
To Radius	10.
So cs. PO 20° 19'	9.972105
To cs. RO 7' 09°, the Deflection	<u>9.996609</u>

4. For the Plain's Diff. of Longitude RPO.

As s. PO 20° 19'	9.54059
To Radius	10.
So s. RO 7° 09'	9.09506
To s. RPO 21° 00', the Plain's Diff. of Longitude,	<u>9.55447</u>

This 21° 00' is but one Hour and 6° 00' of the Equinoctial over, which shews that the Sub-stile (because the Plain declines Westward) will fall between the Hours of 1 and 2; wherefore in the Table between the Hours of 1 and 2 write Sub-stile, and under it against 11 and 1 write 6 deg. in the second Column; and subtract 6 deg. from 15 deg. and there remains 9 deg. so write 9 deg. above Sub-stile in the second Column; then by a continual addition of 15 deg. you will have the rest of the Numbers in the second Column of the Table.

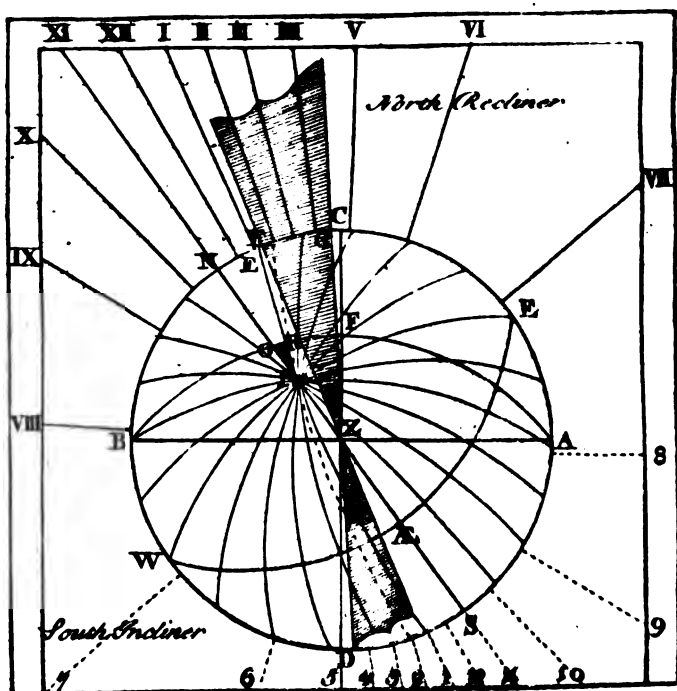
Then for the Hour-Distances upon the Plain, they are found by the usual Canon, *viz*,

As Radius to the s. of RP $19^{\circ} 04'$, the }
 Stile's height, } 9.514028
 So is t. $6^{\circ} 00'$, the Angle at the Pole 9.021620

Tot. of $1^{\circ} 58'$, the Dist. of the Hour 1, 8.535648

And by repeating the same Proportion, you will find the rest of the Hour-Distances in the second Column of the Table.

Latitude		$51^{\circ} 32'$	
Declination		$35^{\circ} 00'$	
Reclination		$53^{\circ} 30'$	
Dist. Mer. and Hor.		$60^{\circ} 38'$	
Stile's height		$19^{\circ} 04'$	
Deflection		$07^{\circ} 09'$	
Diff. of Longitude		$21^{\circ} 00'$	
Hours from Noon.		Angles at the Pole.	Hour-Distances.
5	7	$84^{\circ} 00'$	$72^{\circ} 10'$
6		$69^{\circ} 00'$	$40^{\circ} 23'$
7	5	$54^{\circ} 00'$	$24^{\circ} 12'$
8	4	$39^{\circ} 00'$	$14^{\circ} 49'$
9	3	$24^{\circ} 00'$	$8^{\circ} 16'$
10	2	$9^{\circ} 00'$	$2^{\circ} 58'$
The		Sub-	stile.
11	1	$6^{\circ} 00'$	$1^{\circ} 58'$
12		$21^{\circ} 00'$	$7^{\circ} 09'$
1	11	$36^{\circ} 00'$	$13^{\circ} 21'$
2	10	$51^{\circ} 00'$	$21^{\circ} 58'$
3	9	$66^{\circ} 00'$	$36^{\circ} 16'$
4	8	$81^{\circ} 00'$	$64^{\circ} 08'$



Set $60^{\circ} 38'$, the Distance of the Meridian and Horizon, from B to E, and there draw the Hour-Line 12; and set $7^{\circ} 9'$, the Distance of the Sub-stile from the Meridian, from E to F, and draw ZF the Sub-stile; and set $19^{\circ} 04'$, the Stile's height, from F to G, and draw ZG the Stile: Then set the Hour-Distances from F the Sub-stile, and draw the Hour-Lines, and finish the Dial.

S E C T.

S E C T. X.

How to draw Hour-Lines upon North declining reclining Plains.

As there were three Varieties of South reclining Plains which decline, so are there as many in North Decliners reclining; for to any Declination a Reclination may be fitted, that the Plain shall pass by the Intersection of the Meridian with the Equinoctial Circle. Or the Reclination may be such, that the Plain shall pass between the Zenith and the Equinoctial. Or it may recline so, that the Plain shall pass between the Equinoctial and the Horizon.

The first Variety: Of North declining reclining Plains, the Plain passing thro the Intersection of the Meridian with the Equinoctial.

1. *By the Projection.*

Our first Example shall be of a North Plain declining Eastward 55 deg. and reclining from the Zenith 35° 50'.

First, Draw a Line AB, representing the Base of your reclining Plain, and another at Right-angles thereto, as CD, the Vertical Line of the Plain, crossing each other in Z. Upon Z describe a Circle, and upon the Periphery thereof set 55° 00', the Plain's Declination from C to N Eastward, because the Plain declines Eastward; draw the Line NS the Meridian of the Place, and finish the Projection.

G g

Secondly,

Secondly, Set the half Tangent of $35^{\circ} 50'$, the Reclination from Z to G, and the half Tangent of its Complement from Z to Q the Pole; and let the Secant of the Complement from G upon the Line CD, for the Center; and draw the Plain AGB.

Thirdly, Thro P the Pole of the World, and Q the Pole of the Plain, draw the Arch of a great Circle, which you will find to be the same as the Hour-Circle of six; and therefore the Hour-Line of six will be the Substilar-Line of the Dial.

The Requisites belonging to this Plain are Four;
viz.

1. The Distance of the Sub-stile from the Meridian, BÆ.
2. The height of the Stile above the Plain, PR.
3. The Distance of the Sub-stile from the Meridian, ÆR.
4. The Plain's Difference of Longitude, QPÆ.

All these may be found by the Projection, or

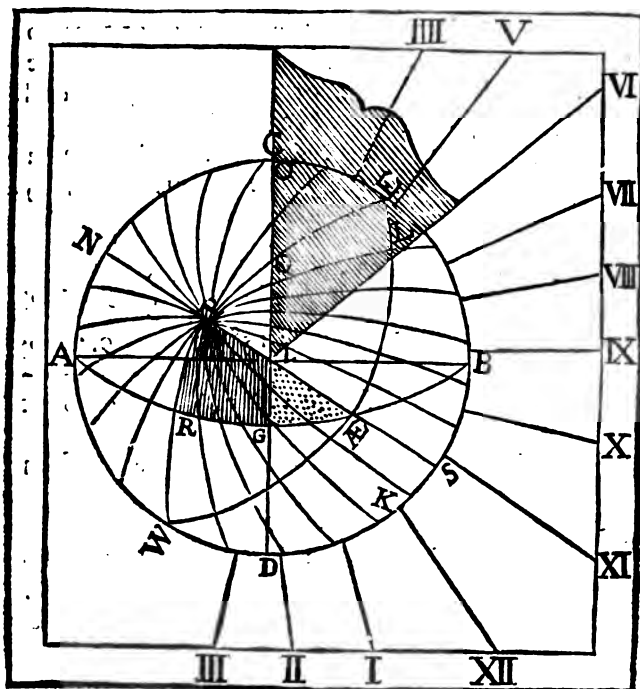
2. *By Trigonometrical Calculation.*

1. For the Distance of the Meridian from the Horizon, AÆ.

In the Triangle ZGÆ, right-angled at G, there is given ZG $35^{\circ} 50'$, the Reclination, and the Angle ÆZG $55^{\circ} 00'$, the Declination; to find ÆG, thus.

Lati-

Latitude	51° 32'	
Declination N.	55 00	
Reclination	35 50	
Dist. Mer. and Hor.	50 06	
Stile's height	48 23	
Deflection	90 00	
Diff. of Longitude	90 00	
Hours from Noon.	Angles at the Pole.	Hour-Dist. from 6.
6	00° 00'	00° 00'
7 5	15 00	11 20
8 4	30 00	23 21
9 3	45 00	36 47
10 2	60 00	52 19
11 1	75 00	70 17
12	90 00	90 00



As Rad. to s. of ZG $35^{\circ} 50'$, the Reclin.	9.767475
So t. of $\angle$ ZG $55^{\circ} 00'$, Declination	10.154773
To t. of $\angle$ EG $39^{\circ} 54'$	9.922248

Whose Complement $50^{\circ} 06'$ is the Distance of the Meridian from the Horizon.

2. For the height of the Pole above the Plain, PR.

In the Quadrantal Triangle PÆR.

As s. of $\angle$ EG $39^{\circ} 54'$	9.807163
To Radius $\angle$ R	10.
So t. of ZG $35^{\circ} 50'$	9.858602
To. t. of PR $48^{\circ} 23'$, the Stile's height	10.051439

The other two Requisites, the Distance of the Substile from the Meridian RÆ, and the Plain's Difference of Longitude QPÆ, or ÆPR, they are each 90 deg. as is evident in the Projection.

And for the Hour-Distances, they are calculated as for an Horizontal Plain, by the general Analogy. Thus,

As Rad. to s. of PR $48^{\circ} 23'$, the Stile's height,	} 9.87367
So t. of the Angle at the Pole $15^{\circ} 00'$	
To t. of the Hour-Distance $11^{\circ} 20'$	9.30172

Which is the Distance of the Hours 5 and 7 from the Substile 6.

To draw the Hour-Lines: First, Set $50^{\circ} 06'$, the Distance of the Meridian from the Horizon, from B to

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to K, and there draw (from Z thro K) the Hour 12; then set 90 deg. from K to L, and draw the Hour-Line of 6, (which is also the Sub-stile.) And from L set off the Hour-Distances, and draw the rest of the Hour-Lines: Then set $48^{\circ} 23'$, the Stile's height, from L to O, and draw ZO the Stile, and it is finished.

The second Variety: Of North declining reclining Plains, the Plain passing between the Zenith and the Equinoctial.

Let an Example be of a North Plain declining Westward $55^{\circ} 00'$, and reclining from the Zenith $20^{\circ} 30'$.

First, Draw a Right-line AB, for the Base or Horizontal Line of your reclining Plain, and at Right-angles thereunto another, CD, for the Vertical Line of the Plain, crossing in Z. Upon Z describe a Circle, and upon it from C to N, set the Plain's Declination 55 deg. 00 min. draw the Line NS for the Meridian of the Place, and the Complement of Latitude from Z to P, and the Latitude from Z to $\text{\AA}$: Then set the half Tangent of 20 deg. 30 min. the Reclination from Z to H, and the half Tangent of the Complement from Z to Q the Pole of the Plain; the Secant Complement set from H, will find the Center to draw the Plain AHB. Then thro P the Pole of the World, and Q the Pole of the Plain, draw the Arch of a great Circle; and find the four Requisites, viz.

1. The Distance of the Meridian from the Horizon, AO.
2. The height of the Pole or Stile, above the Plain, PR.

3. The Distance of the Sub-stile from the Meridian, RO.
4. The Plain's Difference of Longitude, OPR.

All which are found by the Projection, or

2. *By Trigonometrical Calculation*, thus.

1. For the Distance of the Meridian and Horizon, AO.

In the right-angled spherical Triangle ZHO, right-angled at H, there is given the Side ZH 20 deg. 30 min. and the Angle HZO 55 deg. 00 min. the Declination; to find the Side AO.

As Rad. to s. ZH 20° 30', the Reclinat.	9.54432
So t. of HZO 55° 00', the Declination	10.15477

To t. of HO 26° 34'	9.69909
---------------------	---------

Whose Complement 63 deg. 26 min. is the Distance of the Meridian from the Horizon, AO.

2. For the height of the Pole above the Plain, PR.

This must be found in the Triangle PRO, in which there is not yet enough given; wherefore you must get the Side PO, thus.

As s. of HZO 55° 00' the Declination	9.91336
To s. of HO 26° 34'	9.65054
So is Radius	10.

To s. ZO 33° 05'	9.73718
------------------	---------

To

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Of Dialling.

455

To ZO 33 deg. 05 min. add ZP 38 deg. 28 min.
the Sum will be 71 deg. 33 min. fo have you the
whole Side PO.

Now the two Triangles HOZ, and ROP being
proportional, the Side PR may be thus found.

Ass. of OZ 33° 05' Hypoth.	in Triang.	§	0.26282
To s. of HZ 20 30 Perpend.	§	HOZ	9.54433
So s. of OP 71 33 Hypoth.	in Triang.	§	9.97708
To s. of PR 37 29 Perpend.	§	PRO	9.78423

Which 37 deg. 29 min. is the height of the Pole
above the Plain.

3. For the Distance of the Sub-stile from the Me-
ridian, RO.

Ast. of ZH 20° 30' Perpen.	in Triang.	§	0.42726
To s. of HO 26 34 Base	§	HZO	9.65054
So t. of PR 37 29 Perpen.	in Triang.	§	9.88472
To s. of RO 66 32 Base	§	PRO	9.96252

Which 66 deg. 32 min. is the Deflection, or Sub-
stile's Distance from the Meridian.

4. For the Plain's Difference of Longitude, OPR.

Ass. PO 71° 33'	9.97708
To Radius	10.
So s. RO 66° 32'	9.96252

To s. of OPR 75° 15' the Diff. of Long. 9.98544

The Plain's Difference of Longitude being 75 deg.
15 min. which is 5 Hours, there remains 0 deg.
15 min. wherefore the Sub-stile falls between the

fifth and sixth Hours from the Meridian; and under Sub-file in the second Column of the Table write 0 deg. 15 min. and subtract 0 deg. 15 min. from 15 deg. and there remains 14 deg. 45 min. which write above the Sub-file; then by a continual addition of 15 deg. you will have all the Angles at the Pole, as in the second Column of the Table. Then to find the true Hour-Distances, use the general Analogy.

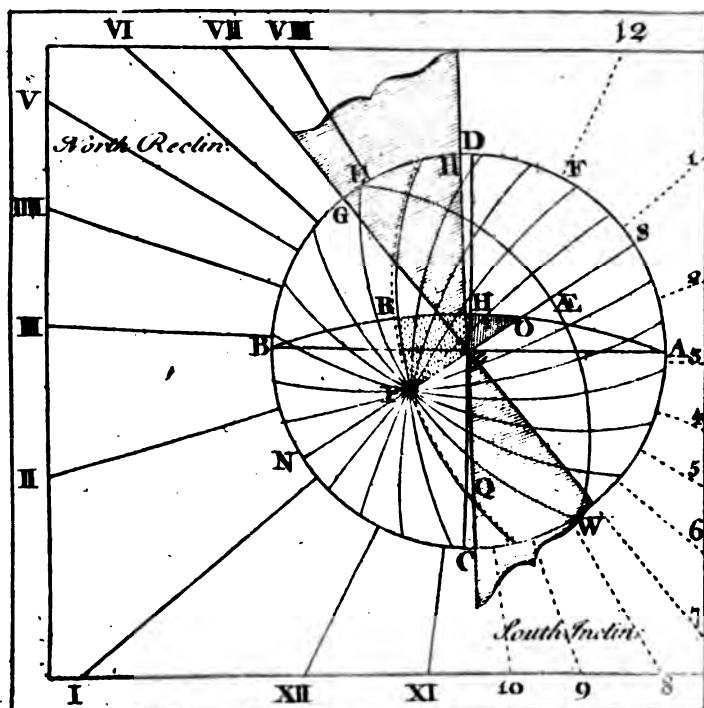
As Rad. to s. of $37^{\circ} 29'$ the Stile's height 9.78428

So is t. of $0^{\circ} 15'$ the Angle at the Pole 7.63982

To t. of the Hour-Distance $0^{\circ} 09'$ 7.42410

And so of all the rest.

Latitude		51° 32'	
Declination		55 00	
Reclination		20 30	
Dist. of the Mer. and Hor.		63 26	
Stile's height		37 29	
Deflection		66 32	
Diff. of Long.		75 15	
Hours from Noon.		Angles at the Pole.	
		Hour-Distances.	
1	11	89° 45'	89° 35'
2	10	74 45	65 54
3	9	59 45	46 13
4	8	44 45	31 06
5	7	29 45	19 11
6		14 45	09 06
The Sub-		file.	
7	5	00 15	00 09
8	4	15 15	09 25
9	3	30 15	19 32
10	2	45 15	31 33
11	1	60 15	46 48
12		75 15	66 36



Set 63 deg. 26 min. (the Distance of the Meridian and Horizon) from A to F, and set 66 deg. 32 min. (the Deflection) from F to G, and draw ZG for the Sub-style ; and set 37 deg. 29 min. (the Stile's height) from G to H, and draw ZH for the Stile. Set off the Hour-Distances from G the Sub-style, and lay a Ruler from Z to each of those Points, and draw the Hour-Lines.

The third Variety of North declining reclining Plains, where the Plain passeth between the Equinoctial and the Horizon.

Suppose a North Plain declines 43 deg. 15 min. Westward, and reclines 49 deg. 20 min. from the Zenith, in the Latitude 53 deg. 00 min.

First, Draw a Right-Line AB for the Horizontal Line, or Base of the declining reclining Plain, and cross it at Right-angles in Z with CD, for the Vertical Line of the Plain. Upon Z describe a Circle, and set the Declination from C to N, and draw NS, the Meridian of the Place; from Z set 38 deg. 28 min. to P, and 51 deg. 32 min. to Æ, and finish the Projection.

Secondly, Set the half Tangent of 49 deg. 20 min. the Reclination; from Z to F, and the half Tangent of its Complement from Z to Q, the Pole of the Plain; set the Secant Complement of Reclination from F upwards in the Line CD, which will give the Center of the Plain, which draw thro the Points AFB.

Thirdly, thro P the Pole of the World, and Q the Pole of the Plain, draw the Arch of a great Circle; then find the Requisites, *viz.*

1. The Distance of the Meridian from the Horizon, AO.
2. The height of the Pole or Stile above the Plain, PR.
3. The Deflection, or Sub-stile's Distance from the Meridian, OR.
4. The Plain's Difference of Longitude, OPR.

All which may be found from the Projection, or

2. *By Trigonometrical Calculation.*

All the Requisites will be found in the two spherical Triangles ZFO, and PRX.

1. For the Distance of the Meridian from the Horizon, AO.

In the Triangle ZFO is given ZF 49 deg. 20 min. the Reclination, and the Angle OZF the Declination; to find OF, thus.

As Rad. to s. of ZF 49° 20' the Reclinat.	9.879963
So t. of OZF the Declination 43° 15'	9.973454
To t. of OF 35° 31'	9.853417

Whose Complement 54 deg. 29 min. is the Distance of the Meridian from the Horizon.

2. For the height of the Pole, or Stile above the Plain, PR.

In the Triangle XPR there is not yet enough given; therefore, first find ZO in the Triangle OZF.

Ass. OZF 43° 15' the Declination	9.83581
To s. OF 35° 31'	9.76413
So Radius	10.
To s. ZO 57° 59'	9.92832

To this 57 deg. 59 min. add ZP 37 deg. 00 min. the Complement of the Latitude, the Sum is PO 94 deg. 59 min. whose Complement to 180 deg. is PX 85 deg. 01 min.

Then

Then in the two proportional Triangles OZF, and PXR.

As s. OZ	57° 59'	Hypoth.	in Triang.	{	0.07168
To s. ZF	49 20	Perpend.	§ OZF.	{	9.87996
So s. PX	85 01	Hypoth.	in Triang.	{	9.99836
To s. PR	63 02	Perpend.	§ PXR.	{	9.95000

So the height of the Pole above the Plain is 63 deg. 02 min.

3. For the Distance of the Sub-stile from the Meridian OR.

In the two Triangles OZF, and PXR, it will be,

As t. of ZF	49° 20', the Reclination	9.934056
To s. of FO	35 31	9.764131
So t. of PR	63 02, the Stile's height	10.293459

To s. of RX	78 48 the Sub-stile's Dist.	{	9.991646
from the N. Meridian		}	

Whose Complement to 180 deg. is OR, the Deflection sought.

4. For the Plain's Difference of Longitude, OPR.

As s. PR 63° 02', the Stile's height	9.95000
To t. RX 78° 48', the Sub-stile's Dist.	{ 10.70332
from the Meridian	
So Radius	10.

To t. of RPX	80° 00'	10.75332
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RPX 80 deg. 00 min. is the Angle counted from the North, therefore the Angle OPR is the Complement

plement to 88 deg. and is 100 deg. 00 min. the Plain's Difference of Longitude reckoned from the South.

The next thing to be done is to find the Hour-Distances; the Difference of Longitude 80 deg. 00 min. is 5 Hours and 5 deg. over, reckoned from the North Meridian; so the Sub-*stile* must stand between the Hours of 6 and 7 in the Evening, and the Angles at the Pole and Hour-Distances will be as in the Table below.

Latitude		53° 00'	
Declination		43 15	
Reclination		49 20	
Dist. Merid. and Hor.		54 29	
Stile's Height		63 02	
Deflection		78 48	
Diff. of Longitude		80 00	
Hours from Noon.		Angles at the Pole.	Hour-Distances.
12		80° 00'	78° 48'
11	1	65 00	62 23
10	2	50 00	46 43
9	3	35 00	31 58
8	4	20 00	17 59
7	5	05 00	4 28
The Sub- <i>stile</i> .			
6		10 00	8 56
5	7	25 00	22 34
4	8	40 00	36 48
3	9	55 00	51 51
2	10	70 00	67 47
1	11	85 00	84 24

Of Dial Plains, and how one Dial may be deduced from another; a North from a South, an East from a West; a North Decliner from a South Decliner; and all inclining Dials from Recliners opposite to them.

As there were six Varieties of direct reclining Plains, and as many Recliners which do also decline; so there are as many of both sorts which do incline, and incline and decline also. To give particular Examples of all those will be needless, if we consider, that when we have drawn a Dial for any Plain, we have also drawn one for the opposite Plain; if the Hour-Lines, Stile, and Sub-stile, be drawn thro' the Center. As for Example, having drawn a North declining reclining Dial, if we draw the Hour-Lines, Stile, and Sub-stile, thro' the Center, we have a South declining inclining Dial, whose Declination is the same but towards the contrary Coast; that is, if one decline from the North Westward, then the other shall decline from the South Eastward, and the Inclination of this be equal to the Reclination of that. But the contrary Pole will be elevated; so that if the Stile of the North Recliner point upwards to the North Pole, the opposite South Incliner will point downwards towards the South Pole. And if any declining Dial be drawn upon Paper, and prick'd thro', and the Hour-Lines, Stile, and Sub-stile drawn upon the back-side of the Paper, it will then be made for the same Declination; but for the contrary Coast; that is, if it declin'd from the South Westward, it will now decline from the South so much Eastward; and the same of declining reclining Dials: so that in making of one Dial, we have made four.

C H A P. IX.

Of Instrumental Dialling.

TH O this Chapter may seem foreign to my Design, yet forasmuch as instrumental Dialling is not very well explain'd in our *English* Authors, (as far as I have seen) especially the referring of reclining Plains that decline to a new Latitude, and new Declination; (Mr. *Leybourn* in his Folio of Dialling, in the 4th and 6th Chapters of the third Tractate, has given us Rules, but false printed, to find the new Latitude and new Declination of such Plains, thereby to find where they will be upright Plains; but mentions nothing of the Demonstration, so that he leaves us wholly in the dark for finding the Reason of those Rules, neither have I seen it in any other Author: I therefore perswade my self such a Demonstration will be acceptable to an *English* Reader, which is what induced me to write this Chapter.

There are several Instruments contriv'd by several Authors for this purpose; but the Instrument I shall make use of shall be the Scale of six Hours, and a Scale of Latitudes agreeable thereunto, such as are placed upon *Collins's* Quadrants.

The Line of Hours is no other but a double Tangent of 45 deg. that is, 45 deg. set both ways from the middle of the Line. The Line of Latitudes being made equal to Radius, or 60 deg. the whole Line of Hours will be 90 deg.

The Line of Latitudes is graduated thus.

As Radius to the Chord of 90 deg. so are the Tangents of the respective Degrees to the Tangents of other Arches.

Then the natural Sines of those Arches are the Numbers, which taken from a Diagonal Scale of equal Parts, shall graduate the Divisions of the Line of Latitudes.

Exam-

Example. Let us calculate for the Division of 10 Degrees.

To the Sine of 45 deg. 9.84948
Add the Log. of 2 0.30103

The Sum is the Chord of 90° 10.15051

As Rad. to the Chord of 90° 10.15051
So is the t. of 10 deg. 9.24632

To t. of 13° 55' 9.39689

The natural Sine against it is 2419.

By the same Method is the following Table calculated.

<i>A Table for dividing the Scale of Latitude.</i>							
1	247	173965	336764	498519	55	9496	
2	493	184176	346900	508600	66	9539	
3	739	194378	357036	518678	67	9578	
4	984	204577	367166	528753	68	9615	
5	1228	214772	377292	538825	69	9651	
6	1470	224961	387417	548895	70	9685	
7	1711	235146	397532	558962	72	9745	
8	1949	245328	407647	569026	74	9801	
9	2186	255500	417758	579088	75	9825	
10	2419	265678	427865	589147	76	9848	
11	2650	275846	437968	599203	78	9888	
12	2879	286010	448068	509258	80	9924	
13	3104	296169	458165	519311	85	9982	
14	3325	306325	468259	529360	90	10000	
15	3543	316475	478348	539408			
16	3758	326622	488436	549454			

S E C T. I.

How to draw Hour-Lines upon an Horizontal Plain.

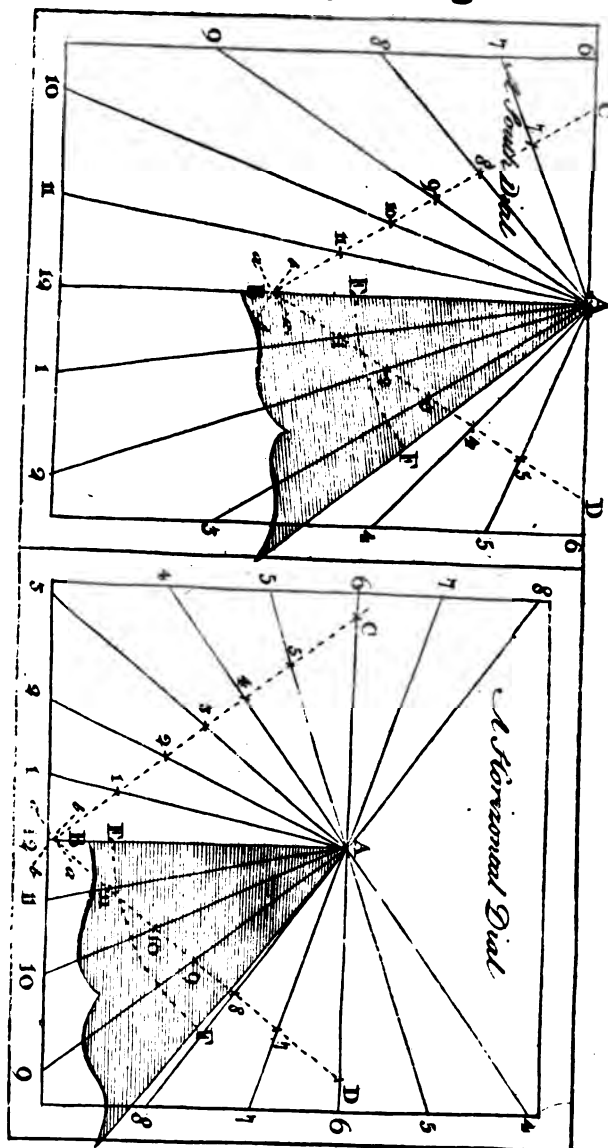
First, Draw the Line CD, for the Hour-Line of 6, then from your Scale of Latitudes take 51 deg. 32 min. (the Latitude of the Place) and set that Distance from the Center A both ways to C and D; then take in your Compasses the whole length of the Line of Hours, and set one foot in C, strike the Arch *aa*, then remove the Compass point to D, and with the other strike the Arch *bb*, and draw the Line AB for the Meridian, or Hour-Line of 12, and draw the Lines CB, and DB. Then from the Hour-Scale take the first Hour-Distance in your Compasses, and set one foot in B, and with the other make marks at 1 and 11, and the same will reach from D to 7, and from C to 5; then take two Hours in your Compasses, and set from B to 2 and 10; and from C to 4, and from D to 8: then take three Hours from your Hour-Scale, and set from B to 9, and 3: Lay a Ruler to the Center A, and to each of the Points in the Lines CB and DB, and draw the Hour-Lines.

Note. That the Hour-Lines 4 and 5, and 7 and 8, must be drawn thro the Center.

With 60 deg. of Chords describe the Arch EF, and upon that, from E to F, set the Stile's height 51 deg. 32 min. and draw the Stile AF.

The erect direct South Dial is drawn after the same manner, only you must set the Complement of Latitude from A to C and D; and the Stile's height must be equal to the Complement of the Latitude also.

By



By this one general way of working, you may make any direct North or South reclining, or inclining Dial; for if you find the height of the Stile by *Self*. 7 and 8. and take that from the Line of Latitudes, and set from the Center both ways to C and D, the rest of the Work will be the same. But if a South Plain reclines equal to the Complement of the Latitude, or if a North Plain reclines equal to the Latitude, then is the one an Equinoctial Plain, and the other a Polar, and must be drawn by the Direction given in the afore said Sections.

S E C T. II.

How to make an upright South or North Dial, declining East or West.

We will take the same Example as in *Self*. 4. of the last *Chap.* of a South Plain declining Westward 30 deg. in the Latitude 51 deg. 32 min.

First, You must find the Requisites, as is taught in the 4th *Section* of the last *Chapter*; but more readily by Scales for that purpose, thus. Take the Declination with your Compasses from the Line of Chords, and apply that extent, first to the Line of the Stile's height, and you will find it to fall upon 32 deg. 36 min. and so much is the Stile's height: and apply the same extent to the Line of the Distance of the Sub-stile from the Meridian, and you will find that to fall upon 21 deg. 40 min. Again, apply the same extent to the Line of the Plain's Difference of Longitude, and you will find it to fall upon 36 deg. 25 min.

Having

Having found the Requisites; the Difference of Longitude being reduced into Time, makes 2 h. 25 min. $\frac{2}{3}$, which is $34\frac{1}{3}$ min. short of 3 hours: which shews that the Sub-stile falls between the hours of 2 and 3.

Being thus far prepared, draw an Horizontal Line CD, and from the Center A draw AB at Right-angles, for the Meridian or Hour-Line of 12. Then with 60 deg. of Chords, upon A describe the Arch EG, and upon it set 21 deg. 40 min. the Distance of the Sub-stile from the Meridian, from E to F, always to the side that is contrary to the Plain's Declination; that is, if the Plain declines Westward (as in this Example) set off the Sub-stile on the East-side of the Meridian, and if the Declination be Eastward, set it upon the West-side: then from F set the Stile's height 32 deg. 36 min. to G, and draw AF the Sub-stile, and AG the Stile.

Then thro the Center A, draw the Line HI, at Right-angles to the Sub-stile; and take 32 deg. 36 min. the Stile's height, out of the Line of Latitudes, and set it from A both ways to H and I; then take in your Compasses the whole Line of Hours, and set one foot in H, with the other strike the Arch *cc*, and remove it to I, and strike the Arch *dd*; and draw the Lines HB, and IB: then from the Hour-Scale take $25\frac{2}{3}$ min. and set from B to 2, (upon the Line HB) again, take 1 hour $25\frac{2}{3}$ min. and set from B to 1; and take 2 hours $25\frac{2}{3}$ min. and set from B to 12, and so set 3 h. $25\frac{2}{3}$ min., 4 h. $25\frac{2}{3}$ min., 5 h. $25\frac{2}{3}$ min., and 6 h. $25\frac{2}{3}$ min., from B to 11, 10, 9, and 8; then take $34\frac{1}{3}$ min. from the Scale of Hours, and set on the other side of the Sub-stile from B to 3, and take 1 h. $34\frac{1}{3}$ min., and set from B to 4; and take 2 h. $34\frac{1}{3}$ min., 3 h. $34\frac{1}{3}$ min., and 4 h. $34\frac{1}{3}$ min., and set from B to 5, 6, and 7; and lay a Ruler to the Center A, and to each of those Points, and draw the Hour-Lines.

H h 3

Note,

S E C T. III.

How to draw Hour-Lines upon East and West-Recliners.

The best way is to reduce these Plains to a new Latitude, and new Declination, where they may be upright Plains.

The new Latitude of all reclining Plains, is an Arch of the Meridian of the Place contained between the Equinoctial and the reclining Plain. Now these East and West Recliners cut the Meridian just in the Horizon, so the Distance of the Horizon and Equinoctial (SÆ in the Figure of the East Recliner, *Sett.* 6. of the last *Chap.*) is the new Latitude, which is equal to the Complement of the old Latitude, or Latitude of the Place given.

The new Declination of all reclining Plains, is an Arch of a Circle contained between the Pole of the reclining Plain, and the Meridian of the Place, and cutteth the Meridian at Right-angles; which in the East Recliner, *Sett.* 6. is represented by ZQ, which is equal to the Complement of ZO the Reclination.

So if you make a declining Dial by the Directions of the last *Section*, for the new Latitude, and to the new Declination; such a Dial will fit the East or West reclining Plain, only you must always remember to place the Hour-Line of 12 parallel to the Horizon in all those Plains.

Example. Let it be required to draw Hour-Lines upon a West Plain reclining 35 deg. in the Latitude 51 deg. 32 min.

The new Latitude will be 38 deg. 28 min. and the new Declination will be 55 deg. for which make a declining Dial, as is directed in the last *Section*.

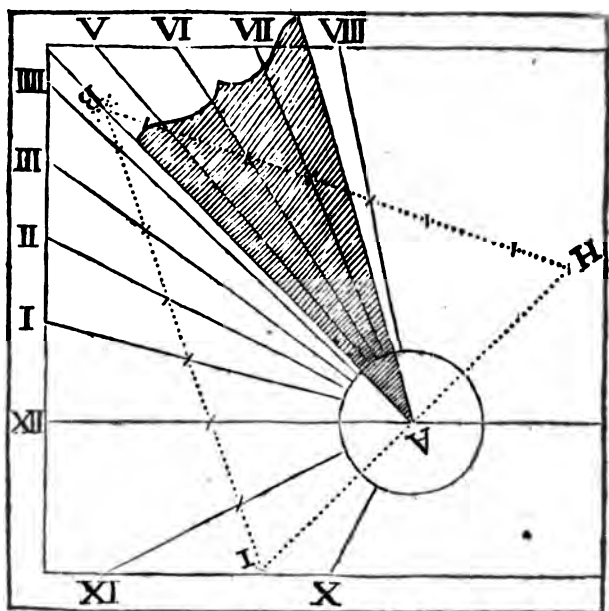
The height of the Stile is found to be $26^{\circ} 41'$

The Deflection

45 53

Difference of Longitude

66 29



S E C T. IV.

*How to draw Hour-Lines upon declining reclining
Plains; and first of South Recliners,*

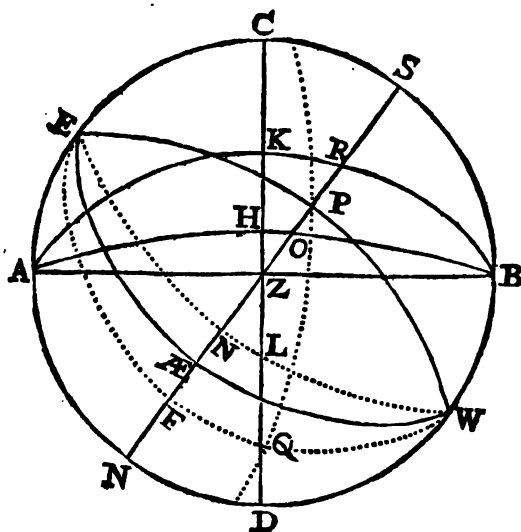
The best way is to refer these Plains to a new Latitude, and new Declination, where they may be upright Plains,

Sap:

Suppose a Plain in the Latitude 51 deg. 32 min. should decline from the South East-ward 35 deg. and recline from the Zenith 18 deg. 30 min. It is required to find in what Latitude it shall be an upright Plain, and what Declination it shall have in that new Latitude.

As Rad. to cs. of Declinat. BS 55° 00'	9.91336
So ct. of Reclination SBO 71° 30'	10.47548
To t. of OS 67° 47'	10.38884

Because OS 67 deg. 47 min. is greater than the Latitude, you must subtract PS 51 deg. 32 min. the Latitude from it, and there will remain OP 16 deg. 15 min. whose Complement OE 73 deg. 45 min. is the new Latitude required: for the new Latitude (as was shewed in the last *Section*) is an Arch of the Meridian, contained between the reclining Plain AHB, and the Equinoctial, and such is OE.



To find the new Declination.

In the Triangle QZF, there are given, ZQ 71 deg. 30 min. the Complement of Reclination, and the Angle QZF 35 deg. the Declination, to find QF the new Latitude.

As Rad. to s. ZQ 71° 30'	9.97696
So s. QZF 35° 00'	9.75859
To s. QF 32° 57'	9.73555

The new Declination is an Arch of a Circle comprehended between the Pole of the reclining Plain, and the Meridian; therefore QF 32 deg. 57 min. is the new Latitude required.

Again, Suppose a South Plain declines 35 deg. Eastward, and reclines from the Zenith 53 deg. 30 min. It is required to find the new Latitude, and new Declination.

First, To find the new Latitude.

In the Triangle RSB, there are given SB 55 deg. 00 min. the Complement of Declination, and the Angle RBS 36 deg. 30 min. to find RS.

As Rad. to s. of BS 55° 00', the Complement of Declination	} 9.91336
So t. of RBS 36° 30', the Complement of Reclination	
	} 9.86921
To t. of RS 31° 13'	9.78257

Because 31 deg. 13 min. is less than the Latitude SP 51 deg. 32 min. subtract it from the Latitude, and there remains 20 deg. 19 min. RP, whose Complement 69 deg. 41 min. is the Distance of R (the Intersection of the Plain and Meridian) from the

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Northern Intersection of the Equinoctial with the Meridian, which is the new Latitude required.

To find the new Declination.

In the Triangle LZN, there are given LZ 36 deg. 30 min. the Complement of Reclination, and the Angle LZN 35 deg. 00 min. the Declination; to find LN.

As Rad. to s. of ZL 36° 30', the Com-	}	9.77439
plement of Reclination		
So s. of LZN 35° 00', the Declination	}	9.75859
To s. of LN 19° 57', the new Declina-	}	9.53298
tion required		

Of North Recliners declining.

1. If a North Plain decline Westward 55 deg. and recline from the Zenith 20 deg. 30 min. it is required to find in what Latitude it will be an upright Plain, and what Declination it shall have in that new Latitude.

In the Triangle AOS, there are given AS the Complement of Declination, and the Angle SAO the Complement of Reclination; to find SO, thus.

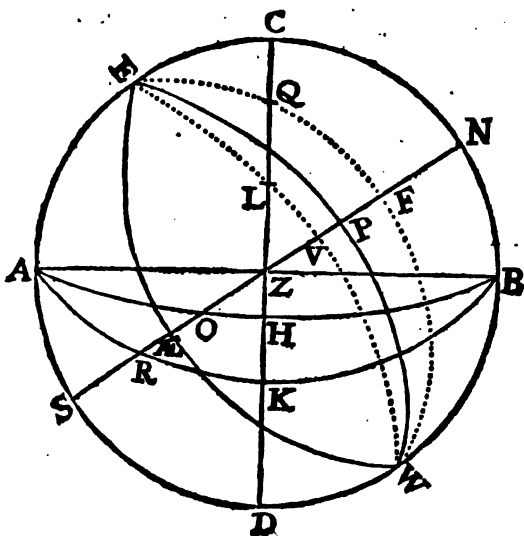
As Rad. to s. of AS 35° 00', the Com-	}	9.75859
plement of Declination		
So t. of SAO 69° 30', the Complement	}	10.42726
of Reclination		
To t. of SO 56° 54'		10.18585

From

From which subtract SE 38 deg. 28 min. the Complement of the Latitude, and there remains EO 18 deg. 26 min. the new Latitude required.

To find the new Declination.

In the Triangle ZQF , are given ZQ the Complement of Reclination, and the Angle QZF the old Declination, to find QF the new Declination.



As Radius

To s. ZQ $69^{\circ} 30'$

So s. QZF $55^{\circ} 00'$

10.00000

9.97159

9.91336

To s. QF $50^{\circ} 06'$

9.88495

Which 50 deg. 06 min. is the new Declination required.

2. If a Plain declines from the North Westward 55 deg. and reclines from the Zenith 50 deg. it is required to find the new Latitude, and new Declination.

To find the new Latitude.

In the Triangle ARS, there are given AS, the Complement of Declination, and the Angle SAR, the Complement of Reclination; to find the Side SR.

As Rad. to s. of AS $35^{\circ}00'$, the Com-	}	9.75859
plement of Declination		
So t. of SAR $40^{\circ}00'$, the Complement	}	9.92381
of Reclination		

To t. of SR $25^{\circ}42'$	9.68240
-----------------------------	---------

Subtract SR 25 deg. 42 min. from SÆ 38 deg. 28 min. the Complement of the Latitude, and there will remain RÆ 12 deg. 46 min. the new Latitude required.

To find the new Declination.

In the Triangle LVZ, there are given ZL 40 deg. 00 min. the Complement of Reclination, and the Angle LZV 55 deg. 00 min. the Declination; to find LV, the new Declination.

As Rad. to s. ZL $40^{\circ}00'$, the Comple-	}	9.80807
ment of Reclination		
So s. of LZV $55^{\circ}00'$, the old Declinat.	}	9.91336

To s. of LV $31^{\circ}47'$, the new Declinat.	9.72143
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Thus have I shewed you the Rules to find the new Latitude, and new Declination, and demonstrated them by Examples in all the Cases that can happen; only when the Reclination and Declination is such that the Plain passes thro the Pole, and so has 90 deg.

new Latitude; or when it so reclines and declines from the North, as to fall in the Intersection of the Meridian and Equinoctial, and then the new Latitude is 00 deg.

I shall now proceed to shew how to find the Requisites, and to draw the Dial.

1. For the Distance of the Meridian from the Horizon.

As Rad. to s. of ZK $50^{\circ} 00'$, the Reclination	9.88425
So t. of RZK $55^{\circ} 00'$, the old Declinat.	10.15477
To t. of RK $47^{\circ} 34'$	10.03902

Whose Complement 42 deg. 26 min. Is the Angle the Meridian makes with the Horizon.

The other Requisites are found by the Proportions in Sect. the 4th, working with the new Latitude and new Declination.

2. For the height of the Pole or Stile above the Plain.

As Rad. to cs. of the new Lat. $12^{\circ} 46'$	9.98913
So cs. of new Declination $31^{\circ} 47'$	9.92944
To s. of the Stile's height $56^{\circ} 00'$	9.91857

3. For the Distance of the Sub-stile from the Meridian.

As Rad. to s. of new Declination $31^{\circ} 47'$	9.72557
So ct. of the new Latitude $12^{\circ} 46'$	10.64477
To t. of the Deflection $66^{\circ} 44'$	10.36634

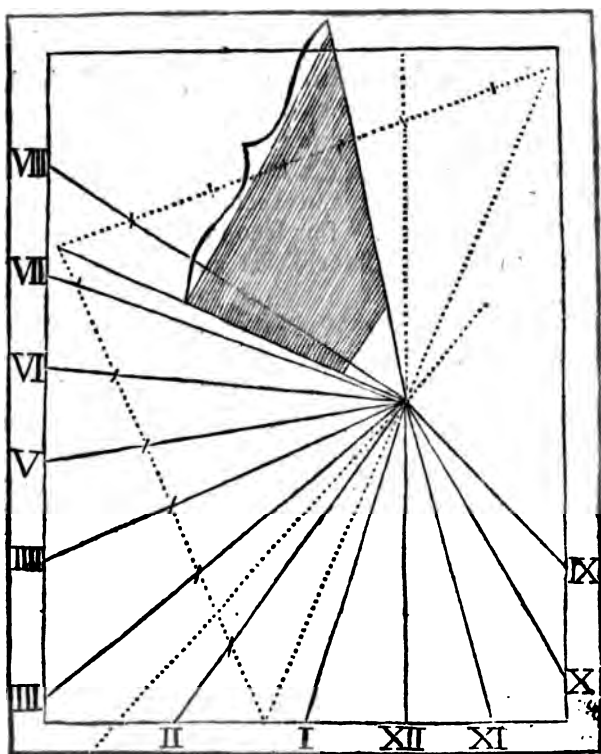
4. For

4. For the Plain's Difference of Longitude.

As cs. of the new Latitude $12^{\circ} 46'$	9.98913
To Radius	10.
So s. of the Deflection $66^{\circ} 44'$	9.96316

To s. of the Plain's Diff: of Long. $70^{\circ} 23'$ 9.97403

Which makes 4 hours 41 min. $\frac{1}{2}$.



F I N I S.

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